Name_____

Math 221 First Midterm Exam Thursday October 5 2006

Circle your section:

301	7:45 MW	B305 VAN VLECK	Murcko
302	8:50 MW	B333 VAN VLECK	Ganguly
303	8:50 MW	B329 VAN VLECK	Andrejko
304	9:55 MW	B219 VAN VLECK	Andrejko
305	9:55 MW	B105 VAN VLECK	Hu
306	11:00 MW	B105 VAN VLECK	Hu
308	12:05p MW	B329 VAN VLECK	Ganguly
310	2:25p MW	B203 VAN VLECK	Murcko
312	9:55-11:50 MWF	277 BASCOM	Rouse
313	12:15-2:10 MWF	3349 ENGR HALL	Owen

I	40 Points	
II	30 Points	
III	30 Points	
IV	30 Points	
V	25 Points	
VI	25 Points	
VII	20 Points	
Total	200 Points	

SHOW YOUR REASONING.

I. (40 points.)   (a) Find $\dfrac{d^2y}{dx^2}$ if $y = 5^x$.

(b) Find $\dfrac{dw}{dx}$ if $w = x^x$.

(c) Find $\dfrac{du}{dx}$ if $u = \sin(\ln x)$.

(d) Find $\displaystyle\lim_{x \to \infty} \dfrac{e^x - 1}{e^x + 1}$.

II. (30 points.) Consider the parametric curve

$$x = \frac{1 - t^2}{1 + t^2}, \quad y = \frac{2t}{1 + t^2}.$$

(i) Show that each point on this curve lies on the unit circle.

(ii) What point corresponds to the parameter value $t = 2$?

(iii) Write the equation of the tangent line at this point in parametric form:

$$x = x_0 + a(t - t_0), \qquad y = y_0 + b(t - t_0).$$

(iv) Write the equation of the tangent line at this point in point slope form:

$$y = y_0 + m(x - x_0).$$

III. (30 points.)   You are videotaping a race from a stand 120 feet from the track, following a car that is moving at a constant velocity along a straight track. When the car is directly in front of you the camera angle is changing at a rate of $\pi/3$ radians per second. How fast is the car going? How fast will the camera angle be changing a half second later?

IV. (30 points.) Denote the linear approximations to f near a and to g near b by

$$L(x) = f(a) + f'(a)(x - a), \qquad M(y) = g(b) + g'(b)(y - b),$$

and assume $b = f(a)$. Show that

$$M \circ L(x) = (g \circ f)(a) + (g \circ f)'(a)(x - a).$$

(This says that *The linear approximation to the composition is the composition of the linear approximations.*)

V. (25 points.)　Water runs out of a cylindrical tank from a drain in the bottom. The water level in the tank t hours after the tank starts to drain is

$$y = 6 \left(1 - \frac{t}{12} \right)^2 \text{ meters}$$

and the tank drains completely after 12 hours. Show that the rate at which the water level drops is proportional to the square root of the water level and find the constant of proportionality.

VI. (25 points.) **(i)** Write the polynomial of degree four which best approximates e^x near $x = 0$.

(i) What is the precise definition of the phrase "the polynomial of degree four which best approximates e^x near $x = 0$".

(i) Write the polynomial of degree eight which best approximates e^{x^2} near $x = 0$.

VII. (20 points.)   **(a)** The function

$$f(x) = \begin{cases} x + c & \text{for } x < 1 \\ 3^x & \text{for } x \geq 1 \end{cases}$$

is continuous. What is c?

(b) Draw a crude graph of the equation $y = f(x)$ where $f(x)$ is as in part (a).

(c) Give a formula (like the above formula for $f(x)$) for the inverse function $x = f^{-1}(y)$ of the function $y = f(x)$ of part (a).