

Math 221 – Exam III (50 Minutes) – Friday, April 16

Answers

I. (20 points.) Write the formula for a Riemann sum S for a function $f(x)$ on the interval $a \leq x \leq b$ and explain in what sense it approximates the definite integral $\int_a^b f(x) dx$.

Answer: A Riemann sum for a function $f(x)$ on the interval $a \leq x \leq b$ is an expression of form

$$S = \sum_{k=1}^n f(\bar{x}_k)(x_k - x_{k-1})$$

where

$$a = x_0 \leq \bar{x}_1 \leq x_1 \leq \bar{x}_2 \leq x_2 \leq \cdots \leq x_{n-1} \leq \bar{x}_n \leq x_n = b.$$

It approximates the definite integral in the sense that

$$S \approx \int_a^b f(x) dx$$

when all the $x_k - x_{k-1}$ are small.

II. (30 points.) (a) Find $\int_{-1}^2 (3x^{-2} - 2x^{10} + 3) dx$.

Answer: This is Problem 3 on page 256.

(b) Find $\int_1^3 \sqrt{7 + 2t^2}(8t) dt$.

Answer: This is Problem 21 on page 256.

III. (25 points.) Find y as a function of x if

$$\frac{dy}{dx} = -y^3(x^2 + 2)^2$$

and $y = 1$ when $x = 0$.

Answer: This is Problem 14 on page 220.

IV. (25 points.) (a) Find $\sum_{n=1}^5 n \cos(n\pi)$. (An exact integer answer is required here.)

Answer: This is problem 7 page 226

(b) Write the sum $\sum_{i=3}^{19} i(i-2)$ in sigma notation with $k = i-2$ as the dummy variable.

Answer: This is problem 31 p 226.

V. (25 points.) Use Riemann sums with four intervals of length one to find positive numbers m and M with

$$0 < m \leq \int_1^5 \left(3 + \frac{1}{x}\right) dx \leq M.$$

Answer: This is Problem 35 page 250.

VI. (25 points.) Starting from rest, a train increases speed at constant acceleration a_1 , then travels at a constant speed v_m , and finally brakes to a stop at constant de-acceleration $-a_2$. It took 7 minutes to travel 3 miles from Addison to Howard and 3 minutes to travel 1 mile from Howard to Dempster. The train spent one minute in the Howard station.

(a) Sketch the graph of the speed v as a function of time t for $0 \leq t \leq 11$. (The time $t = 0$ corresponds to the moment when the train leaves Addison.)

(b) Find the maximum speed v_m and the accelerations a_1 and a_2 if the train takes $1/2$ minute to accelerate to its maximum speed and $1/3$ of minute to decelerate from the maximum speed to rest. **SHOW YOUR REASONING.**)

Answer: this is like Problem Problem 33 page 221.

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There are 194 scores

grade	range	count	percent
A	120...150	19	9.8%
AB	110...119	27	13.9%
B	100...109	32	16.5%
BC	90... 99	41	21.1%
C	75... 89	41	21.1%
D	60... 74	21	10.8%
F	0... 59	13	6.7%

Mean score = 93.9. Mean grade = 2.43.

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