Chapter 18
Growth and Form

Chapter Objectives

Check off these skills when you feel that you have mastered them.

☐ Determine the scaling factor when given the original dimensions of an object and its scaled dimensions.

☐ Given the original dimensions of an object and its scaling factor, determine its scaled dimensions.

☐ Calculate the change in area of a scaled object when its original area and the scaling factor are given.

☐ Calculate the change in volume of a scaled object when its original volume and the scaling factor are given.

☐ Determine whether two given geometric objects are similar.

☐ Locate on a number line the new location of a scaled point.

☐ When given the two-dimensional coordinates of a geometric object, its center, and the scaling factor, calculate its new coordinates after the scaling has taken place.

☐ Calculate from given formulas the perimeter and area of a two-dimensional object.

☐ Calculate from given formulas the surface area and volume of a three-dimensional object.

☐ Describe the concept of area-volume tension.

☐ Explain why objects in nature are restricted by a potential maximum size.

Guided Reading

Introduction

We examine how a variety of physical dimensions of an object − length, area, weight, and so on − are changed by proportional growth of the object. These changes influence the growth and development of an individual organism and the evolution of a species.

Section 18.1 Geometric Similarity

Key idea

Two objects are **geometrically similar** if they have the same shape, regardless of their relative sizes. The **linear scaling factor** relating two geometrically similar objects *A* and *B* is the ratio of the length of a part of *B* to the length of the corresponding part of *A*.

Example A

Compare a cube *A*, which is 2 inches on a side, to a similar cube *B*, which is 12 inches on a side. What is the scaling factor of the enlargement from *A* to *B* and what is the ratio of the diagonal of *B* to the diagonal of *A*?

Solution

The scaling factor r is the ratio of any two corresponding linear dimensions. Thus r = ratio of sides = $\frac{12}{2} = 6,$ which is also the ratio of diagonals when comparing the second object to the original.

Key idea

The area of the surface of a scaled object changes according to the square of the linear scaling factor.

Example B

Compare a cube *A*, which is 2 inches on a side, to a similar cube *B*, which is 12 inches on a side. What is the ratio of the surface area of the large cube *B* to that of the small cube *A*?

Solution

You do not need to calculate the surface areas of the cubes. Area scales according to the square of the scaling factor. Since $r = 6$, $r^2 = 36$, and so we get $\frac{\text{area } B}{\text{area } A} = 6^2 = 36.$

Key idea

The volume (and weight) of a scaled object changes according to the cube of the linear scaling factor.

Example C

Cube *A*, which is 2 inches on a side, is similar to cube *B*, which is 12 inches on a side. If the small cube *A* weighs 3 ounces, how much does the large cube *B* weigh?

Solution

Weight, like volume, scales by the cube of the scaling factor. Since $\frac{\text{weight } B}{\text{weight } A} = 6^3 = 216,$ weight *B* $= 216 \times 3$ oz = 648 oz, or 40.5 pounds.

Question 1

A model of a water tower is built to a scale of 1 to 59. If the model holds 8 cubic inches, how much will the actual water tower hold?

Answer

Approximately 951 cubic feet

Key idea

In describing a growth situation or a comparison between two similar objects or numbers, the phrase "*x* is increased by" a certain percentage means that you must add the amount of the increase to the current value of *x*. The phrase "*x* is decreased by" a percentage means that you must subtract the amount of decrease from *x*.

Example D

a) This year, the value of the stock of the ABC Corporation rose by 25%, from 120 to ____.

b) This year, the value of the stock of the XYZ Corporation fell by 25%, from 150 to ____.

Solution

a) The new stock value is $\left(1+\frac{25}{100}\right)\times 120 = 120 + 30 = 150$.

b) The new stock value is $\left(1-\frac{25}{100}\right)\times 150 = 150 - 37.5 = 112.5$.

Question 2

a) This year, the value of the stock of the Dippy Dan Corporation fell by *x*%, from 100 to 80. What is *x*?

b) This year, the value of the stock of the Buckaroo Corporation rose by *x*%, from 80 to 100. What is *x*?

Answer

a) 20

b) 25

Section 18.2 How Much Is That in …?

Key idea

Some basic dimensional units in the US system of measurement are: foot (length), gallon (volume), pound (weight). Some comparable units in the metric system are: meter (length), liter (volume), kilogram (weight). Refer to Table 18.3 on page 671 for conversion between the two measuring systems.

Key idea

Conversions from one system to the other are done according to our rules for scaling. For example, 1 meter = 3.28 feet; that is, we have a scaling factor of 3.28 from meters to feet. Therefore, an area of $5\text{ m}^2 = 5\times(3.281)^2\text{ ft}^2 \approx 53.82\text{ ft}^2$.

Example E

Convert

a) 5 pounds = ________ kilograms
b) 60 kilometers = _________ miles
c) 2000 in^2 = ________ m^2

Solution

These answers are approximate.

a) 1 lb $\approx$ 0.4536 kg, so 5 lb = 5 $\times$ 0.4536 = 2.27 kg.
b) 1 km = 0.621 mi, so 60 km = 60 $\times$ 0.621 = 37.26 mi.
c) 1 in = 2.54 cm = 0.0254 m, so 1 in^2 = $(0.0254)^2$ = 0.000645 m^2;
therefore, 2000 in^2 = 2000 $\times$ 0.000645 = 1.29 m^2.

Question 3

Convert

a) 17 miles = ________ kilometers
b) 100 meters = _________ yards
c) 300 m^2 = ________ in^2

Answer

These answers are approximate.

a) 27.37
b) 109.4
c) 464,999

Section 18.3 Scaling a Mountain

Key idea

The size of a real object or organism is limited by a variety of structural or physiological considerations. For example, as an object is scaled upward in size, the **mass** and weight grow as the cube of the scaling factor, whereas, the surface area grows as the square of the same factor.

Key idea

The **pressure** on the bottom face or base of an object (for example, the feet of an animal or the foundation of a building) is the ratio of the weight of the object to the area of the base; that is, $P = \frac{W}{A}$. The weight may be calculated by multiplying the volume by the density.

Example F

What is the pressure at the base of a block of stone that is 2 ft wide, 3 ft long and 4 ft high, given that the density of the stone is 350 lb per ft^3?

Solution

$P = \frac{W}{A}$. The weight W of the block is volume $\times$ density.

Thus, W = (2 ft $\times$ 3 ft $\times$ 4 ft) $\times$ 350 lb/ft^3 = 8400 lb.

The base area A = 2 $\times$ 3 = 6 ft^2. Therefore, the pressure P = 8400/6 = 1400 lb/ft^2.

Example G

What are the dimensions and the pressure at the base of a block of the same stone that is the same shape but scaled up by a factor of 5?

Solution

The scaling factor is 5. Volume, and therefore weight, scales by $5^3 = 125$, while area scales by $5^2 = 25$. Thus, the pressure is $P = \dfrac{125 \times W}{25 \times A} = \dfrac{125 \times (24 \times 350)}{25 \times 6} = 7000\,\text{lb/ft}^2$.

Example H

What is the pressure at the base of a cylindrical column of the same stone that is 3 feet in diameter and 60 feet high, if the density of the stone is 350 lb/sq ft?

Solution

The diameter of the column is 3, so the radius is 1.5. The column is a cylinder, so its volume is $\pi \times$ (radius)^2 $\times$ height = 3.14 $\times$ 2.25 $\times$ 60 = 423.9 ft^3. Because the density is 350 lb/ft^3, the weight W = volume $\times$ density = 423.9 $\times$ 350 = 148,365 lb. The area A of the base is $\pi \times$ (radius)^2 = 7.065 ft^2. Then $P = \dfrac{W}{A} = \dfrac{148{,}365}{7.065}$ = 21,000 lb/ft^2.

Question 4

The weight of a block of marble that measures $1\text{ ft} \times 2\text{ ft} \times 3\text{ft}$ weighs 1240 lbs. If 10 of these blocks are used to make a wall 6 ft high, 10 ft long, and 1 ft wide, what is the pressure on the bottom faces?

Answer

Approximately $8.61\,\text{lb/in}^2$

Section 18.4 Sorry, No King Kongs

Key idea

Though the weight of an object increases with the cube of the linear scaling factor, the ability to support weight increases only with the square of the linear scaling factor.

Key idea

As an object is scaled up in size, the area of its surface increases as the square of the scaling factor, while the volume increases as the cube of the scaling factor. This **area-volume tension** can strongly influence the development of structural parts of organisms that depend on both dimensions for strength, mobility, heat control, breathing, flight, and so on.

Key idea

Proportional growth does not preserve many organic properties. As individuals grow, or as species evolve, their physiological proportions must change; growth is not proportional.

Section 18.5 Dimension Tension

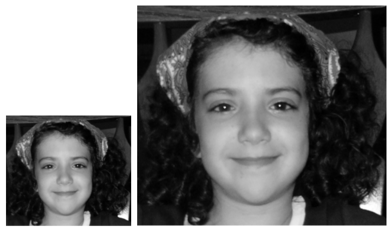
Key idea

Area-volume tension is a result of the fact that as an object is scaled up, the volume increases faster than the surface area and faster than areas of cross sections.

Key idea

Proportional growth is growth according to geometric similarity: the length of every part of the organism enlarges by the same linear scaling factor.

Proportional growth, scaling factor = 2



Disproportionate growth: height scaling factor = 2, length scaling factor = 4

Section 18.6 How to Grow

Key idea

Allometric growth is the growth of the length of one feature at a rate proportional to a power of the length of another.

Key idea

On a graph we can use **orders of magnitude**, such as powers of 10 $\left(10^0, 10^1, 10^2, \ldots\right)$, instead of integer values such as 0, 1, 2, … . When the scale is powers of 10, it is called **base-10 logarithmic scale**. On graph paper when both axes are scaled as such, the graph paper is called **log-log paper**. When only one axis is scaled as such, it is called **semilog paper**.

Key idea

Large changes in scale force a change in either material or form. Thus, limits are imposed on the scale of living organisms.

Key idea

A **power curve** is described by the equation $y = bx^a$. In this case *y* is proportional to *x*.

Example I

The following is data concerning the growth of an ebix (fictitious animal).

Age	Height	Log (Height)	Nose length	Log (Nose length)
3.25	61 in	1.79	20 in	1.30
4.2	100 in	2.00	50 in	1.70

Find the slope of the line from ages 3.25 to 4.2, where the *height* is plotted on the horizontal axis and the *nose length* is on the vertical axis (log of each, on log-log paper).

Solution

The slope for the line from ages 3.25 to 4.2 is the vertical change over the horizontal change in terms of log units.

$$\frac{\log 50 - \log 20}{\log 100 - \log 61} = \frac{1.70 - 1.30}{2.00 - 1.79} = \frac{0.4}{0.21} \approx 1.9$$

Thus, the slope of the line on log-log paper is approximately 1.9.

Homework Help

Exercises 1 – 14
Carefully read Section 18.1 before responding to these exercises.

Exercises 15 – 23
Carefully read Section 18.2 before responding to these exercises.

Exercises 24 – 31
Carefully read Section 18.3 before responding to these exercises.

Exercises 32 – 45
Carefully read Section 18.4 before responding to these exercises.

Exercises 46 – 57
Carefully read Section 18.5 before responding to these exercises.

Exercises 58 – 59
Carefully read Section 18.6 before responding to these exercises.

Do You Know the Terms?

Cut out the following 18 flashcards to test yourself on Review Vocabulary. You can also find these flashcards at http://www.whfreeman.com/fapp7e.

Chapter 18 **Growth and Form** **Allometric growth**	**Chapter 18** **Growth and Form** **Area-volume tension**
Chapter 18 **Growth and Form** **Base-10 logarithmic scale**	**Chapter 18** **Growth and Form** **Crushing strength**
Chapter 18 **Growth and Form** **Density**	**Chapter 18** **Growth and Form** **Dilation**
Chapter 18 **Growth and Form** **Geometrically similar**	**Chapter 18** **Growth and Form** **Isometric growth**

A result of the fact that as an object is scaled up, the volume increases faster than the surface area and faster than areas of cross sections	**A pattern of growth in which the length of one feature grows at a rate proportional to a power of the length of another feature**
The maximum ability of a substance to withstand pressure without crushing or deforming	**A scale on which equal divisions correspond to powers of 10**
A linear scaling	**Weight per unit volume**
Proportional growth	**Two objects are this if they have the same shape, regardless of the materials of which they are made. They need not be of the same size. Corresponding linear dimensions must have the same factor of proportionality**

Chapter 18 **Growth and Form** **Linear scaling factor**	**Chapter 18** **Growth and Form** **Log-log paper**
Chapter 18 **Growth and Form** **Orders of magnitude**	**Chapter 18** **Growth and Form** **Power curve**
Chapter 18 **Growth and Form** **Pressure**	**Chapter 18** **Growth and Form** **Problem of scale**
Chapter 18 **Growth and Form** **Proportional growth**	**Chapter 18** **Growth and Form** **Semilog paper**

Graph paper on which both the vertical and the horizontal scales are logarithmic scales, that is, the scales are marked in orders of magnitude 1, 10, 100, 1000, ..., instead of 1, 2, 3, 4, . .	The number by which each linear dimension of an object is multiplied when it is scaled up or down; that is, the ratio of the length of any part of one of two geometrically similar objects to the length of the corresponding part of the second.
A curve described by an equation $y = bx^a$, so that y is proportional to a power of x	Powers of 10
As an object or being is scaled up, its surface and cross-sectional areas increase at a rate different from its volume, forcing adaptations of materials or shape.	Force per unit area
Graph paper on which only one of the scales is a logarithmic scale	Growth according to geometric similarity, where the length of every part of the organism enlarges by the same linear scaling factor

Chapter 18 **Growth and Form** **Weight**	**Chapter 18** **Growth and Form** **Wing loading**

Weight supported divided by wing area	**Force under gravity** **W = Volume X Density**

Practice Quiz

1. You want to enlarge a small painting, measuring 4 inches by 7 inches, onto a piece of paper, measuring 8.5 inches by 11 inches, so that the image is proportional and as large as possible. What is the scaling factor for the enlargement?

a. 1.571

b. 2.125

c. 3.339

2. A small quilted wall hanging has an area of 15 sq. ft. A larger bed quilt is proportional to the wall hanging with a linear scaling factor of 3. What is the area of the bed quilt?

a. 3 sq. ft.

b. 45 sq. ft.

c. 135 sq. ft.

3. A model airplane is built to a scale of 1 to 50. If the wingspan of the actual airplane is 25 ft, what is the wingspan of the model plane?

a. 2 ft

b. 1/2 ft

c. 1/10 ft

4. At the grocery store, an 8-inch cherry pie costs $4.39 and a similar 10-inch cherry pie costs $6.15. Which is the better buy?

a. the 8-inch pie

b. the 10-inch pie

c. they are about the same

5. A 3rd grade class glues 64 sugar cubes together to form a larger cube. What is the linear scaling factor?

a. 4

b. 8

c. 64

6. 5 cm is approximately _______ in.

a. 1.6

b. 1.97

c. 12.7

7. 9.5 sq. mi is approximately _______ sq. km.

a. 15.30

b. 3.66

c. 24.62

8. A table weighs 125 pounds and is supported by four legs, which are each 0.75 inch by 0.5 inch by 24 inches high. How much pressure do the legs exert on the floor?

a. 333 lb/sq. in

b. 83 lb/sq in

c. 2000 lb/sq in

9. The growth of human bodies is best modeled as _______________.

a. isometric growth

b. proportional growth

c. allometric growth

10. If points $(1,5)$ and $(2,15)$ lie on the graph of $\log y = B + a\log x,$ what is the value of a?

a. $\dfrac{\log 15 - \log 5}{\log 2 - \log 1}$

b. $\dfrac{\log 15 - \log 2}{\log 5 - \log 1}$

c. $\dfrac{\log 15 - \log 1}{\log 5 - \log 2}$

Word Search

Refer to page 694 of your text to obtain the Review Vocabulary. There are 17 hidden vocabulary words/expressions in the word search below. It should be noted that spaces are removed as well as hyphens. Also, *Base-10 logarithmic scale* does not appear in the word search.

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D E E G I S R D N E I F L F H E I L E I P X H F J
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