Name____

Math 221 – Exam II – Tuesday Evening October 31, 2000

Circle your section:

382	Lu	8:50 TR	350 Birge
384	Ores	9:55 TR	215 Ingraham
385	Lu	11:00 TR	B337 Van Vleck
388	Raichev	12:05 TR	B129 Van Vleck
389	Raichev	1:20 TR	B317 Van Vleck
390	Givens	1:20 TR	B129 Van Vleck
391	Chakrabarti	2:25 TR	B317 Van Vleck
392	Givens	2:25 TR	B129 Van Vleck
393	Ores	3:30 TR	B317 Van Vleck
394	Chakrabarti	3:30 TR	B129 Van Vleck

I	25 Points	
II	25 Points	
III	25 Points	
IV	25 Points	
V	25 Points	
VI	25 Points	
Total	150 Points	

Show your reasoning.

I. (25 points.) (1) State the Mean Value Theorem.

(2) Let f be a function, a a number in its domain, and n a nonnegative integer. Define the degree n Taylor polynomial of f centered at a.

(3) The hypothesis of the Extended Mean Value Theorem is that that $f(x)$ is $n+1$ times differentiable and that $f^{(n+1)}$ is continuous on an interval, that a and b are two numbers in that interval, and that $P_n(x)$ is the degree n Taylor polynomial of $f(x)$ centered at a. State the rest of the Extended Mean Value Theorem.

II. (25 points.) Evaluate the following limits. If the limit does not exists write DNE and say why. Distinguish between a limit which is infinite and one which does not exist. EXPLAIN YOUR REASONING.

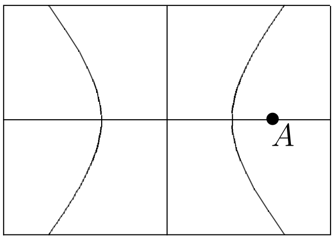
(i) $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x} - 1 - (x/2)}{x^2}$.

(ii) $\lim\limits_{x \to 0} \left(\dfrac{1}{\sin x} - \dfrac{1}{x} \right)$.

III. (25 points.) A ladder 26 ft long leans against a vertical wall. The foot of the ladder is being drawn away from the wall at a rate of 4 ft/sec. How fast is the top of the ladder sliding down the wall at the instant when the foot of the ladder is 10 ft from the wall?

IV. (25 points.) Graph $y = x^2 - x^{-1}$. Determine the intervals where y is increasing (as x increases), and where the graph is concave up. Indicate the turning points and points of inflection on the graph and draw the tangent line at each.

V. (25 points.) Find the points on the curve $y^2 = x^2 - 1$ that are nearest the point $A(a, 0)$ in case (1) $a = \sqrt{8}$ and (2) $a = \sqrt{2}$. Suggestion: The algebra is simpler if you minimize the square of the distance rather than the distance. Do not plug in for a till the end of the problem so you can do both parts at the same time.



VI. (25 points.) (i) Find the polynomial of degree two which best approximates the function $f(x) = 7 + 5x^2 + x^4$ near $x = 1$.

(ii) For $f(x)$ and $P(x)$ as in part (i) prove an inequality of form

$$|f(x) - P(x)| \leq M|x - 1|^3$$

valid in the range $1 < x < 2$. (You are suppose to find a number M which makes the inequality true for all x in the interval $1 < x < 2$.)