Name_____

Math 221 First Midterm Exam Thursday October 5 2006

Circle your section:

301	7:45 MW	B305 VAN VLECK	Murcko
302	8:50 MW	B333 VAN VLECK	Ganguly
303	8:50 MW	B329 VAN VLECK	Andrejko
304	9:55 MW	B219 VAN VLECK	Andrejko
305	9:55 MW	B105 VAN VLECK	Hu
306	11:00 MW	B105 VAN VLECK	Hu
308	12:05p MW	B329 VAN VLECK	Ganguly
310	2:25p MW	B203 VAN VLECK	Murcko
312	9:55-11:50 MWF	277 BASCOM	Rouse
313	12:15-2:10 MWF	3349 ENGR HALL	Owen

I	40 Points	
II	40 Points	
III	35 Points	
IV	40 Points	
V	25 Points	
VI	20 Points	
Total	200 Points	

SHOW YOUR REASONING.

I. (40 points.) In each of the following, find dy/dx.

(a) $y = \sin x$.

(b) $y = (\sin x)^{-1}$.

(c) $y = \sin(x^{-1})$.

(d) $y = \sin^{-1}(x)$.

II. (40 points.) Find the limit. Distinguish between an infinite limit and one which doesn't exist. (Give reasons!)

(a) $\lim_{x \to 0} \dfrac{\sin(6x)}{x}$

(b) $\lim_{x \to 0+} \dfrac{\sin 6}{x}$

(c) $\lim_{x \to \infty} \dfrac{\sin(6x)}{x}$

(d) $\lim_{h \to 0} \dfrac{\sin(6+h) - \sin 6}{h}$

III. (35 points.) Find an equation for the tangent line to the curve

$$x^2 + xy - y^2 = 1$$

at the point $(x, y) = (2, 3)$.

IV. (40 points.) State and prove the formula for the derivative of the product of two functions. In your proof you may use (without proof) the limit laws, the theorem that a differentiable function is continuous, and high school algebra.

V. (25 points.) (a) Find $f'(x)$ and $g'(x)$ if

$$f(x) = \frac{x+1}{x-1}, \qquad g(x) = \frac{x+2}{x-2}.$$

(b) Let $f(x)$ and $g(x)$ be as in part (a) and $p = f \cdot g$, i.e. $p(x) = f(x) \cdot g(x)$. Find the derivative $p'(x)$ of $p(x)$.

(c) Let $f(x)$ and $g(x)$ be as in parts (a) and (b) and let $w = f \circ g$, i.e. $w(x) = f(g(x))$. Find the value $g(5)$ of $g(u)$ when $u = 5$ and the value $w'(5)$ of the derivative $w'(x)$ when $x = 5$.

VI. (20 points.) **(a)** Draw the graph $y = f(x)$ where $f(x)$ is the function defined by

$$f(x) = \begin{cases} 2x & \text{for } x < 0; \\ 3x & \text{for } 0 \leq x < 1; \\ 4x - 1 & \text{for } 1 \leq x. \end{cases}$$

(b) Give a formula (like the above formula for $f(x)$) for the inverse function $x = f^{-1}(y)$.