Math 221, Quiz 5, 5 April 2002

Answers

I (5 points) Fill in the boxes so as to complete the following statement:

A definite integral can be approximated by a Riemann sum. If

$$a = x_0 < x_1 < x_2 < \cdots x_{n-1} < x_n = b,$$

$$\Delta x_k = \boxed{x_k - x_{k-1}},$$

$$\boxed{x_{k-1}} \le c_k \le \boxed{x_k},$$

and

$$\boxed{\max\{\Delta x_1, \Delta x_2, \ldots, \Delta x_n\}} \approx 0,$$

then

$$\sum_{k=1}^{n} f(c_k)\,\Delta x_k \approx \int_a^b f(x)\,dx.$$

II (15 points) A continuous function f satisfies $f(1) = 0.59$, $f(1.8) = 0.9$, $f(2.5) = 1$, $f(3.4) = 0.84$, $f(4) = 0.59$, $f(x)$ is increasing for $1 \le x \le 2.5$ and $f(x)$ is decreasing for $2.5 \le x \le 4$. Find a Riemann sum S such that

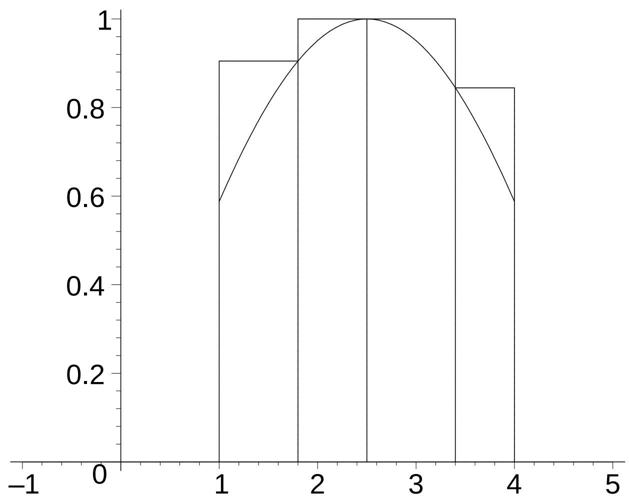
$$\int_1^4 f(x)\,dx \le S$$

and S is as small as possible given the above information. DO NOT DO ANY ARITHMETIC. Sketch a possible graph and also draw the area represented by your Riemann sum.

Answer: Take $x_0 = 1$, $x_1 = 1.8$, $x_2 = 2.5$, $x_3 = 3.4$, $x_4 = 4$, and $c_1 = x_1$, $c_2 = x_2$, $c_3 = x_2$, $c_4 = x_3$ so that $f(x)$ attains its maximum on the interval $[x_{k-1}, x_k]$ when $x = c_k$. Then

$$\begin{aligned} S &= f(c_1)(x_1 - x_0) + f(c_2)(x_2 - x_1) + f(c_3)(x_3 - x_2) + f(c_4)(x_4 - x_3) \\ &= 0.9(1.8 - 1) + 1(2.5 - 1.8) + 1(3.4 - 1.5) + 0.84(4 - 3.4) \end{aligned}$$

(The graph is on the reverse side.)



Comments.

Despite the fact that I gave the definition of the Riemann Integral four times in lecture, distributed a one page handout on Wednesday containing the definition, and wrote the answer to question I on the board a half hour before the quiz, only about 25% of the students filled in the boxes correctly on question I. This is more evidence that very little learning goes on in lecture. I wish I had pointed out that the same definition appears at the end of Chapter 4 in the book. I did not penalize students who took $\Delta x_k = (b-a)/n$ *since this is the first definition in the book. Perhaps one third of the students showed a complete lack of ability to use the notation correctly.*

In question II many students found a sum which is $\leq$ *the integral. I deducted only two points for this error. The next time I ask a question like question II, I will drop the phrase "and* S *is as small as possible given the above information." The intention of this phrase is to exclude the answer* $S = f(c_2)(b-a)$ *which is a trivial Riemann sum that is* $\geq$ *the integral. I think it is better to give full credit to any one who writes this.*