Math 221, Quiz 7, 3 May 2002

Answers

I (10 points) Find the following integral. Simplify your answer.

$$\int_{2/\sqrt{3}}^{2/\sqrt{2}} \frac{dx}{x\sqrt{x^2-1}}$$

Answer: Recall that $\dfrac{d}{dx}\sec^{-1}(x) = \dfrac{1}{x\sqrt{x^2-1}}$ and for $0 \le \theta < \pi/2$ we have

$$\theta = \sec^{-1} x \iff x = \sec\theta \iff \frac{1}{x} = \cos\theta \iff \theta = \cos^{-1}\left(\frac{1}{x}\right).$$

Hence

$$\begin{aligned}\int_{2/\sqrt{3}}^{2/\sqrt{2}} \frac{dx}{x\sqrt{x^2-1}} &= \sec^{-1}(2/\sqrt{2}) - \sec^{-1}(2/\sqrt{3}) \\ &= \cos^{-1}\left(\sqrt{2}/2\right) - \cos^{-1}\left(\sqrt{3}/2\right) \\ &= \pi/4 - \pi/6 = \pi/12.\end{aligned}$$

II (10 points) Find the equation of the line tangent to the curve $2e^{xy} = x + y$ at the point $(0, 2)$.

Answer: By implicit differentiation

$$2e^x y\left(y + x\frac{dy}{dx}\right) = 1 + \frac{dy}{dx},$$

so at $(0, 2)$,

$$4 = 1 + \left.\frac{dy}{dx}\right|_{x=0}.$$

Then

$$\left.\frac{dy}{dx}\right|_{x=0} = 3,$$

and the equation for the tangent line at $(0, 2)$ is

$$\frac{y-2}{x-0} = 3,$$

Grader's Comments

I realized after grading the first problem, that I should have formulated it in a different way, dividing it in two parts and asking for the derivative of $sec^{-1}x$ first. Students didn't do well on this part, tough many of them get it correct. I gave 3 points for evaluating $sec^{-1}x$ at the appropriate values.

To my surprise, the second problem wasn't easy for many of them, I would say that it was as bad as the first one. There were a lot of mistakes about properties of exponential and logarithmic function, and also in the application of the chain rule and the product rule.