

Math 234, Final Exam, May 15, 2003

I. (50 points.) (a) Find an equation for the tangent plane to the surface $x^2 - y^2 + z^2 = 4$ at the point $(2, -3, 3)$.

(b) Find an equation for the tangent plane to the surface $z = 1 + x^2 + y^3$ at the point $(x, y, z) = (2, 1, 6)$.

II. (50 points.) Find the flux of the field $\mathbf{F} = 2x\mathbf{i} - 3y\mathbf{j}$ outward across the ellipse with parametric equations

$$x = \cos t, \quad y = 4 \sin t, \quad 0 \leq t \leq 2\pi.$$

III. (50 points.) Consider the function $f(x, y) = (x - 3)^2 + (x - 3)(y - 2) + (y - 2)^2$ on the square $0 \leq x \leq 1, 0 \leq y \leq 1$.

(a) At what point in the square is the function smallest?

(b) At what point in the boundary of the square is the function smallest?

IV. (50 points.) Find the area of the surface $y^2 + z^2 = 2x$ cut off by the plane $x = 1$.

V. (50 points.) (a) Find the outward unit normal vector \mathbf{n} to the ellipse C with equation $9x^2 + y^2 = 1$.

(b) Find the line integral

$$\oint_C \mathbf{F} \cdot \mathbf{n} \, ds$$

(taken with the counter clockwise orientation) where the vector field \mathbf{F} is given by

$$\mathbf{F} = \mathbf{i} + 2xy\mathbf{j}.$$

(As usual, the ds in the integral indicates arclength.)

VI. (50 points.) A force is given by $\mathbf{F} = (x^2 - y)\mathbf{i} + (y^2 - z)\mathbf{j} + (z^2 - x)\mathbf{k}$.

(a) Find the work done by the force as the particle moves from $(0, 0, 0)$ to $(1, 1, 1)$ along a straight line.

(b) Find the work done by the force as the particle moves from $(0, 0, 0)$ to $(1, 1, 1)$ along the curve

$$x = t, \quad y = t^2, \quad z = t^3, \quad 0 \leq t \leq 1.$$

(c) Is \mathbf{F} a gradient? (Give a reason.)

VII. (50 points.) Let C be the triangle with vertices $(1, 2)$, $(3, 2)$, $(2, 5)$ and let the vector fields \mathbf{F} and \mathbf{G} be defined by

$$\mathbf{F} = x \mathbf{i} - y \mathbf{j}, \quad \mathbf{G} = y \mathbf{i} - x \mathbf{j}.$$

Evaluate the line integrals $\oint_C \mathbf{F} \cdot \mathbf{T} ds$ and $\oint_C \mathbf{G} \cdot \mathbf{T} ds$. Both integrals are to be traversed in the counterclockwise direction and, as usual, \mathbf{T} denotes the unit tangent vector to the boundary and ds denotes the arclength element.

VIII. (50 points.) Explain Stokes' Theorem. Your explanation should include a precise statement, including the definitions of the notations used in the formula. Your grade will be based in part on the clarity of your writing.

----- Thu May 15 15:02:43 2003

There are 192 scores

grade	range	count	percent
A	360...400	17	8.9%
AB	320...359	26	13.5%
B	280...319	38	19.8%
BC	240...279	32	16.7%
C	200...239	29	15.1%
D	160...199	23	12.0%
F	0...159	27	14.1%

Mean score = 253.9. Mean grade = 2.26.

--