Name\_\_\_\_\_

Math 221 First Midterm Exam Thursday October 5 2006

Circle your section:

301 302 303 304 305 306 308 310 312 313	7:45 MW 8:50 MW 8:50 MW 9:55 MW 9:55 MW 11:00 MW 12:05p MW 2:25p MW 9:55-11:50 MWF 12:15 2:10 MWF	B305 VAN VLECK B333 VAN VLECK B329 VAN VLECK B219 VAN VLECK B105 VAN VLECK B105 VAN VLECK B329 VAN VLECK B203 VAN VLECK 277 BASCOM 3340 ENCE HALL	Murcko Ganguly Andrejko Hu Hu Ganguly Murcko Rouse Owon
313	12:15-2:10 MWF	3349 ENGR HALL	Owen

Ι	40 Points	
II	40 Points	
III	35 Points	
IV	40 Points	
V	25 Points	
VI	20 Points	
Total	200 Points	

Show your reasoning.

**I.** (40 points.) In each of the following, find dy/dx. (a)  $y = \sin x$ .

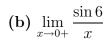
(b)  $y = (\sin x)^{-1}$ .

(c)  $y = \sin(x^{-1})$ .

(d)  $y = \sin^{-1}(x)$ .

**II.** (40 points.) Find the limit. Distinguish between an infinite limit and one which doesn't exist. (Give reasons!)

(a) 
$$\lim_{x \to 0} \frac{\sin(6x)}{x}$$



(c) 
$$\lim_{x \to \infty} \frac{\sin(6x)}{x}$$

(d) 
$$\lim_{h \to 0} \frac{\sin(6+h) - \sin 6}{h}$$

III. (35 points.) Find an equation for the tangent line to the curve

$$x^2 + xy - y^2 = 1$$

at the point (x, y) = (2, 3).

**IV.** (40 points.) State and prove the formula for the derivative of the product of two functions. In your proof you may use (without proof) the limit laws, the theorem that a differentiable function is continuous, and high school algebra.

**V.** (25 points.) (a) Find f'(x) and g'(x) if

$$f(x) = \frac{x+1}{x-1}, \qquad g(x) = \frac{x+2}{x-2}.$$

(b) Let f(x) and g(x) be as in part (a) and  $p = f \cdot g$ , i.e.  $p(x) = f(x) \cdot g(x)$ . Find the derivative p'(x) of p(x).

(c) Let f(x) and g(x) be as in parts (a) and (b) and let  $w = f \circ g$ , i.e. w(x) = f(g(x)). Find the value g(5) of g(u) when u = 5 and the value w'(5) of the derivative w'(x) when x = 5.

**VI.** (20 points.) (a) Draw the graph y = f(x) where f(x) is the function defined by

$$f(x) = \begin{cases} 2x & \text{for } x < 0; \\ 3x & \text{for } 0 \le x < 1; \\ 4x - 1 & \text{for } 1 \le x. \end{cases}$$

(b) Give a formula (like the above formula for f(x)) for the inverse function  $x = f^{-1}(y)$ .