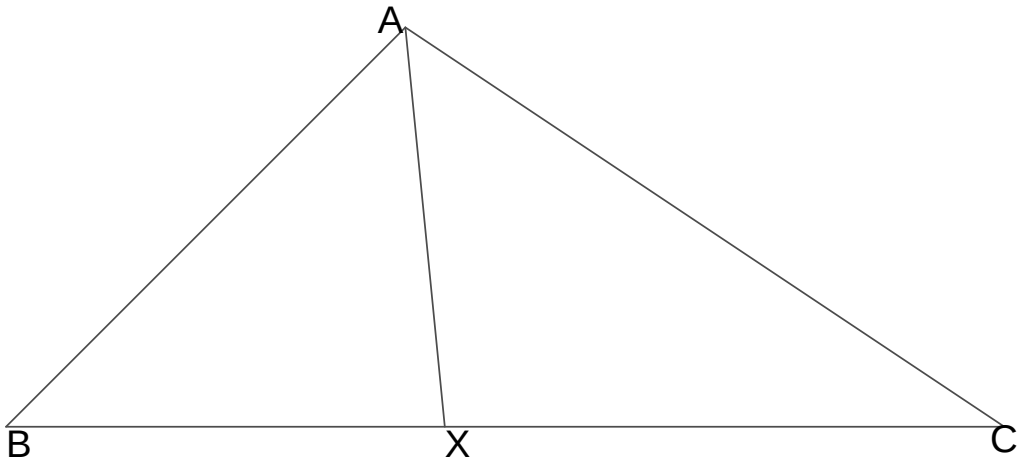
Every Triangle is Isosceles!? -II

In a triangle ABC, let X be the point at which the angle bisector of the angle at A meets the segment BC. By Theorem 1.12 in the text we have

$$\frac{XB}{AB} = \frac{XC}{AC}. \tag{1}$$



Now $\angle AXB = \angle ACX + \angle CAX = C + \frac{1}{2}A$ since the angles of a triangle sum to 180 degrees. By the law of sines (page 20 of the text) applied to triangle AXB we have

$$\frac{XB}{AB} = \frac{\sin BAX}{\sin AXB} = \frac{\sin \frac{1}{2}A}{\sin(C + \frac{1}{2}A)} \tag{2}$$

Similarly $\angle AXC = \angle ABX + \angle BAX = B + \frac{1}{2}A$ so

$$\frac{XC}{AC} = \frac{\sin \frac{1}{2}A}{\sin(B + \frac{1}{2}A)}. \tag{3}$$

From (1-3) we get $\sin(C + \frac{1}{2}A) + \sin(B + \frac{1}{2}A)$ so $C + \frac{1}{2}A = B + \frac{1}{2}A$ so $C = B$ so $AB = AC$ (by Exercise 1B.1) so ABC is isosceles.