Math 221, Quiz 4, 8 March 2002

Answers

The Amazing Yarko can shrink a giant golden globe to a point at the rate of 10 m^3 per second. Assume the globe is perfectly spherical and that she preserves its shape in the process. Recall that the volume contained by a sphere is $\frac{4}{3}\pi(\text{radius})^3$, and the surface area of a sphere is $4\pi(\text{radius})^2$.

1 How fast is the globe's radius decreasing when it is 10 m? (Don't forget physical units!)

Answer: This is very much like problems 4 and 15 of section 3-2. Define

$V :=$ the volume of the globe at time t.

$r :=$ the radius of the globe at time t.

$A :=$ the surface area of the globe at time t.

In this notation, the job is to find the absolute value of $\left.\frac{dr}{dt}\right|_{r=10}$. Well,

$$V = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\Rightarrow \quad \frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$$

$$\Rightarrow \quad \left.\frac{dr}{dt}\right|_{r=10} = \frac{1}{4\pi(10)^2}(-10) = \frac{-1}{40\pi}.$$

So the radius is decreasing at a rate of $\frac{1}{40\pi}$ m/sec.

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2 Also, how fast is the globe's surface area decreasing at that instant? (Don't forget units!)

Answer: The job is to find the absolute value of $\left.\frac{dA}{dt}\right|_{r=10}$. Well,

$$A = 4\pi r^2 \quad \Rightarrow \quad \frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$
$$\Rightarrow \quad \left.\frac{dA}{dt}\right|_{r=10} = 8\pi(10)(\frac{-1}{40\pi}) = -2.$$

So the surface area is decreasing at a rate of $2\text{m}^2/\text{sec}$.

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-- Sun Mar 10 20:26:45 2002
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There are  202 scores
grade      range       count      percent
   A      18... 20     48         23.8%
  AB      16... 17     52         25.7%
   B      14... 15     26         12.9%
  BC      12... 13     11          5.4%
   C      10... 11     15          7.4%
   D       8...  9     16          7.9%
   F       0...  7     34         16.8%
Mean score =  13.3. Mean grade = 2.60.
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