Final Exam Information

Math 221

December 1999

Your final grade will be computed as follows: (1) We will add the curves on the exams. to produce an exam curve where 800 points is perfect. (2) We will calculate a preliminary grade for each student by adding the scores on the exams and using this curve. (3) Each TA will decide a quiz grade curve for his/her sections. It takes into account the preliminary grades for his/her students. (4) The TA adds the quiz grade curve to the exam curve to produce a final curve (where 1000 points is perfect) for his/her sections.

The final will be worth 300 points. It is guaranteed that a score of 240 is worth a B on the final. It is guaranteed that 240 points worth of questions on the final will be essentially chosen from the following:

(1) The homework assigned this semester. (I have reproduced the list of problems below in case you have lost your syllabus.)

(2) The three exams given this semester.

(3) The additional problems below.

The qualifier *essentially* is added because I may change the wording of a problem slightly to make it more appropriate for an exam. In problems where I have given a hint, I might not give the hint on the exam.

You will be given the formulas

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta,$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta,$$

if they are needed. You will not be given the formulas for the area of a circle (but see Problem 99 below), the circumference of a circle (but see Problem 46 below), the volume of a sphere (but see Problem 282 below), the area of a sphere (but see Problem 47 below), the area of a sector (but see Problem 98 below), or the volume of a cone (but see Problem 286 below).

For graphing problems you may be asked to determine (a) where $f(x)$ is defined, (b) where $f(x)$ is continuous, (c) where $f(x)$ is differentiable, (d) where $f(x)$ is increasing and where it is decreasing, (e) where $f(x)$ is concave up and where it is concave down, (f) what the critical points of $f(x)$ are, (g) where the points of inflection are, (h) what (if any) the horizontal asymptotes to $f(x)$ are, and (i) what (if any) the verical asymptotes to $f(x)$ are.

For proofs the question will be carefully worded to indicate what you may assume in your proof. (See Problem 10 for example.) In this document you may use without proof any previously asserted fact. For example, you may use the fact that $\sin'(\theta) = \cos(\theta)$ to prove that $\cos'(\theta) = -\sin(\theta)$ since the former question precedes the latter below. (See Problems 12 and 13.) You may always use high school algebra (like $\cos(\theta) = \sin(\pi/2 - \theta)$) in your proofs.

1 Homework assigned this semester

0.1: 1,3,4,,8,10,11,15,25,39,41,51-55,61,63,87,88,89,99. **1.2**: 1,3. **0.4**: 2,3. **1.1**: 2, 6, 8. **1.2**: 1, 9, 15, 16, 21-23. **1.3**: 1, 13, 15, 19, 27, 34, 35, 36, 39, 45=49,59, 60, 61, 65, 71. **1.5**: 1,3, 8, 19-21, 37, 39, 45, 59. **1.6**: 1-3, 8, 9, 11, 13, 15, 23, 31, 34, 44-46. **1.7**: 1, 3, 15, 17, 18. **1 Review**: 17, 18, 41.

2.1: 1, 3, 30, 32, 45, 51, 57, 59, 60, 61, 62, 64. **2.2**: 1, 3, 5, 7, 9, 15, 17, 31, 35, 37, 39, 41, 49, 51, 53, 60, 61, 75, 76. **2.3**: 1, 7, 9, 10, 13-16, 22. **2.4**: 1, 7, 9, 11, 20, 32, 35, 45, 57. **2.5**: 1, 3, 5, 17, 35, 53, 56, 57, 64, 66. **2.6**: 1, 5, 17, 21, 25, 27, 45. **2.7**: 1, 2, 5, 10, 15, 39, 41-43, 52, 53. **2.8**: 1-4, 7, 10, 11, 16,19, 21, 23, 26, 29, 30, 34. **2.9**: 1, 3, 7, 13, 39, 51, 54, 55. **2 Review**: 1-12, 13, 35, 45-47, 58, 86, 90-94. **2 Problems Plus**: 1, 2*, 3*, 11, 22*, 25*.

3.1: 1-7, 13, 14, 23, 25, 27, 29, 39, 43, 49, 52. **3.2**: 1-6, 7, 9, 1, 13, 15, 17, 19, 21, 23, 25, 33, 41. **3.3**: 1-21(odd), 22, 25, 35-41(odd), 47, 49, 63, 65, 71, 75, 81, 83. **3.4**: 1, 7, 9, 11, 13, 35, 37, 45, 47, 49.66, 67. **3.5**: 1, 5, 9, 11, 12, 13, 19, 21. **3.6**: 1, 3, 9, 10, 13, 15, 18, 21, 23, 31, 49, 50, 61, 65. **3.7**: 1, 2, 7, 8, 11, 12, 14, 23. **3.8**: 1, 3, 5, 7, 21, 22, 29, 51, 57, 65, 71, 74-78. **3 Review**: 1-16, 27, 29.61, 62, 63, 111. **3 Applications Plus**: 4.

4.1: 1-10, 17, 21, 27, 39, 41, 43, 67-70. **4.2**: 1, 5, 7, 11, 17-19. **4.3**: 1, 2, 3, 5, 7, 33, 35, 37, 41, 43, 45, 47. **4.4**: 1, 2, 3, 5, 15, 23, 25, 29, 33, 34. **4.5**: 1, 2, 5, 9, , 11, 12, 45, 60, 62. **4.6**: 1. **4.7**: 1, 5, 7, 10-11, 15, 20, 21, 23, 25, 26, 31, 33, 34, 37, 38, 43, 49. **4.9**: 1, 9, 11, 17, 23, 41, 42, 61-65. **4 Review**: 1-16, 54. **4 Problems Plus**: 4, 5.

5.1: 3, 4, 10, 11, 21-31(odd). **5.2**: 1-6, 11, 13. **5.3**: 1-8, 21-24, 27, 29, 31, 33, 34, 45, 47, 49, 51, 55, 57. **5.4**: 3, 4, 5, 9, 11, 17, 18, 19, 25, 35, 49, 50, 57, 59, 77, 79, 81, 83, 84, 95, 98, 100. **5.5**: 1, 2, 3, 7, 9, 27, 29, 41, 43. **5.6**: 1. **5 Review**: 1-20, 23, 39. **5 Applications Plus**: 1, 3, 6, 7.

6.1: 1, 5, 7, 9, 37, 39. **6.2**: 1, 2, 5, 9, 11, 13-16, 33, 35, 49-51. **6.3**: 1, 3, 9, 11, 21, 23, 42. **6.5**: 1, 3, 19. **8.4**: 1-5, 10, 21-23. **6 Review**: 1, 6, 28, 33. **6 Problems Plus**: 1, 3, 10, 13, 23.

2 Additional Problems

1. State and prove the Sum Rule for derivatives. You may use (without proof) the Limit Laws.

2. State and prove the Product Rule for derivatives. You may use (without proof) the Limit Laws.

3. State and prove the Quotient Rule for derivatives. You may use (without proof) the Limit Laws.

4. State and prove the Chain Rule for derivatives. You may use (without proof) the Limit Laws. You may assume (as the proof in the Stewart text does) that the inner function has a nonzero derivative.

5. State the Squeeze Theorem.

6. Prove that $\dfrac{dx^n}{dx} = nx^{n-1}$, for all positive integers n.

7. Prove that $\dfrac{dx^n}{dx} = nx^{n-1}$, for $n = 0$.

8. Prove that $\dfrac{dx^n}{dx} = nx^{n-1}$, for all negative integers n.

9. Prove that $\dfrac{de^x}{dx} = e^x$.

10. Prove that

$$\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1.$$

You may assume without proof the Squeeze Theorem, the Limit Laws, and that the sin and cos are continuous. Hint: See Problem 98.

11. Prove that

$$\lim_{\theta \to 0} \frac{1 - \cos(\theta)}{\theta} = 0.$$

12. Prove that $\dfrac{d \sin x}{dx} = \cos x$.

13. Prove that $\dfrac{d \cos x}{dx} = -\sin x$.

14. Prove that $\dfrac{d \tan x}{dx} = \sec^2 x$.

15. Prove that $\dfrac{d \cot x}{dx} = -\csc^2 x$.

16. Prove that $\dfrac{d \ln x}{dx} = \dfrac{1}{x}$.

17. Prove that $\dfrac{d \sin^{-1} x}{dx} = \dfrac{1}{\sqrt{1 - x^2}}$.

18. Prove that $\dfrac{d\cos^{-1}x}{dx} = -\dfrac{1}{\sqrt{1-x^2}}$.

19. Prove that $\dfrac{d\tan^{-1}x}{dx} = \dfrac{1}{1+x^2}$.

20. True or false? A differentiable function must be continuous. Explain.

21. True or false? A continuous function must be differentiable. Explain.

22. Explain why $\lim\limits_{x\to 0} 1/x$ does not exist.

23. Explain why $\lim\limits_{\theta\to\pi/2} \tan\theta$ does not exist.

24. Explain why $\lim\limits_{\theta\to\pi/2} \sec\theta$ does not exist.

25. Explain why $\lim\limits_{\theta\to 0} \csc\theta$ does not exist.

26. Explain why $\lim\limits_{x\to 0} \sin(1/x)$ does not exist.

27. Explain why $\lim\limits_{\theta\to\infty} \cos\theta$ does not exist.

28. Let $\operatorname{sgn}(x)$ be the sign function. This function is given by

$$\operatorname{sgn}(x) = \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{if } x = 0, \\ -1, & \text{if } x < 0. \end{cases}$$

Explain why $\lim\limits_{x\to 0} \operatorname{sgn}(x)$ does not exist.

29. Explain why $\lim\limits_{y\to 0} 2^{1/y}$ does not exist.

30. Explain why $\lim\limits_{x\to 1} 2^{1/(x-1)}$ does not exist.

31. Calculate $\lim\limits_{\Delta x\to 0} \dfrac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}$ when $f(x) = \sin 2x$.

32. Calculate $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{(x+h)-x}$ when $f(x) = \cos 2x$.

33. Calculate $\lim\limits_{x\to a} \dfrac{f(x)-f(a)}{x-a}$ when $f(x) = \sin(x^2)$.

34. Calculate $\lim\limits_{x\to x_0} \dfrac{f(x)-f(x_0)}{x-x_0}$ when $f(x) = \cos(x^2)$.

35. Calculate $\lim\limits_{\Delta x\to 0} \dfrac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}$ when $f(x) = \sqrt{\sin x}$.

36. Calculate $\lim\limits_{\Delta x\to 0} \dfrac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}$ when $f(x) = x\sin x$.

37. Calculate $\lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$ when $f(x) = e^{\sqrt{x}}$.

38. Calculate $\lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$ when $f(x) = e^{\sin x}$.

39. Calculate $\lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$ when $f(x) = \ln(ax+b)$.

40. Calculate $\lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$ when $f(x) = e^{\cos x}$.

41. Calculate $\lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$ when $f(x) = x^x$.

42. Calculate $\lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$ when $f(x) = \dfrac{\sin x}{x}$.

43. Calculate $\lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$ when $f(x) = \sqrt{ax+b}$.

44. Calculate $\lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$ when $f(x) = (mx+c)^n$.

45. Use differentiation to estimate the number $\dfrac{127^{4/3} - 125^{4/3}}{2}$ approximately without a calculator. Your answer should have the form p/q where p and q are integers. Hint: $5^3 = 125$.

46. What is the derivative of the area of a circle with respect to its radius?

47. What is the derivative of the volume of a sphere with respect to its radius?

48. Find the slope of the tangent to the curve $y = x^3 - x$ at $x = 2$.

49. Find the equations of the tangent and normal to the curve $y = x^3 - 2x + 7$ at the point $(1, 6)$.

50. Find the equation of the tangentline to the curve $3xy^2 - 2x^2y = 1$ at the point $(1, 1)$. Find d^2y/dx^2 at this point.

51. Find the equations of the tangent and normal to the curve $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ at the point $(a\cos\theta, b\sin\theta)$.

52. Find the equations of the tangent and normal to the curve $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ at the point $(a\sec\theta, b\tan\theta)$.

53. Find the equations of the tangent and normal to the curve $c^2(x^2 + y^2) = x^2y^2$ at the point $(c/\cos\theta, c/\sin\theta)$.

54. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

55. Show that the equation of the tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ at the point (p, q) is $\dfrac{xp}{a^2} - \dfrac{yq}{b^2} = 1$

56. Find the equations of the tangent and normal to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at the point where $x = 1$.

57. Find the linear and quadratic approximations to $f(x) = \dfrac{1}{\sqrt{4+x}}$ at $x = 0$.

58. Find the linear and quadratic approximations to $f(x) = \sqrt{1+x}$ at $x = 0$.

59. Find the linear and quadratic approximations to $f(x) = \dfrac{1}{(1+2x)^4}$ at $x = 0$.

60. Find the linear and quadratic approximations to $f(x) = (1+x)^3$ at $x = 0$.

61. Find the linear and quadratic approximations to $f(x) = \sec x$ at $x = 0$.

62. Find the linear and quadratic approximations to $f(x) = x \sin x$ at $x = 0$.

63. Find the linear and quadratic approximations to $f(x) = x^3$ at $x = 1$.

64. Find the linear and quadratic approximations to $f(x) = x^{1/3}$ at $x = -8$.

65. Find the linear and quadratic approximations to $f(\theta) = \sin\theta$ at $\theta = \pi/6$.

66. Find the linear and quadratic approximations to $f(x) = x^{-1}$ at $x = 4$.

67. Find the linear and quadratic approximations to $f(x) = x^3 - x$ at $x = 1$.

68. Find the linear and quadratic approximations to $f(x) = \sqrt{x}$ at $x = 4$.

69. Find the linear and quadratic approximations to $f(x) = \sqrt{x^2+9}$ at $x = -4$.

70. Use quadratic approximation to find the approximate value of $\sqrt{401}$ without a calculator. Hint: $\sqrt{400} = 20$.

71. Use quadratic approximation to find the approximate value of $(255)^{1/4}$ without a calculator. Hint: $256^{1/4} = 4$.

72. Use quadratic approximation to find the approximate value of $\dfrac{1}{(2.002)^2}$ without a calculator.

73. Approximate $(1.97)^6$ without a calculator without a calculator. (Leave arithmetic undone.)

74. Let f be a function such that $f(1) = 2$ and whose derivative is known to be $f'(x) = \sqrt{x^3+1}$. Use a linear approximation to estimate the value of $f(1.1)$. Use a quadratic approximation to estimate the value of $f(1.1)$.

75. Find the second derivative of x^7 with respect to x.

76. Find the second derivative of $\ln x$ with respect to x.

77. Find the second derivative of 5^x with respect to x.

78. Find the second derivative of $\tan\theta$ with respect to θ.

79. Find the second derivative of x^2e^{3x} with respect to x.

80. Find the second derivative of $\sin 3x \cos 5x$ with respect to x.

81. Find the third derivative of u^4 with respect to u.

82. Find the third derivative of $\ln x$ with respect to x.

83. Find the second derivative of $\tan x$ with respect to x.

84. If $\theta = \sin^{-1} y$ show that $\dfrac{d^2\theta}{dy^2} = \dfrac{y}{(1-y^2)^{3/2}}$.

85. If $y = e^{-t}\cos t$ show that $\dfrac{d^2y}{dt^2} = 2e^{-t}\sin t$.

86. If $u = t + \cot t$ show that $\sin^2 t \cdot \dfrac{d^2u}{dt^2} - 2u + 2t = 0$.

87. If $y = e^{\tan x}$ show that $\cos^2 x \cdot \dfrac{d^2y}{dx^2} - (1 + \sin 2x)\dfrac{dy}{dx} = 0$.

88. State L'Hôpital's rule and give an example which illustrates how it is used.

89. Explain why L'Hôpital's rule works. Hint: Expand the numerator and the denominator in terms of Δx.

90. Give three examples to illustrate that a limit problem that looks like it is coming out to $0/0$ could be really getting closer and closer to almost anything and must be looked at a different way.

91. Give three examples to illustrate that a limit problem that looks like it is coming out to 1^∞ could be really getting closer and closer to almost anything and must be looked at a different way.

92. Give three examples to illustrate that a limit problem that looks like it is coming out to 0^0 could be really getting closer and closer to almost anything and must be looked at a different way.

93. Give three examples to illustrate that a limit problem that looks like it is coming out to $\infty - \infty$ could be really getting closer and closer to almost anything and must be looked at a different way.

94. Explain how limit problems that come out to ∞/∞ can always be converted into limit problems that come out to $0/0$ and why doing such a conversion is useful.

95. Explain how limit problems that come out to $\infty - \infty$ can be converted into limit problems that come out to $0/0$ and why doing such a conversion is useful.

96. Explain how limit problems that come out to 0^0 can be converted into limit problems that come out to $0/0$ and why doing such a conversion is useful.

97. Explain how limit problems that come out to 1^∞ can be converted into limit problems that come out to $0/0$ and why doing such a conversion is useful.

98. Use calculus to show that the area A of a sector of a circle with central angle θ is $A = (\theta/2)R^2$ where R is the radius and θ is measured in radians. Hint: Divide the sector into n equal sectors of central angle $\Delta\theta = \theta/n$ and area ΔA. As in the proof (see Problem 10) that

$$\lim_{\Delta\theta \to 0} \frac{\sin(\Delta\theta)}{\Delta\theta} = 1,$$

the area ΔA lies between the areas of two right triangles whose areas can be expressed in terms of R and trig functions of $\Delta\theta$. Apply the Squeeze Theorem to $A = n\Delta A$ and use l'Hôpital's rule or Problem 10.

99. Use calculus to show that the area of a circle of radius R is πR^2. Hint: The area of a sector is a more general problem. (See problem 98.)

100. For which values of x is the function $f(x) = x^2 + 3x + 4$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

101. For which values of x is the function $f(x) = \begin{cases} \frac{x^2-x-6}{x-3}, & \text{if } x \neq 3, \\ 5, & \text{if } x = 3, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

102. For which values of x is the function $f(x) = \begin{cases} \frac{\sin 3x}{x}, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

103. Determine the value of k for which the function

$$f(x) = \begin{cases} \frac{\sin 2x}{5x}, & \text{if } x \neq 0, \\ k, & \text{if } x = 0, \end{cases}$$

is continuous at $x = 0$. Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

104. What does it mean for a function $f(x)$ to be continuous at $x = a$?

105. What does it mean for a function $f(x)$ to be differentiable at $x = a$?

106. What does $f'(a)$ indicate you about the graph of $y = f(x)$? Explain why this is true.

107. What does it mean for a function to be increasing? Explain how to use calculus to tell if a function is increasing. Explain why this works.

108. What does it mean for a function to be concave up? Explain how to use calculus to tell if a function is concave up. Explain why this works.

109. What is a horizontal asymptote of a function $f(x)$? Explain how to justify that a given line $y = b$ is a horizontal asymptote of $f(x)$.

110. What is a vertical asymptote of a function $f(x)$? Explain how to justify that a given line $x = a$ is an vertical asymptote of $f(x)$.

111. If $f(x) = |x|$, what is $f'(-2)$?

112. Find the values of a and b so that the function

$$f(x) = \begin{cases} x^2 + 3x + a, & \text{if } x \leq 1, \\ bx + 2, & \text{if } x > 1, \end{cases}$$

is differentiable for all values of x.

113. Graph $f(x) = \begin{cases} 2 - x, & \text{if } x \geq 1, \\ x, & \text{if } 0 \leq x \leq 1. \end{cases}$

114. Graph $f(x) = \begin{cases} 2 + x, & \text{if } x \geq 0, \\ 2 - x, & \text{if } x < 0. \end{cases}$

115. Graph $f(x) = \begin{cases} 1 - x, & \text{if } x < 1, \\ x^2 - 1, & \text{if } x \geq 1. \end{cases}$

116. Graph $f(x) = x + 1/x$.

117. Graph $f(x) = \dfrac{x^2 + 2x - 20}{x - 4}$ for $5 < x < 9$.

118. Graph $f(x) = \dfrac{1}{x^2 + 1}$.

119. Graph $f(x) = xe^x$.

120. State Rolle's theorem and draw a picture which illustrates the statement of the theorem.

121. State the Mean Value Theorem and draw a picture which illustrates the statement of the theorem.

122. Explain why Rolle's theorem is a *special case* of the Mean Value Theorem.

123. Let $f(x) = 1 - x^{2/3}$. Show that $f(-1) = f(1)$ but that there is no number c in the interval $(-1, 1)$ such that $f'(c) = 0$. Why does this not contradict Rolle's theorem?

124. Let $f(x) = (x - 1)^{-2}$. Show that $f(0) = f(2)$ but that there is no number c in the interval $(0, 2)$ such that $f'(c) = 0$. Why does this not contradict Rolle's theorem?

125. Show that the Mean Value Theorem is not applicable to the function $f(x) = |x|$ in the interval $[-1, 1]$.

126. Show that the Mean Value Theorem is not applicable to the function $f(x) = 1/x$ in the interval $[-1, 1]$.

127. Find a point on the curve $y = x^3$ where the tangent is parallel to the chord joining $(1, 1)$ and $(3, 27)$.

128. Show that the equation $x^5 + 10x + 3 = 0$ has exactly one real root.

129. Find the local maxima and minima of $f(x) = (5x-1)^2 + 4$ without using derivatives.

130. Find the local maxima and minima of $f(x) = -(x-3)^2 + 9$ without using derivatives.

131. Find the local maxima and minima of $f(x) = -|x+4| + 6$ without using derivatives.

132. Find the local maxima and minima of $f(x) = \sin 2x + 5$ without using derivatives.

133. Find the local maxima and minima of $f(x) = |\sin 4x + 3|$ without using derivatives.

134. Find the local maxima and minima of $f(x) = x^4 - 62x^2 + 120x + 9$.

135. Find the local maxima and minima of $f(x) = (x-1)(x+2)^2$.

136. Find the local maxima and minima of $f(x) = -(x-1)^3(x+1)^2$.

137. Find the local maxima and minima of $f(x) = x/2 + 2/x$ for $x > 0$.

138. Find the local maxima and minima of $f(x) = 2x^3 - 24x + 107$ in the interval $[1, 3]$.

139. Find the local maxima and minima of $f(x) = \sin x + (1/2)\cos x$ in $0 \leq x \leq \pi/2$.

140. Show that the maximum value of $\left(\dfrac{1}{x}\right)^x$ is $e^{1/e}$.

141. Show that $f(x) = x + 1/x$ has a local maximum and a local minimum, but the value at the local maximum is less than the value at the local minimum.

142. Find the maximum profit that a company can make if the profit function is given by $p(x) = 41 + 24x - 18x^2$.

143. A train is moving along the curve $y = x^2 + 2$. A girl is at the point $(3, 2)$. At what point will the train be at when the girl and the train are closest?

144. Find the local maxima and minima of $f(x) = -x + 2\sin x$ in $[0, 2\pi]$.

145. Divide 15 into two parts such that the square of one times the cube of the other is maximum.

146. Suppose the sum of two numbers is fixed. Show that their product is maximum exactly when each one of them is half of the total sum.

147. Divide a into two parts such that the pth power of one times the qth power of the other is maximum.

148. Which fraction exceeds its pth power by the maximum amount?

149. Find the dimensions of the rectangle of area 96 cm^2 which has minimum perimeter. What is this minimum perimeter?

150. Show that the right circular cone with a given volume and minimum surface area has altitude equal to $\sqrt{2}$ times the radius of the base.

151. Show that the altitude of the right circular cone with maximum volume that can be inscribed in a sphere of radius R is $4R/3$.

152. Show that the height of a right circular cylinder with maximum volume that can be inscribed in a given right circular cone of height h is $h/3$.

153. A cylindrical can is to be made to hold 1 liter of oil. Find the dimensions of the can which will minimize the cost of the metal to make the can.

154. An open box is to be made out of a given quantity of cardboard of area p^2. Find the maximum volume of the box if its base is square.

155. Find the dimensions of the maximum rectangular area that can be fenced with a fence 300 yards long.

156. Show that the triangle of the greatest area with given base and vertical angle is isosceles.

157. Show that a right triangle with a given perimeter has greatest area when it is isosceles.

158. What do distance, speed and acceleration have to do with calculus? Explain thoroughly.

159. A particle, starting from a fixed point P, moves in a straight line. Its distance from P after t seconds is $s = 11 + 5t + t^3$ meters. Find the distance, velocity and acceleration of the particle after 4 seconds, and find the distance it travels during the 4th second.

160. The displacement of a particle at time t is given by $x = 2t^3 - 5t^2 + 4t + 3$. Find (i) the time when the acceleration is 8cm/s^2, and (ii) the velocity and displacement at that instant.

161. A particle moves along a straight line according to the law $s = t^3 - 6t^2 + 19t - 4$. Find (i) its displacement and acceleration when its velocity is 7m/s, and (ii) its displacement and velocity when its acceleration is 6m/s^2.

162. A particle moves along a straight line so that after t seconds its distance from a fixed point P on the line is s meters, where $s = t^3 - 4t^2 + 3t$. Find

(i) when the particle is at P, and (ii) its velocity and acceleration at these times t.

163. A particle moves along a straight line according to the law $s = at^2 - 2bt + c$, where a, b, c are constants. Prove that the acceleration of the particle is constant.

164. The displacement of a particle moving in a straight line is $x = 2t^3 - 9t^2 + 12t + 1$ meters at time t. Find (i) the velocity and acceleration at $t = 1$ second, (ii) the time when the particle stops momentarily, and (iii) the distance between two stops.

165. The distance s in meters traveled by a particle in t seconds is given by $s = ae^t + be^{-t}$. Show that the acceleration of the particle at time t is equal to the distance the particle travels in t seconds.

166. A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2 ft/s, how fast is the angle between the top of the ladder and the wall changing when the angle is $\pi/4$ radians?

167. A ladder 13 meters long is leaning against a wall. The bottom of the ladder is pulled along the ground away from from the wall at the rate of 2 m/s. How fast is its height on the wall decreasing when the foot of the ladder is 5 m away from the wall?

168. A television camera is positioned 4000 ft from the base of a rocket launching pad. A rocket rises vertically and its speed is 600 ft/s when it has risen 3000 feet. (a) How fast is the distance from the television camera to the rocket changing at that moment? (b) How fast is the camera's angle of elevation changing at that same moment?

169. Explain why exponential functions arise in computing radioactive decay.

170. Explain why exponential functions are used as models for population growth.

171. Radiocarbon dating works on the principle that ^{14}C decays according to radioactive decay with a half life of 5730 years. A parchment fragment was discovered that had about 74% as much ^{14}C as does plant material on earth today. Estimate the age of the parchment.

172. After 3 days a sample of radon-222 decayed to 58% of its original amount. (a) What is the half life of radon-222? (b) How long would it take the sample to decay to 10% of its original amount?

173. Polonium-210 has a half life of 140 days. (a) If a sample has a mass of 200 mg find a formula for the mass that remains after t days. (b) Find the mass after 100 days. (c) When will the mass be reduced to 10 mg? (d) Sketch the graph of the mass as a function of time.

174. If the bacteria in a culture increase continuously at a rate proportional to the number present, and the initial number is N_0, find the number at time t.

175. If a radioactive substance disintegrates at a rate proportional to the amount present how much of the substance remains at time t if the initial amount is Q_0?

176. Current agricultural experts believe that the world's farms can feed about 10 billion people. The 1950 world population was 2.517 billion and the 1992 world population was 5.4 billion. When can we expect to run out of food?

177. The Archer Daniel Midlands company runs two ads on Sunday mornings. One says that "when this baby is old enough to vote, the world will have one billion new mouths to feed" and the other says "in thirty six years, the world will have to set eight billion places at the table." What does ADM think the population of the world is at present? How fast does ADM think the population is increasing? Use units of billions of people so you can write 8 instead of $8,000,000,000$. (Hint: $36 = 2 \times 18$.)

178. The population of California grows exponentially at a rate of 2% per year. The population of California on January 1, 1990 was 20,000,000. (a) Write a formula for the population $N(t)$ of California t years after January 1, 1990. (b) Each Californian consumes pizzas at the rate of 70 pizzas per year. At what rate is California consuming pizzas t years after 1990? (c) How many pizzas were consumed in California from January 1, 1995 to January 1, 1999?

179. The population of the country of Slobia grows exponentially. (a) If its population in the year 1980 was 1,980,000 and its population in the year 1990 was 1,990,000, what will be its population in the year 2000? (b) How long will it take the population to double? (Your answer may be expressed in terms of exponentials and natural logarithms.)

180. If $f'(x) = x - 1/x^2$ and $f(1) = 1/2$ find $f(x)$.

181. $\displaystyle\int 2^x \, dx$

182. $\displaystyle\int (6x^5 - 2x^{-4} - 7x + 3/x - 5 + 4e^x + 7^x) \, dx$

183. $\displaystyle\int (x/a + a/x + x^a + a^x + ax) \, dx$

184. $\displaystyle\int \left(\sqrt{x} - \sqrt[3]{x^4} + \frac{7}{\sqrt[3]{x^2}} - 6e^x + 1 \right) dx$

185. $\displaystyle\int_{-2}^{4} (3x - 5) \, dx$

186. $\displaystyle\int_{1}^{2} x^{-2} \, dx$

187. $\displaystyle\int_{0}^{1} (1 - 2x - 3x^2) \, dx$

188. $\displaystyle\int_1^2 (5x^2 - 4x + 3)\, dx$

189. $\displaystyle\int_{-3}^0 (5y^4 - 6y^2 + 14)\, dy$

190. $\displaystyle\int_0^1 (y^9 - 2y^5 + 3y)\, dy$

191. $\displaystyle\int_0^4 \sqrt{x}\, dx$

192. $\displaystyle\int_0^1 x^{3/7}\, dx$

193. $\displaystyle\int_1^3 \left(\frac{1}{t^2} - \frac{1}{t^4}\right) dt$

194. $\displaystyle\int_1^2 \frac{t^6 - t^2}{t^4}\, dt$

195. $\displaystyle\int_1^2 \frac{x^2 + 1}{\sqrt{x}}\, dx$

196. $\displaystyle\int_0^2 (x^3 - 1)^2\, dx$

197. $\displaystyle\int_0^1 u(\sqrt{u} + \sqrt[3]{u})\, du$

198. $\displaystyle\int_{-1}^1 \frac{3}{t^4}\, dt$

199. $\displaystyle\int_1^2 (x + 1/x)^2\, dx$

200. $\displaystyle\int_3^3 \sqrt{x^5 + 2}\, dx$

201. $\displaystyle\int_1^{-1} (x - 1)(3x + 2)\, dx$

202. $\displaystyle\int_1^4 (\sqrt{t} - 2/\sqrt{t})\, dt$

203. $\displaystyle\int_1^8 \left(\sqrt[3]{r} + \frac{1}{\sqrt[3]{r}}\right) dr$

204. $\displaystyle\int_{-1}^0 (x + 1)^3\, dx$

205. $\displaystyle\int_{-5}^{-2} \frac{x^4-1}{x^2+1}\,dx$

206. $\displaystyle\int_{1}^{e} \frac{x^2+x+1}{x}\,dx$

207. $\displaystyle\int_{4}^{9} \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2\,dx$

208. $\displaystyle\int_{0}^{1} \left(\sqrt[4]{x^5}+\sqrt[5]{x^4}\right)\,dx$

209. $\displaystyle\int_{1}^{8} \frac{x-1}{\sqrt[3]{x^2}}\,dx$

210. $\displaystyle\int_{\pi/4}^{\pi/3} \sin t\,dt$

211. $\displaystyle\int_{0}^{\pi/2} (\cos\theta+2\sin\theta)\,d\theta$

212. $\displaystyle\int_{\pi/2}^{\pi} \sec x\tan x\,dx$

213. $\displaystyle\int_{\pi/3}^{\pi/2} \csc x\cot x\,dx$

214. $\displaystyle\int_{\pi/6}^{\pi/3} \csc^2\theta\,d\theta$

215. $\displaystyle\int_{\pi/4}^{\pi} \sec^2\theta\,d\theta$

216. $\displaystyle\int_{1}^{\sqrt{3}} \frac{6}{1+x^2}\,dx$

217. $\displaystyle\int_{0}^{0.5} \frac{dx}{\sqrt{1-x^2}}\,dx$

218. $\displaystyle\int_{4}^{8} (1/x)\,dx$

219. $\displaystyle\int_{\ln 3}^{\ln 6} 8e^x\,dx$

220. $\displaystyle\int_{8}^{9} 2^t\,dt$

221. $\displaystyle\int_{-e^2}^{-e} \frac{3}{x}\,dx$

222. $\displaystyle\int_{-2}^{3} |x^2 - 1|\,dx$

223. $\displaystyle\int_{-1}^{2} |x - x^2|\,dx$

224. $\displaystyle\int_{-1}^{2} (x - 2|x|)\,dx$

225. $\displaystyle\int_{0}^{2} (x^2 - |x - 1|)\,dx$

226. $\displaystyle\int_{0}^{2} f(x)\,dx$ where $f(x) = \begin{cases} x^4, & \text{if } 0 \le x < 1, \\ x^5, & \text{if } 1 \le x \le 2. \end{cases}$

227. $\displaystyle\int_{-\pi}^{\pi} f(x)\,dx$ where $f(x) = \begin{cases} x, & \text{if } -\pi \le x \le 0, \\ \sin x, & \text{if } 0 < x \le \pi. \end{cases}$

228. Explain what a Riemann sum is and write the definition of $\displaystyle\int_a^b f(x)dx$ as a limit of Riemann sums.

229. Give an example which shows that $\displaystyle\int_a^b f(x)dx$ is not always the true area bounded by the curves $y = f(x)$, $y = 0$, $x = a$, and $x = b$ even though $f(x)$ is continuous between a and b.

230. State the Fundamental Theorem of Calculus.

231. Let $f(x)$ be a function which is continuous and let $A(x)$ be the area under $f(x)$ from a to x. Compute the derivative of $A(x)$ by using limits.

232. Why is the Fundamental Theorem of Calculus true? Explain carefully and thoroughly.

233. Give an example which illustrates the Fundamental Theorem of Calculus. It order to do this compute an area by summing up the areas of tiny boxes and then show that applying the Fundamental Theorem of Calculus gives the same answer.

234. Sketch the graph of the curve $y = \sqrt{x+1}$ and determine the area of the region enclosed by the curve, the x-axis and the lines $x = 0$, $x = 4$.

235. Make a sketch of the graph of the function $y = 4 - x^2$ and determine the area enclosed by the curve, the x-axis and the lines $x = 0$, $x = 2$.

236. Find the area under the curve $y = \sqrt{6x+4}$ and above the x-axis between $x = 0$ and $x = 2$. Draw a sketch of the curve.

237. Graph the curve $y = x^3$ and determine the area enclosed by the curve and the lines $y = 0$, $x = 2$ and $x = 4$.

238. Graph the function $f(x) = 9 - x^2$, $0 \le x \le 3$, and determine the area enclosed between the curve and the x-axis.

239. Graph the curve $y = 2\sqrt{1 - x^2}$, $x \in [0, 1]$, and find the area enclosed between the curve and the x-axis.

240. Determine the area under the curve $y = \sqrt{a^2 - x^2}$ and between the lines $x = 0$ and $x = a$.

241. Graph the curve $y = 2\sqrt{9 - x^2}$ and determine the area enclosed between the curve and the x-axis.

242. Graph the area between the curve $y^2 = 4x$ and the line $x = 3$. Find the area of this region.

243. Find the area bounded by the curve $y = 4 - x^2$ and the lines $y = 0$ and $y = 3$.

244. Find the area bounded by the curve $y = x(x - 3)(x - 5)$, the x-axis and the lines $x = 0$ and $x = 5$.

245. Find the area enclosed between the curve $y = \sin 2x$, $0 \le x \le \pi/4$ and the axes.

246. Find the area enclosed between the curve $y = \cos 2x$, $0 \le x \le \pi/4$ and the axes.

247. Find the area enclosed between the curve $y = 3\cos x$, $0 \le x \le \pi/2$ and the axes.

248. Find the area enclosed between the curve $y = \cos 3x$, $0 \le x \le \pi/6$ and the axes.

249. Find the area enclosed between the curve $y = \tan^2 x$, $0 \le x \le \pi/4$ and the axes.

250. Find the area enclosed between the curve $y = \csc^2 x$, $0 \le x \le \pi/4$ and the axes.

251. Find the area of the region bounded by $y = -1$, $y = 2$, $x = y^3$, and $x = 0$.

252. Find the area of the region bounded by the parabola $y = 4x^2$, $x \ge 0$, the y-axis, and the lines $y = 1$ and $y = 4$.

253. Find the area of the region bounded by the curve $y = 4 - x^2$ and the lines $y = 0$ and $y = 3$.

254. Graph $y^2 + 1 = x$, $x \le 2$ and find the area enclosed by the curve and the line $x = 2$.

255. Find the area bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$.

256. Graph the curve $y = x/\pi + 2\sin^2 x$ and write a definite integral whose value is the area between the x-axis, the curve and the lines $x = 0$ and $x = \pi$. *Do not* evaluate the integral. *Do* specify the limits of integration.

257. Find the area bounded by $y = \sin x$ and the x-axis between $x = 0$ and $x = 2\pi$.

258. Find the area of the region bounded by the parabola $y^2 = 4x$ and the line $y = 2x$.

259. Find the area bounded by the curve $y = x(2 - x)$ and the line $x = 2y$.

260. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

261. Calculate the area of the region bounded by the parabolas $y = x^2$ and $x = y^2$.

262. Find the area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$.

263. Find the area of the region bounded by the curves $y = \sqrt{x}$ and $y = x$.

264. Use integration to find the area of the triangular region bounded by the lines $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

265. Find the area bounded by the parabola $x^2 - 2 = y$ and the line $x + y = 0$.

266. Graph the curve $y = (1/2)x^2 + 1$ and the straight line $y = x + 1$ and find the area between the curve and the line.

267. Find the area of the region between the parabolas $y^2 = x$ and $x^2 = 16y$.

268. Find the area of the region enclosed by the parabola $y^2 = 4ax$ and the line $y = mx$.

269. Write a definite integral whose value is the area of the region between the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$. Find this area. If you cannot evaluate the integral by calculus you may use geometry to find the area. Hint: The part of a circle cut off by a line is a circular sector with a triangle removed.

270. Write a definite integral whose value is the area of the region between the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$. *Do not* evaluate the integral. *Do* specify the limits of integration.

271. Write a definite integral whose value is the area of the region between the curves $x^2 + y^2 = 2$ and $x = y^2$ *Do not* evaluate the integral. *Do* specify the limits of integration.

272. Write a definite integral whose value is the area of the region between the curves $x^2 + y^2 = 2$ and $x = y^2$. Find this area. If you cannot evaluate the integral by calculus you may use geometry to find the area. Hint: Divide the region into two parts.

273. Write a definite integral whose value is the area of the part of the first quadrant which is between the parabola $y^2 = x$ and the circle $x^2 + y^2 - 2x = 0$. Find this area. If you cannot evaluate the integral by calculus you may use geometry to find the area. Hint: Draw a careful graph. Divide a semicircle in two.

274. Find the area bounded by the curves $y = x$ and $y = x^3$.

275. Graph $y = \sin x$ and $y = \cos x$ for $0 \le x \le \pi/2$ and find the area enclosed by them and the x-axis.

276. Write a definite integral whose value is the area inside the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. Evaluate this area. Hint: After a suitable change of variable, the definite integral becomes the definite integral whose value is the area of a circle.

277. Using integration find the area of the triangle with vertices $(-1, 1)$, $(0, 5)$ and $(3, 2)$.

278. Find the volume that results by rotating the triangle $1 \le x \le 2$, $0 \le y \le 3x - 3$ around the x axis.

279. Find the volume that results by rotating the triangle $1 \le x \le 2$, $0 \le y \le 3x - 3$ around the y axis.

280. Find the volume that results by rotating the triangle $1 \le x \le 2$, $0 \le y \le 3x - 3$ around the line $x = -1$.

281. Find the volume that results by rotating the triangle $1 \le x \le 2$, $0 \le y \le 3x - 3$ around the line $y = -1$.

282. Find the volume that results by rotating the semicircle $y = \sqrt{R^2 - x^2}$ about the x-axis.

283. A triangle is formed by drawing lines from the two endpoints of a line segment of length b to a vertex V which is at a height h above the line of the line segment. Its area is then $A = \int_{y=0}^{h} dV$ where dV is the area of the strip cut out by two parallel lines separated by a distance of dz and at a height of z above the line containing the line segment. Find a formula for dA in terms of b, z, and dz and evaluate the definite integral.

284. A pyramid is formed by drawing lines from the four vertices of a rectangle of area A to a vertex V which is at a height h above the plane of the rectangle. Its volume is then $V = \int_{z=0}^{h} dV$ where dV is the volume of the slice cut out by two planes parallel to the plane of the rectangle and separated by a distance of dz and at a height of z above the plane of the rectangle. Find a formula for dV in terms of A, z, and dz and evaluate the definite integral.

285. A tetrahedron is formed by drawing lines from the three vertices of a triangle of area A to a vertex V which is at a height h above the plane of the triangle. Its volume is then $V = \int_{z=0}^{h} dV$ where dV is the volume of the slice cut out by two planes parallel to the plane of the triangle and separated by a distance of dz and at a height of z above the plane of the rectangle. Find a formula for dV in terms of A, z, and dz and evaluate the definite integral.

286. A cone is formed formed by drawing lines from the perimeter of a circle of area A to a vertex V which is at a height h above the plane of the circle.

Its volume is then $V = \int_{z=0}^{h} dV$ where dV is the volume of the slice cut out by two planes parallel to the plane of the circle and separated by a distance of dz and at a height of z above the plane of the rectangle. Find a formula for dV in terms of A, z, and dz and evaluate the definite integral.

287. A vase is constructed by rotating the curve $y = 10\sqrt{x-1}$ for $0 \le y \le 6$ around the y axis. It is filled with water to a height $y = h$ where $h < 6$. (a) Find the volume of the water in terms of h. (b) If the vase is filling with water at the rate of 2 cubic units per second, how fast is the height of the water increasing when this height is 5 units?