Math 221 – Quiz 3 – February 14, 2002

Answers

1. (10 pts.) For $1 < x < \infty$ the function f is defined by

$$f(x) = 4 - (x-1)^2.$$

Let g be the inverse function to f. You are given that the point $(0,3)$ lies on $y = g(x)$. Find the slope of the tangent line to the graph $y = g(x)$ at point $(x,y) = (0,3)$. [**Hint.** Find $g'(0)$.]

Answer: The curves $y = g(x)$ and $x = f(y)$ are the same so at any point

$$(x, y) = (x, g(x)) = (f(y), y)$$

on this curve we have

$$\frac{dy}{dx} = g'(x), \qquad \frac{dx}{dy} = f'(y).$$

In particular at the point $(0,3) = (0, g(0)) = (f(3), 3)$ we have that

$$g'(0) = \frac{1}{f'(3)}.$$

We compute:

$$f(y) = 4 - (y-1)^2 = 3 + 2y - y^2, \qquad f'(y) = 2 - 2y.$$

Thus $g'(0) = 1/(2 - 2(3)) = -1/4$, which is the required slope.

2. (10 pts.) Let $y = u^7 + 7u$; $u = 3x + 71$. Find dy/dx.

Answer: By chain rule

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (7u^6 + 7) \cdot 3 = 21u^6 + 21 = 21(3x + 71)^6 + 21.$$