# A counterexample to L'Hopital's rule

EB

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The following is a counterexample to the naive rule that whenever the limit of $f'/g'$ exists, so does the limit of $f/g$

$$f(x) = x + \cos(x)\sin(x), \qquad g(x) = \exp(\sin(x))f(x).$$

This is from K. Stromberg, *Introduction to Classical Real Analysis*, page 188. The proof of L'Hopital's rule uses the hypothesis that $g'(x) \neq 0$ for $x \neq a$ although it allows $\lim_{x\to a} g'(x) = 0$. In this example, $f'(x)$ and $g'(x)$ both vanish at the zeros of $\cos x$. The ratio $f'/g'$ extends to a smooth function, but is strictly speaking undefined at these zeros.