Chapter 23
The Economics of Resources

For All Practical Purposes: Effective Teaching

Students should find the topics in this course stimulating and real-life applications a refreshing departure from other traditional mathematics courses. Students should be seeing many, if not all, of these topics for the first time. Because the mathematics course for which this text has been designed does not lead to a next level of mathematics, you have a freedom not often experienced in your field. Because of this, you may be able to vary your teaching style greatly and experiment with different ways to motivate this group of liberal arts student.

Chapter Briefing

This chapter explores models for the decay or depletion of resources. As individuals and as a society, we are faced with the need to find a balance between using resources and conserving them. Some of these resources replenish themselves but some are nonrenewable.

Being well prepared for class discussion with examples is essential. In order to facilitate your preparation, the **Chapter Topics to the Point** has been broken down into **Growth Models for Biological Populations**, **How Long Can a Nonrenewable Resource Last?**, **Sustaining Renewable Resources**, **The Economics of Harvesting Resources**, and **Dynamical Systems and Chaos**. The material in this chapter of the *Teaching Guide* is presented in the same order as the text. Examples with solutions for these topics that do not appear in the text nor study guide are included in the *Teaching Guide*. You should feel free to use these examples in class, if needed.

The last section of this chapter of *The Teaching Guide for the First-Time Instructor* is **Solutions to Student Study Guide ✎ Questions**. These are the complete solutions to the two questions included in the *Student Study Guide*. Students only have the answers to these questions, not the solutions.

Chapter Topics to the Point

Growth Models for Biological Populations

If we use a geometric growth model to describe and predict human (or other species) populations, the effective rate of growth is the difference between population increase caused by births and decrease caused by deaths. This difference is called the **rate of natural increase**.

Example

Predict the population of a country that has a current (2006) population of 37 million in the year 2016 given r = 0.45%. What would happen at this rate in 155 years?

Solution

With annual compounding, we can predict that the population 20 years later, in 2016 would be the following.

$$\text{Population in 2016} = 37\text{ million} \times (1+0.0045)^{20} \approx 40.5\text{ million}$$

In 155 years the population would be as follows.

$$37\text{ million} \times (1+0.0045)^{155} \approx 74.2\text{ million}$$

At this rate, the population would double in about 155 years.

Small changes in the birth or death rate will affect the rate of natural increase, and this changes our prediction significantly.

Teaching Tip

Give students different rates of natural increase and through trial and error have them determine when a population would double. You might find it interesting to plot these values on a graph. Try it again with a population tripling.

A population cannot keep growing without limit. The resources available to the population limit the size of that population. A population limit in a particular environment is called the **carrying capacity**.

The closer a population gets to its carrying capacity, the more slowly the population will grow. The logistics model for population growth takes carrying capacity into account by reducing the annual increase of rP by a factor of how close the population size P is to the carrying capacity M. It is given by the following.

$$\text{growth rate } P' = rP\left(1 - \frac{\text{population size}}{\text{carrying capacity}}\right) = rP\left(1 - \frac{P}{M}\right)$$

Example

A fishery has a carrying capacity of 85,000 fish. The natural rate of increase for the population is 2.4% per year. What is the growth rate of the population if the population is at 43,000 fish?

Solution

$\text{growth rate} = 0.024\left(1 - \frac{43{,}000}{85{,}000}\right) = 0.0119$, 1.19% per year.

Teaching Tip

Redo the last example with several different present populations to make a generalization about what happens to the growth rate.

How Long Can a Nonrenewable Resource Last?

A **nonrenewable resource**, such as a fossil fuel or a mineral ore deposit, is a natural resource that does not replenish itself.

A growing population is likely to use a nonrenewable resource at an increasing rate. The regular and increasing withdrawals from the resource pool are analogous to regular deposits in a sinking fund with interest, and the same formula applies to calculate the accumulated amount of the resource that has been used, and is thus gone forever. The **static reserve** is the time the resource will last with constant use; the **exponential reserve** is the time it will last with use increasing geometrically with the population.

The formula for the exponential reserve of a resource with supply *S*, initial annual use *U*, and usage growth rate *r* is as follows.

$$n = \frac{\ln\left[1+\left(\frac{S}{U}\right)r\right]}{\ln\left[1+r\right]}$$

Here, ln is the natural logarithm function, available on your calculator.

Teaching Tip

Students may notice that there is also a common logarithm key on their calculator. For the type of calculation described above, they can use either. You may choose to have them explore this in order to lead to a brief discussion of the meaning of logarithms.

Example

Imagine that a certain iron ore deposit will last for 120 years at the current usage rate. How long would that same deposit last if usage increases at the rate of 5% each year?

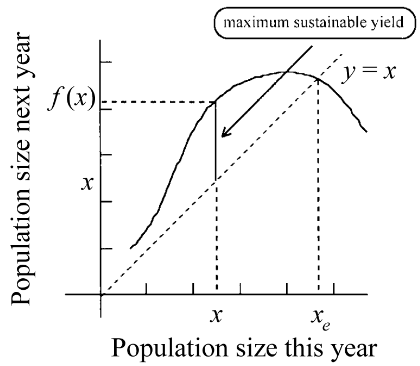
Solution

The static reserve, $\frac{S}{U}$, is 120 years, so we can substitute that value into the formula to solve for *n*, using the assumed 5% = 0.05 for *r*. We obtain $n = \frac{\ln\left[1+120(0.05)\right]}{\ln\left[1+0.05\right]} = \frac{\ln 7}{\ln 1.05} \approx 40$ years.

Sustaining Renewable Resources

A **renewable natural resource** replenishes itself at a natural rate and can often be harvested at moderate levels for economic or social purposes without damaging its regrowth. Since heavy harvesting may overwhelm and destroy the population, economics and conservation are crucial ingredients in formulating proper harvesting policies.

We keep track of the population (measured in biomass) from one year to the next using a **reproduction curve**. Under normal conditions, natural reproduction will produce a geometrically growing population, but too high a population level is likely to lead to overcrowding and to strain the available resources, thus resulting in a population decrease. This model leads to a reproduction curve looking something like this:



The dotted 45° diagonal line is the set of points where the population would be unchanged from year to year, and any point where it intersects the reproduction curve is an **equilibrium population size**.

The marked population value x is the level which produces maximum natural increase or yield in a year, and the difference between x and $f(x)$ (the population level a year later) is the maximum sustainable yield (or harvest) $f(x) - x$. This amount is the maximum that may be harvested each year without damaging the population, and represents a good choice for a sustained-yield harvesting policy.

The Economics of Harvesting Resources

If our main concern is profit, we must take into account the economic value of our harvest and the cost of harvesting. If we also include in our model **economy of scale** (denser populations are easier to harvest), then the sustainable harvest which yields a maximum profit may be smaller than the maximum sustainable yield.

Finally, if we also take into account the economic value of capital and consider profit as our only motivation, it may be most profitable to harvest the entire population, effectively killing it, and invest the profits elsewhere. The history of the lumbering and fishing industries demonstrates this unfortunate fact.

Dynamical Systems and Chaos

In some populations, the state of the population depends only on its state at previous times. This kind of system is called a **dynamical system**. For example, a population's size in a given year may depend entirely upon its size in the previous year.

Behavior that is determined by preceding events but is unpredictable in the long run is called **chaos**.

In some systems a small change in the initial conditions can make a huge difference later on. This is the **butterfly effect.**

The logistic population model can illustrate chaos in biological population. Consider the current year's population as a fraction *x* of the carrying capacity, and next year's population as a fraction $f(x)$. The amount by which the population is multiplied each year is $\lambda = 1 + r$, where *r* is the population's annual growth rate. Then the logistic model can be written as $f(x) = \lambda x(1 - x)$.

Example

A population grows according to a logistic growth model, with population parameter $\lambda = 1.2$ and $x = 0.60$ for the first year. What is the next population fraction?

Solution

$1.2(0.60)(1 - 0.60) = 0.288$

The logistic model illustrates chaotic behavior when the population parameter λ is equal to 4. In this case, for any starting population fraction, the population fraction changes year after year in no predictable pattern.

Solutions to Student Study Guide ✎ Questions

Question 1

In 2005, the Acme corporation had non-renewable resources of 3479 million pounds of materials. The annual consumption was 98 million pounds. The projected company consumption will increase 2.3% per year, through 2020.

a) What is the static reserve in 2005?

b) What is the exponential reserve in 2005?

Solution

a) The static reserve will be $\frac{3479}{98} = 35.5$ years.

b) The exponential reserve will be $\frac{\ln\left[1+(35.5)(0.023)\right]}{\ln\left[1+0.023\right]} \approx 26.25$ years

Question 2

Let $f(n) =$ the sum of the cubes of the digits of *n*. Start with 371 and apply *f* repeatedly. Start with 234 and apply *f* repeatedly. Start with 313 and apply *f* repeatedly. Are the three behaviors the same?

Solution

If you start with 371, the sequence of outcomes is as follows.

$$371, 371, 371, \ldots$$

If you start with 234, the sequence of outcomes is as follows.

$$234, 99, 1458, 702, 351, 153, 153, 153, \ldots$$

If you start with 313, the sequence of outcomes is as follows.

$$313, 55, 250, 133, 55, 250, 133, 55, \ldots$$

The behaviors are different. In the first case, we have the same outcome as the initial value (fixed point). In the second case, the sequence eventually stabilized on one value. In the last case the sequence oscillates between four values.