Math 221 – Quiz 2 – February 8, 2002

Answers

Let $f(x) = \frac{x^3+1}{x}$.

a) (10 points) Find $f'(x)$, the derivative of f at x.

Answer: We can use the quotient rule, along with the addition and power rules:

$$\begin{aligned} f'(x) &= \frac{(x^3+1)'x - (x^3+1)(x)'}{(x)^2} = \frac{(3x^2+0)x - (x^3+1)(1)}{x^2} \\ &= \frac{3x^3 - (x^3+1)}{x^2} = \frac{2x^3-1}{x^2}, \end{aligned}$$

or we can do some algebra first:

$$f(x) = \frac{x^3+1}{x} = \frac{x^3}{x} + \frac{1}{x} = x^2 + x^{-1}$$

and then take the derivative using the addition and power rules:

$$f'(x) = (x^2 + x^{-1})' = (x^2)' + (x^{-1})' = 2x - x^{-2} = 2x - \frac{1}{x^2}.$$

b) (10 points) Find all the points $P(a, f(a))$ where the tangent line to the curve $y = f(x)$ is horizontal, **and** write down the equation of the tangent line at such points.

Answer: The tangent line to the curve $y = f(x)$ at the point $(a, f(a))$ has slope $f'(a)$. Since horizontal lines are precisely the lines with slope 0, we are interested in the points of the form (a,f(a)) with $a \neq 0$ and $f'(a) = 0$.

$$f'(a) = 0 \quad \Leftrightarrow \quad \frac{2a^3-1}{a^2} = 0 \quad \Leftrightarrow \quad 2a^3 - 1 = 0 \quad \Leftrightarrow \quad a = \frac{1}{\sqrt[3]{2}}.$$

As $f(\frac{1}{\sqrt[3]{2}}) = \frac{3\sqrt[3]{2}}{2}$, it follows that the only point where the tangent line to the curve $y = f(x)$ is horizontal is $(\frac{1}{\sqrt[3]{2}}, \frac{3\sqrt[3]{2}}{2})$, and the equation of the tangent line at that point is

$$y = \frac{3\sqrt[3]{2}}{2}.$$

points	# students	points	# students
0	5	11	7
1	0	12	10
2	2	13	9
3	1	14	15
4	2	15	12
5	1	16	6
6	1	17	29
7	12	18	8
8	7	19	4
9	2	20	29
10	51		