Math 541

Professor Robbin

September 6, 2000

Lecture 2 9:55 MWF, 113 Van Vleck

My Office and Phone 313 Van Vleck, 263-4698

e-mail `robbin@math.wisc.edu`

Home Page `http://www.math.wisc.edu/~robbin`

My Office Hours 3:30 Monday, 9:00 Wednesday, 12:00 Friday.

Text I. N. Herstein: *Abstract Algebra* Third edition.

1 Important Dates

Labor Day:	Monday	Sep 4	
Evening Exam:	5:30-7:00 PM	Thursday	Oct 26
Last Day to Drop:	Friday	Nov 3	
Thanksgiving Recess:	Thursday-Sunday	Nov 23-26	
Last Class Day:	Friday	Dec 15	
Final Exam:	12:25 PM	Thursday	Dec 21

2 Policy

There will be homework due (almost) every Monday. At least one problem will be graded and returned with an answer written by me on Wednesday. There will be a quiz on the corresponding material on Friday. Homework will count for 20% of your grade, quizzes for 25%, the evening exam for 25%, and the final exam for 30%.

The quality of your writing will constitute an important component of your grade. You should never assume that your reader knows anything about a problem beyond what appears on the work you hand in. In particular, when you write homework you should not assume that the reader has access to the book – if you write only a problem number the reader will not know what the problem asks. The sample worked problem in the handout *Logic, Proofs, and Sets* illustrates the style that I want.

3 Sample Evening Exam

1. Prove that the composition of onto functions is onto.

2. Prove that the composition of one-to-one functions is one-to-one.

3. Suppose that the set S has at least 3 elements. Prove that there permutations $f, g \in A(S)$ with $f \circ g \neq g \circ f$.

4. Let a, b, and d be positive integers with $d|a$ and $d|b$. Assume that $d = \alpha a + \beta b$ for some $\alpha, \beta \in \mathbb{Z}$. Show that $d = (a, b)$.

5. Let a, b, and m be positive integers with $a|m$ and $b|m$. Assume that $(a, b) = 1$. Show that $(ab)|m$.

6. Prove that there are infinitely many primes.

7. Show that the set $\mathbb{R} \setminus \{-1\}$ of all real numbers except -1 is an Abelian group under the operation $a * b = a + b + ab$.

8. Let $G = \{1, a, b, c\}$ be a group of order 4 with identity element 1. The two possible multiplication tables for G are partially filled in below. In each case explain how the ab and ba entries are obtained and then fill in the rest of the table in your exam booklet.

$*$	1	a	b	c
1	1	a	b	c
a	a	1	?	
b	b	?	1	
c	c			1

$*$	1	a	b	c
1	1	a	b	c
a	a	b	?	
b	b	?		
c	c			

4 Sample Final Exam

1. Let $\phi : S \to T$. Define "ϕ is one-to-one" and "ϕ is onto".

2. What is $A(S)$ What are its basic properties?

3. State the axioms for an equivalence relation.

4. Define "equivalence class". How do they partition a set?

5. State the Unique Factorization Theorem for the integers $\mathbb{Z}$.

6. In $\mathbb{Z}$ define "congruence mod n". What are its properties?

7. State the axioms for a (multiplicative) group.

8. Define "subgroup H of G". State an easy test for a subgroup.

9. Where do the cosets of H come from?

10. State Lagrange's Theorem.

11. Define "group homomorphism" and "kernel".

12. Define "normal subgroup". What are the elements of G/N?

13. State Cauchy's Theorem.

14. In S_n describe the cycle $\alpha = (a_1\ a_2\ \cdots\ a_r)$ as a mapping.

15. What is the order of the cycle α? Is it even or odd?

16. Define "even permutation", "odd permutation", and "A_n".

17. State the axioms for a ring R.

18. Define "integral domain" and "field".

19. Define "ideal" and "congruence mod I".

20. Define "ring homomorphism" and "kernel".

21. State the First Homomorphism Theorem for rings.

22. Suppose R is a commutative ring. When is R/I a field?

23. Define "degree" in $F[x]$. What are its basic properties?

24. Define "principal ideal domain R".

25. Give two examples of a PID.

26. In R define "greatest common divisor of a and b".

27. State the result on the existence and uniqueness of the g.c.d.

28. Define "irreducible polynomial".

29. State Eisenstein's criteria for irreducibility.

30. Describe the elements in the fraction field of an integral domain.

31. In the symmetric group S_9 let

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 6 & 5 & 7 & 3 & 1 & 9 & 4 & 8 \end{pmatrix}, \qquad \beta = (1\ 5)(4\ 7).$$

Write α and $\alpha\beta$ as products of cycles and find the order of each.

32. In $\mathbb{Z}_5[x]$ find the g.c.d. of $f(x) = 3x^4 + x^3 + 3x^2 + 3x + 2$ and $g(x) = x^3 + 4x^2 + 2x + 1$.