Exercise A

Complete the following tables using the functions

$$f(x) = x^3, \qquad f'(x) = 3x^2.$$

Hint: $2.001 = 2 + 10^{-3}$, $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. Use the first table to fill in the second. (The purpose is to accustom the student to the various forms the difference quotient takes.)

x_0	x	$f(x_0)$	$f(x)$	$x - x_0$	$f(x) - f(x_0)$	$\frac{f(x) - f(x_0)}{x - x_0}$	$f'(x_0)$
2	2.2	8	10.648	0.2	2.648	13.24	12
2	2.1	8	9.261	0.1	1.261	12.61	12
2	2.01	8	8.1206001	0.01	0.1206001	12.06001	12
2	2.001	8	8.012006001	0.001	0.012006001	12.006001	12
2	2.0001	8	8.001200060001	0.0001	0.001200060001	12.00060001	12

x	h	$f(x)$	$f(x+h)$	$f(x+h) - f(x)$	$\frac{f(x+h) - f(x)}{h}$	$f'(x)$
2	0.2	8	10.648	2.648	13.24	12
2	0.1	8	9.261	1.261	12.61	12
2	0.01	8	8.1206001	0.1206001	12.06001	12
2	0.001	8	8.012006001	0.012006001	12.006001	12
2	0.0001	8	8.001200060001	0.001200060001	12.00060001	12

Exercise B

Complete the following table using the functions

$$f(x) = \sin x, \qquad f'(x) = \cos x.$$

Use a calculator. Be sure to set the calculator to accept angles in radians, *not* in degrees.

x	h	$f(x)$	$f(x+h)$	$\dfrac{f(x+h)-f(x)}{h}$	$f'(x)$
$\pi/3$	0.2	.8660254040	.9480972191	.410359076	0.5
$\pi/3$	0.1	.8660254040	.9116155922	.455901882	0.5
$\pi/3$	0.01	.8660254040	.8709820194	.49566154	0.5
$\pi/3$	0.001	.8660254040	.8665249706	.4995666	0.5
$\pi/3$	0.0001	.8660254040	.8660753994	.499954	0.5

Exericise C

In the following let

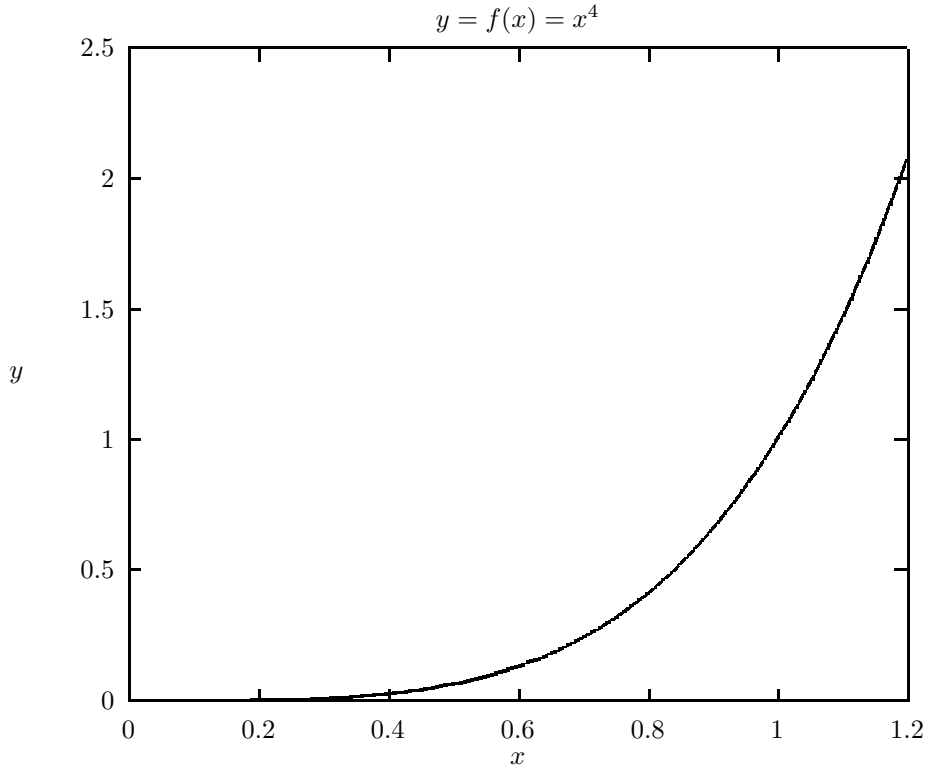
$$f(x) = x^4, \qquad f'(x) = 4x^3,$$

$$\begin{array}{lllll} x_1 = 1.2, & x_2 = 1.1, & x_3 = 1.05, & x_4 = 1.01, & x_0 = 1; \\ y_1 = f(x_1), & y_2 = f(x_2), & y_3 = f(x_3), & y_4 = f(x_4), & y_0 = f(x_0). \end{array}$$

Consider the five points

$$P_1(x_1, y_1), \quad P_2(x_2, y_2), \quad P_3(x_3, y_3), \quad P_4(x_4, y_4), \quad P_0(x_0, y_0).$$

Note that the points P_1, P_2, P_3, P_4 are getting closer to P_0. Plot the points on the graph.



$P_0 = (1,\ 1);$ $P_1 = (1.2,\ 2.0736);$ $P_2 = (1.1,\ 1.4641);$

$P_3 = (1.05,\ 1.220625);$ $P_4 = (1.01,\ 1.04060401).$

Exericise D

In the following continue the notation of Exercise C on the proceeding page. Consider the four lines

$$P_0P_1, \qquad P_0P_2, \qquad P_0P_3, \qquad P_0P_4.$$

For $i = 1, 2, 3, 4$ let m_i be the slope of the line P_0P_i, in other words,

$$m_1 = \frac{y_1 - y_0}{x_1 - x_0}, \qquad m_2 = \frac{y_2 - y_0}{x_2 - x_0}, \qquad m_3 = \frac{y_3 - y_0}{x_3 - x_0}, \qquad m_4 = \frac{y_4 - y_0}{x_4 - x_0}.$$

Calculate these slopes and draw the lines on the graph. What do you notice about the numbers m_1, m_2, m_3, m_4 and the quantity $f'(x_0)$?

$$\begin{aligned} m_1 &= 5.368 \\ m_2 &= 4.641 \\ m_3 &= 4.4125 \\ m_4 &= 4.060401 \\ f'(x_0) &= 4 \end{aligned}$$

Hint: The binomial formula

$$(a + h)^4 = a^4 + 4a^3h + 6a^2h^2 + 4ah^3 + h^4$$

(which you should have learned in high school) with $a = 1$ and h taking the four values

$$h = 0.2, \quad 0.1, \quad 0.05, \quad 0.01,$$

will make the arithmetic easier.