Math 221, Quiz IV, October 20, 2000

Answers

I (8 points) Use the Mean Value Theorem to prove that $|\sin b - \sin a| \leq |b - a|$.

Answer. By the Mean Value Theorem, we know there is at least one number c between a and b such that

$$\sin b - \sin a = \sin'(c)(b-a) = (\cos c)(b-a).$$

Therefore,

$$|\sin b - \sin a| = |\cos c||b-a| \leq |b-a|.$$

II (12 points) Let $f(x) = x^3 - 3x + 1$.

a) $\lim\limits_{x\to\infty} f(x) = ?$ $\qquad \lim\limits_{x\to-\infty} f(x) = ?$

Answer. $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = -\infty$.

b) Are there any absolute maximum or minimum points? Where are they if they exist?

Answer. By part a) there is no absolute maximum or minimum point.

c) Find the local maximum point(s) and local minimum point(s), respectively.

Answer. A local minimum of a function is a point where the function is decreasing slightly to the left and increasing slightly to the right. A local maximum of a function is a point where the function is increasing slightly to the left and decreasing slightly to the right, Since $f'(x) = 3x^2 - 3$ we have $f'(x) = 0$ for $x = \pm 1$. As $f'(-2) = 9 > 0$ we have $f'(x) > 0$ for $-\infty < x < -1$ and so f is increasing on this interval. As $f'(0) = -3 < 0$ we have $f'(x) < 0$ for $-1 < x < 1$and so f is decreasing on this interval. As $f'(2) = 9 > 0$ we have $f'(x) > 0$ for $1 < x < \infty$ and so f is increasing on this interval. Hence there is a local maximum point at $x = -1$, and a local minimum point at $x = 1$.

d) Find the point(s) of inflection.

Answer.A point of inflection is a point where the concavity of the function changes. The second derivative is $f''(x) = 6x$. Thus $f''(0) = 0$. As $f''(x) < 0$ for $x < 0$ the function is concave down on the interval $x < 0$. As $f''(x) > 0$ for $x < 0$ the function is concave up on the interval $x > 0$. Hence $x = 0$ is the unique point of inflection.

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There are 193 scores

range	count	percent
18... 20	130	67.4%
16... 17	19	9.8%
14... 15	19	9.8%
12... 13	4	2.1%
10... 11	7	3.6%
8... 9	7	3.6%
0... 7	7	3.6%

Mean score = 17.3.

The lecturer goofed and said the Mean Value Theorem would not be on the test. He therefore gave the students the option of basing their score entirely on problem II. Only six students took the option of doing problem I.