Chapter 18
Growth and Form

For All Practical Purposes: Effective Teaching

- Being a Graduate Teaching Assistant can be viewed as an apprenticeship experience. As such, you should have your work assessed by a faculty supervisor on a continuing basis. You should feel comfortable in articulating your goals early on in the term when you are appointed and work with your faculty supervisor to achieve these goals.
- Besides the formal student evaluations that your institution may require or recommend for your course, you may choose to include an informal evaluation which includes targeted questions to help you evaluate student interest in cooperative learning projects or other activities you included in your course. If you have a thought of including a new component in the future, you could ask for their opinion to help you determine if student interest would be great. You may ask for their feedback regarding homework length and difficulty. Avoid questions, however, that would be asked on the formal survey instruments.

Chapter Briefing

In this chapter, you will be mainly examining applications where you find an optimal solution. We examine how a variety of physical dimensions of an object − length, area, weight, and so on − are changed by proportional growth of the object. These changes influence the growth and development of an individual organism and the evolution of a species.

Being well prepared for class discussion with examples is essential. In order to facilitate your preparation, the **Chapter Topics to the Point** has been broken down into **Geometric Similarity**, **How Much Is That in …?**, **Scaling a Mountain**, **Sorry**, **No King Kongs**, **Dimension Tension**, and **How to Grow**. The material in this chapter of the *Teaching Guide* is presented in the same order as the text. Examples with solutions for these topics that do not appear in the text nor study guide are included in the *Teaching Guide*. You should feel free to use these examples in class, if needed.

The last section of this chapter of *The Teaching Guide for the First-Time Instructor* is **Solutions to Student Study Guide ✎ Questions**. These are the complete solutions to the four questions included in the *Student Study Guide*. Students only have the answers to these questions, not the solutions.

Chapter Topics to the Point

Geometric Similarity

Two objects are **geometrically similar** if they have the same shape, regardless of their relative sizes. The **linear scaling factor** relating two geometrically similar objects *A* and *B* is the ratio of the length of a part of *B* to the length of the corresponding part of *A*.

The area of the surface of a scaled object changes according to the square of the linear scaling factor.

Example

Compare a sphere *A*, which has a 5-inch radius, to another sphere *B*, which has a 8-inch diameter. What is the ratio of the surface area of the large sphere *A* to that of the small sphere *B*?

Solution

You do not need to calculate the surface areas of the spheres. Area scales according to the square of the scaling factor. The radius of the sphere *B* is 4 inches. Since $r = \frac{5}{4}$, $r^2 = \frac{25}{16}$, and so we get $\frac{\text{area } A}{\text{area } B} = \frac{25}{16} = 1.5625.$

The volume (and weight) of a scaled object changes according to the cube of the linear scaling factor.

Teaching Tip

In this example, you can emphasize that in doing such comparisons you must "compare apples with apples." In other words, you must compare corresponding linear measurements.

Example

Sphere *A*, which has a 5-inch radius, is similar to sphere *B*, which has a 4-inch radius. If the small sphere *B* weighs 8 ounces, how much does the large sphere *A* weigh?

Solution

Weight, like volume, scales by the cube of the scaling factor. Since $\frac{\text{weight } A}{\text{weight } B} = \left(\frac{5}{4}\right)^3 = \frac{125}{64}$, weight $A = \frac{125}{64} \times 8 \text{ oz} = \frac{125}{8} \text{ oz} = 15.625 \text{ oz}.$

In describing a growth situation or a comparison between two similar objects or numbers, the phrase "*x* is increased by" a certain percentage means that you must add the amount of the increase to the current value of *x*. The phrase "*x* is decreased by" a percentage means that you must subtract the amount of decrease from *x*.

Example

This year, the value of the stock of the ACME Corporation fell by 35%, from 180 to ____.

Solution

The new stock value is $\left(1-\frac{35}{100}\right)\times 180 = 180 - 63 = 117.$

Teaching Tip

Students should readily be able to compare "*x* is increased by" to the addition of sales tax to an item and "*x* is decreased by" to an item on sale at a store.

➢ How Much Is That in …?

Some basic dimensional units in the US system of measurement are: foot (length), gallon (volume), pound (weight). Some comparable units in the metric system are: meter (length), liter (volume), kilogram (weight). Refer to Table 18.3 on page 671 for conversion between the two measuring systems.

Conversions from one system to the other are done according to our rules for scaling. For example, 1 in = 2.54 cm; that is, we have a scaling factor of 2.54 from in. to cm. Therefore, an area of $3 \text{ in}^2 = 3\times(2.54)^2 \text{ cm}^2 \approx 19.35 \text{ cm}^2$.

☝Teaching Tip
In dealing with approximate values, it may be a good opportunity to talk about significant digits.

☝Teaching Tip
In doing conversions, point out to students that units cancel like numbers. With that they know whether to multiply or divide by a conversion factor.

☝Teaching Tip
In order to make a handout of the conversion tables that appear in this chapter, Tables 18.1-18.3 appear on the next page.

Table 18.1 Units of the U.S Customary System (Page 669)

Distance:

1 mile (mi) = 1760 yards (yd) = 5280 feet (ft) = 63,360 inches (in.)
1 yard (yd) = 3 feet (ft) = 36 inches (in.)
1 foot (ft) = 12 inches (in.)

Area:

1 square mile = 1 mi $\times$ 1 mi = 5280 ft $\times$ 5280 ft = 27,878,400 $\text{ft}^2 \approx 28\times10^6\ \text{ft}^2$
= 63,360 in. $\times$ 63,360 in. = 4,014,489,600 $\text{in.}^2 \approx 4\times10^9\ \text{in.}^2$
= 640 acres

1 acre = 43,560 ft^2

Volume:

1 cubic mile = 1 mi $\times$ 1 mi $\times$ 1 mi = 5280 ft $\times$ 5280 ft $\times$ 5280 ft
= 147,197,952,000 $\text{ft}^3 \approx 147\times10^9\ \text{ft}^3$
= 63,360 in.$\times$ 63,360 in.$\times$ 63,360 in. $\approx 2.5\times10^{14}\ \text{in.}^3$

1 U.S. gallon (gal) = 4 U.S. quarts (qt) = 231 in.^3, exactly

Mass:

1 ton (t) = 2000 pounds (lb)

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Table 18.2 Units of the Metric System (Page 670)

Distance:

1 meter (m) = 100 centimeters (cm)
1 kilometer (km) = 1000 meters (m) = 100,000 centimeters (cm) = 1×10^5 cm

Area:

1 square meter (m^2) = 1 m $\times$ 1 m = 100 cm $\times$ 100 cm = 10,000 (cm^2) = $1\times10^4\ \text{cm}^2$

1 hectare (ha) = 10,000 m^2

Volume:

1 liter (L) = 1000 cm^3 = 0.001 m^3

1 cubic meter (m^3) = 1 m$\times$ 1 m $\times$ 1 m = 100 cm$\times$ 100 cm$\times$ 100 cm = 1,000,000 cm^3 = $1\times10^6\ \text{cm}^3$ (or cc)

Mass:

1 kilogram (kg) = 1000 grams (g)

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Table 18.3 Conversions between U.S Customary System and Metric System (Page 671)

Distance:

1 in. = 2.54 cm, exactly
1 ft = 12 in. = 12$\times$ 2.54 cm = 30.48 cm = 0.3048 m, exactly
1 yd = 0.9144 m, exactly
1 mi = 5280 ft = 5280 $\times$ 30.48 cm = 160,934.4 cm, exactly $\approx$ 1.61 km
1 cm $\approx$ 0.393701 in. $\approx$ 0.4 in.
1 m $\approx$ 39.37 in. $\approx$ 3.281 ft
1 km $\approx$ 0.621 mi

Area:

1 hectare (ha) $\approx$ 2.47 acres

Volume:

1 cubic meter (m^3) = 1000 liters $\approx$ 264.2 U.S. gallons $\approx$ 35.31 ft^3

1 liter (L) = 1000 cm^3 $\approx$ 1.057 U.S. quarts (qt) $\approx$ 0.2642 U.S. gallons

Mass:

1 lb = 0.45359237 kg, exactly
1 kg $\approx$ 2.205 lb

Scaling a Mountain

The size of a real object or organism is limited by a variety of structural or physiological considerations. For example, as an object is scaled upward in size, the **mass** and weight grow as the cube of the scaling factor, whereas, the surface area grows as the square of the same factor.

The **pressure** on the bottom face or base of an object (for example, the feet of an animal or the foundation of a building) is the ratio of the weight of the object to the area of the base; that is, $P = \frac{W}{A}$. The weight may be calculated by multiplying the volume by the density.

Example

What is the pressure at the base of a block of stone that is 3 ft wide, 4 ft long and 5 ft high, given that the density of the stone is 420 lb per ft^3?

Solution

$P = \frac{W}{A}$. The weight W of the block is volume $\times$ density.
Thus, W = (3 ft$\times$ 4 ft $\times$ 5 ft) $\times$ 420 lb/ft^3 = 25200 lb.
The base area $A = 3 \times 4 = 12$ ft^2. Therefore, the pressure P = 25200/12= 2100 lb/ft^2.

Teaching Tip

Question 4 in the *Student Study Guide* asks the following question (the solution is in this *Guide*).

"The weight of a block of marble that measures $1\text{ ft}\times 2\text{ ft}\times 3\text{ft}$ weighs 1240 lbs. If 10 of these blocks are used to make a wall 6 ft high, 10 ft long, and 1 ft wide, what is the pressure on the bottom faces?"

You can address this question in class and ask students to determine how the answer changes (if at all) if the wall is rotated. Asking this question will also help for students to think about rotations which is in the next chapter.

Sorry, No King Kongs

Though the weight of an object increases with the cube of the linear scaling factor, the ability to support weight increases only with the square of the linear scaling factor.

As an object is scaled up in size, the area of its surface increases as the square of the scaling factor, while the volume increases as the cube of the scaling factor. This **area-volume tension** can strongly influence the development of structural parts of organisms that depend on both dimensions for strength, mobility, heat control, breathing, flight, and so on.

Proportional growth does not preserve many organic properties. As individuals grow, or as species evolve, their physiological proportions must change; growth is not proportional.

Dimension Tension

Area-volume tension is a result of the fact that as an object is scaled up, the volume increases faster than the surface area and faster than areas of cross sections.

Proportional growth is growth according to geometric similarity: the length of every part of the organism enlarges by the same linear scaling factor.

How to Grow

Allometric growth is the growth of the length of one feature at a rate proportional to a power of the length of another.

On a graph we can use **orders of magnitude**, such as powers of 10 $\left(10^0, 10^1, 10^2, \ldots\right)$, instead of integer values such as 0, 1, 2, … . When the scale is powers of 10, it is called **base-10 logarithmic scale**. On graph paper when both axes are scaled as such, the graph paper is called **log-log paper**. When only one axis is scaled as such, it is called **semilog paper**.

Large changes in scale force a change in either material or form. Thus, limits are imposed on the scale of living organisms.

A **power curve** is described by the equation $y = bx^a$. In this case *y* is proportional to *x*.

Example

The following is data concerning the growth of an bahbooma (fictitious animal).

Age	Height	Log (Height)	Tail length	Log (Tail length)
4.1	12 in	1.08	6 in	0.78
5.6	42 in	1.62	12 in	1.08

Find the slope of the line from ages 3.25 to 4.2, where the *height* is plotted on the horizontal axis and the *tail length* is on the vertical axis (log of each, on log-log paper).

Solution

The slope for the line from ages 4.1 to 5.6 is the vertical change over the horizontal change in terms of log units.

$$\frac{\log 12 - \log 6}{\log 42 - \log 12} = \frac{1.08 - 0.78}{1.62 - 1.08} = \frac{0.30}{0.54} = \frac{5}{9} \approx 0.56$$

Thus, the slope of the line on log-log paper is $\frac{5}{9}$.

Teaching Tip

You may choose to introduce students to the different types of graph paper and plot actual data. Log-log graph paper can easily be found on the Internet. One site at the time of printing of this *Guide* is http://www.csun.edu/~vceed002/ref/measurement/data/graph_paper.html. This site also has semilog graph paper available.

Solutions to Student Study Guide ✎ Questions

Question 1

A model of a water tower is built to a scale of 1 to 59. If the model holds 8 cubic inches, how much will the actual water tower hold?

Solution

The actual water tower would hold $8 \cdot 59^3 = 1,643,032$ cubic inches of water. This would be $\dfrac{1,643,032 \text{ cubic inches}}{12^3 \text{ cubic inches per cubic foot}} \approx 950.83$ cubic feet or approximately 951 cubic feet of water.

Question 2

a) This year, the value of the stock of the Dippy Dan Corporation fell by x%, from 100 to 80. What is x?

b) This year, the value of the stock of the Buckaroo Corporation rose by x%, from 80 to 100. What is x?

Solution

a) Since $\left(1-\frac{x}{100}\right)\times 100 = 80,$ we have $100 - x = 80.$ Thus, x is 20.

b) Since $\left(1+\frac{x}{100}\right)\times 80 = 100,$ we have $80 + \frac{4x}{5} = 100.$ This implies $\frac{4x}{5} = 20$ or $4x = 100.$ Thus, x is 25.

Question 3

Convert

a) 17 miles = ________ kilometers

b) 100 meters = _________ yards

c) 300 m^2 = ________ in^2

Solution

a) Since $1\,\text{mi} \approx 1.61\,\text{km},$ we have $17\,\text{mi} \cdot \dfrac{1.61\,\text{km}}{1\,\text{mi}} = 27.37$ km.

b) Since $1 \text{ yd} = 0.9144 \text{ m},$ we have $100\,\text{m} \cdot \dfrac{1 \text{ yd}}{0.9144\,\text{m}} \approx$ 109.4 m.

c) Since $1 \text{ m} \approx 39.37 \text{ in},$ we have $1 \text{ m}^2 \approx 39.37^2 \text{ in}^2$. Thus we have the following.

$$300\,\text{m}^2 \cdot \frac{39.37^2 \text{ in}^2}{1\,\text{m}^2} \approx 464{,}999 \text{ in}^2.$$

Question 4

The weight of a block of marble that measures $1 \text{ ft} \times 2 \text{ ft} \times 3 \text{ ft}$ weighs 1240 lbs. If 10 of these blocks are used to make a wall 6 ft high, 10 ft long, and 1 ft wide, what is the pressure on the bottom faces?

Solution

In order to have a wall 6 ft high, 10 ft long, and 1 ft wide, there must be two rows of five blocks. The area on the bottom is $10 \text{ ft} \times 1 \text{ ft} = 10 \text{ ft}^2$, or 1440 in^2. Since $P = \dfrac{W}{A}$, we have the following.

$$P = \frac{W}{A} = \frac{1240 \times 10}{1440} \approx 8.61 \text{ lb/in}^2$$