Calculus 223 First Midterm Exam

Thursday February 14, 1996

I. (14%) (a) Find an equation for the tangent plane to the surface $z = -11 + x^2 + y^3$ at the point $(x, y, z) = (4, 0, 5)$.

(b) Find an equation for the tangent plane to the surface $x + 2y + 3z - \cos(xyz) = 18$ at the point $(x, y, z) = (4, 0, 5)$.

II. (10%) (a) Evaluate $\lim_{(x,y)\to(0,0)} \tan^{-1} \dfrac{1}{x^2 + y^2}$ or show that the limit does not exist.

(b) Evaluate $\lim_{(x,y)\to(0,0)} \tan^{-1} \dfrac{xy}{x^2 + y^2}$ or show that the limit does not exist.

III. (15%) (a) Find the derivative of $f(x, y, z) = x^2 + xy + xyz$ at the point $(1, 2, 3)$ in the direction

$$\mathbf{u} = \frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}.$$

(b) In what direction is this function increasing the fastest at the point $(1, 2, 3)$?

IV. (10%) Find the quadratic polynomial which best approximates the function $f(x, y) = x^3 + x^2y + y^3$ near the point $(x, y) = (1, 2)$.

V. (15%) Let $f(x, y) = 2x^2 + 2xy + y^2 - 8x - 6y$.

(a) What is the smallest value $f(x, y)$ can take?

(b) What is the largest value $f(x, y)$ can take?

VI. (15%) Minimize the function $f(x, y) = x^2 - 5xy + y^2$ on the square $-1 \le x \le 1$, $-1 \le y \le 1$.

VII. (15%) A is curve given parametrically by the equations

$$x = 4 + t, \qquad y = u(t), \qquad z = v(t).$$

The curve lies in both of the surfaces

$$z = -11 + x^2 + y^3, \qquad x + 2y + 3z - \cos(xyz) = 18,$$

of problem I and passes through the point $P_0 = (4, 0, 5)$ at $t = 0$. Find the velocity vector of this curve at $t = 0$.

VIII. (6%) (True or false: Circle your choice. One point for each correct answer, -2 points for each incorrect answer.)

Consider the function $f(x, y) = x^2 + 3xy + 2y^2$.

True False The level curves of f are ellipses.

True False The level curves of f are hyperbolas.

True False The function f takes only values which are greater than or equal to zero.

True False The function f takes all values.