Math 221, Quiz 6, 17 Nov 2000
Answers

1 If f and g are two continuous functions on $[a, b]$ and if for all x in $[a, b]$ $f(x) \geq g(x)$, then the area of the region between f and g over $[a, b]$ is

(a) $\int_b^a [g(x) - f(x)]\, dx$ (b) $\int_a^b [g(x) - f(x)]\, dx$ (c) $\int_a^b f(x)g(x)\, dx$ (d) $2\int_0^b [g(x) - f(x)]\, d$

Answer: The area of the region is $\int_a^b [f(x) - g(x)]\, dx = \int_b^a [g(x) - f(x)]\, dx$. So (a) is the correct answer.

2 Gilfong's favorite region in the whole wide world is over $[0, \frac{\pi}{4}]$ bounded above by $\cos x$ and below by $\sin x$. He would like to know the area of this region so that he can shout it from rooftops. What should Gilfong shout?

(a) $\sqrt{2} - 1$ (b) $1 - \sqrt{2}$ (c) $1 + \sqrt{2}$ (d) $\sqrt{2}$

Answer: The area of the region is

$$\begin{aligned}
\int_0^{\frac{\pi}{4}} [\cos x - \sin x]\, dx &= \sin x + \cos x]_0^{\frac{\pi}{4}} \\
&= \sin \frac{\pi}{4} + \cos \frac{\pi}{4} - (\sin 0 + \cos 0) \\
&= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - (0 + 1) \\
&= \sqrt{2} - 1.
\end{aligned}$$

So (a) is the correct answer.

3 Rikitikitambonosarambocheriberibuchipitperipambo skateboards around town with velocity $v(t) = 69(t - 1)$ $\frac{\text{cm}}{\text{sec}}$ and boasts, "I go real far real fast." How far does Riki really go between $t = 0$ and $t = 2$, that is, what's his total distance traveled in that interval?

(a) 69 cm (b) $\frac{1}{2} \cdot 69$ cm (c) 0 cm (d) 69 $\frac{\text{cm}}{\text{sec}^2}$

Answer: The total distance traveled is

$$\begin{aligned}\int_0^2 |v(t)|\,\mathrm{d}t &= \int_0^2 |69(t-1)|\,\mathrm{d}t \\ &= 69\int_0^2 |t-1|\,\mathrm{d}t \\ &= 69(\frac{1}{2}+\frac{1}{2}) \qquad \text{(by graphing } |t-1| \text{ or just computing)} \\ &= 69.\end{aligned}$$

So (a) is the correct answer.

4 Hmm, I seem to be in a muddle. Could you set me straight, and tell me the average value of the function $f(x) = a - b$ on $[a, b]$?

(a) $a-b$ (b) $b-a$ (c) c (d) $-a-b$

Answer: By computing $\frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x$ or remembering that the average value of a constant function on an interval is that constant, you get an average value of $a - b$. So (a) is the correct answer.

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There are  169 scores
  score       count      percent
  20          18         10.7%
  15          46         27.2%
  10          52         30.8%
   5          39         23.1%
   0          14          8.3%
Mean score =  10.4.
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