NAME____

Calculus 221 Final Exam

Wednesday, December 22, 1999

Circle your section.

321	Alkan	7:45 TR	B129 Van Vleck
323	Mulaire	9:55 TR	B211 Van Vleck
325	Slepcev	11:00 TR	23 Ingraham
326	Slepcev	12:05 TR	2323 Sterling
328	Laghi	1:20 TR	B131 Van Vleck
329	Mulaire	1:20 TR	B305 Van Vleck
330	Alkan	2:25 TR	23 Ingraham
331	Laghi	3:30 TR	B129 Van Vleck

I	20 Points	
II	30 Points	
III	30 Points	
IV	40 Points	
V	40 Points	
VI	30 Points	
VII	20 Points	
VIII	30 Points	
IX	30 Points	
X	30 Points	
Total	300 Points	

Show your reasoning. No Calculators or Notes.
You may leave numerical expressions unevaluated.

I. (20 points.) **(a)** Find $g'(x)$ where $g(x) = x^2 + \dfrac{1}{x^2}$.

(b) Find $\displaystyle\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$ where $f(x) = e^{\sqrt{x}}$.

II. (30 points.) **(a)** Evaluate $\int_1^2 (x + 1/x)^2 \, dx$.

(b) Evaluate $\int_8^9 2^t \, dt$.

III. (30 points.) **(a)** Evaluate $\int_{-1}^{2} (x - 2|x|)\, dx$.

(b) Find $h'(x)$ where $h(x) = \int_{2}^{1/x} \sin^4 t \, dt$.

IV. (40 points.) **(a)** Find the equation of the tangent line to the curve $xy - y^3 = 3$ at the point $(4, 1)$.

(b) Find d^2y/dx^2 at this point.

V. (40 points.) Consider the function $f(x) = xe^x$.

(a) Find $f'(x)$ and $f''(x)$.

(b) Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.

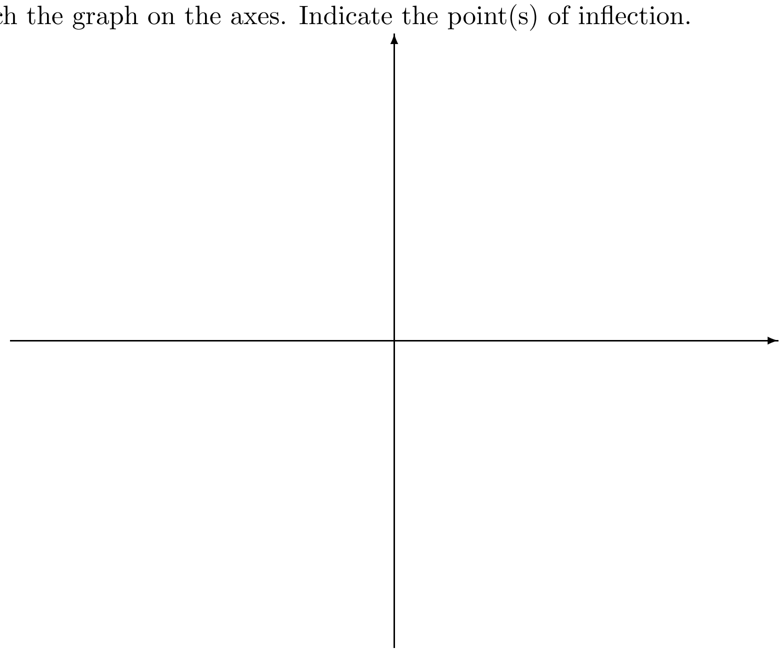
(V continued) **(c)** Find all points x where the tangent line to the graph $y = f(x)$ is horizontal.

(d) Find all inflection points x for the graph $y = f(x)$.

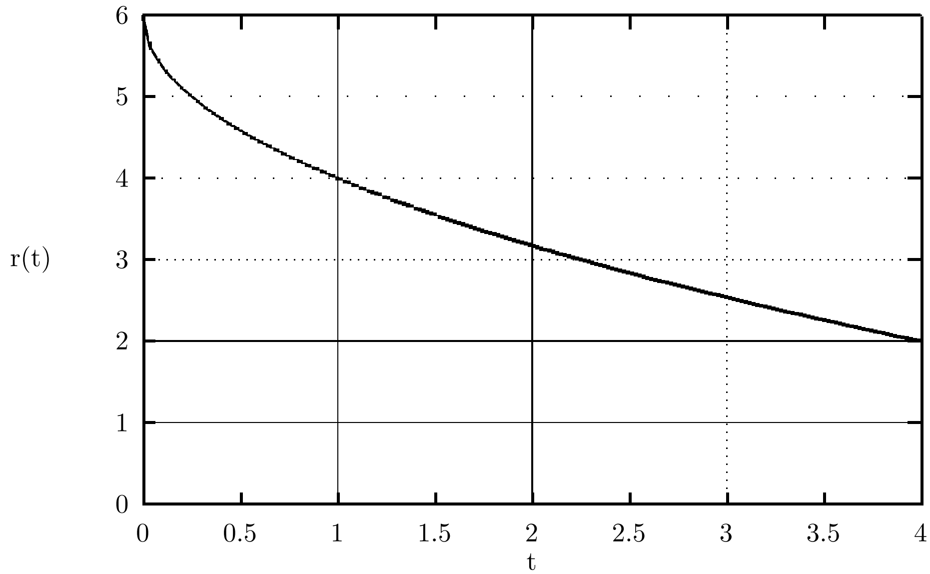
(V continued) **(e)** On which intervals is the function f increasing?

(f) On which intervals is the function f concave up?

(g) Sketch the graph on the axes. Indicate the point(s) of inflection.



VI. (30 points.) Water leaked from a tank at a rate of $r(t)$ liters per hour where the graph of r is as shown. Use the graph to estimate the volume V of water that leaked out during the first four hours. For full credit you should find positive numbers A and B so that $0 < A \le V \le B$ and say why your answer is correct and what it has to do with calculus. *Do not try to find a formula for $r(t)$.*



VII. (20 points.) Interpret $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{1+(i/n)^2} \frac{1}{n}$ as a limit of Riemann sums and evaluate the corresponding definite integral.

VIII. (30 points.) **(a)** Find the volume that results by rotating the triangle $1 \leq x \leq 2$, $0 \leq y \leq 3x - 3$ around the x axis.

b) Find the volume that results by rotating the triangle $1 \leq x \leq 2$, $0 \leq y \leq 3x - 3$ around the y axis.

IX. (30 points.) State and prove the Product Rule for differentiation. You may use the Limit Laws without proving them.

X. (30 points.) **(a)** True or false? A differentiable function must be continuous. If true, give a proof; if false, illustrate with an example.

(b) True or false? A continuous function must be differentiable. If true, give a proof; if false, illustrate with an example.