Chapter 9
Social Choice: The Impossible Dream

Chapter Objectives

Check off these skills when you feel that you have mastered them.

- ☐ Analyze and interpret preference list ballots.
- ☐ Explain three desired properties of Majority Rule.
- ☐ Explain May's theorem.
- ☐ Explain the difference between majority rule and the plurality method.
- ☐ Discuss why the majority method may not be appropriate for an election in which there are more than two candidates.
- ☐ Apply the plurality voting method to determine the winner in an election whose preference list ballots are given.
- ☐ Explain the Condorcet winner criterion (CWC).
- ☐ Rearrange preference list ballots to accommodate the elimination of one or more candidates.
- ☐ Structure two alternative contests from a preference schedule by rearranging preference list ballots; then determine whether a Condorcet winner exists.
- ☐ Apply the Borda count method to determine the winner from preference list ballots.
- ☐ Explain independence of irrelevant alternatives (IIA).
- ☐ Apply the sequential pairwise voting method to determine the winner from preference list ballots.
- ☐ Explain Pareto condition.
- ☐ Apply the Hare system to determine the winner from preference list ballots
- ☐ Explain monotonicity.
- ☐ Apply the plurality runoff method to determine the winner from preference list ballots.
- ☐ Explain Arrow's impossibility theorem.
- ☐ Recognize the application of the law of transitivity in the interpretation of individual preference schedules and its possible nonvalidity in group preferences (the Condorcet paradox).
- ☐ Apply the process of approval voting and discuss its consideration in political races.

Guided Reading

Introduction

Voting occurs in many situations, such as in elections of public officials, officers of a club, or among a group of friends who have to decide in which restaurant to eat. While elections involving just two choices are quite simple, the opposite is true for elections with three or more alternatives, in which many complications and paradoxes arise. Social choice theory was developed to analyze the various types of voting methods, to discover the potential pitfalls in each, and to attempt to find improved systems of voting.

In this chapter, you will be interpreting and altering preference list ballots. A **preference list ballot** consists of a rank ordering of candidates. Throughout the chapter these vertical lists will have the most preferential candidate on top and the least preferential on the bottom. An example of a voter's ballot for four candidates (say *A*, *B*, *C*, and *D*) could be as follows.

Rank	
First	*A*
Second	*D*
Third	*B*
Fourth	*C*

For a particular vote, we can summarize the preference list ballots in a single table as follows.

	Number of voters (15)			
Rank	**5**	**2**	**7**	**1**
First	*A*	*C*	*B*	*D*
Second	*D*	*B*	*C*	*A*
Third	*B*	*A*	*D*	*B*
Fourth	*C*	*D*	*A*	*C*

For this example, there are four candidates (namely *A*, *B*, *C*, and *D*). There are a total of 15 voters. The number above the four different preference list ballots represents how many voters identified that particular column as the ordering of the candidates as their preference.

Throughout this chapter it will be assumed that there are an **odd number of voters** for any given discussion.

Section 9.1 Majority Rule and Condorcet's Method

🗝 Key idea

In a **dictatorship** all ballots except that of the dictator are ignored.

🗝 Key idea

In **imposed rule** candidate *X* wins regardless of who vote for whom.

🗝 Key idea

In **minority rule**, the candidate with the fewest votes wins.

🗝 Key idea

When there are only two candidates or alternatives, May's theorem states that **majority rule** is the only voting method that satisfies three desirable properties, given an odd number of voters and no ties.

Question 1

What are three properties satisfied by majority rule?

Answer

The three properties are:

1. All voters are treated equally.
2. Both candidates are treated equally.
3. If a single voter who voted for the loser, *B*, changes his mind and votes for the winner, *A*, then *A* is still the winner. This is what is called **monotone.**

Key idea

Condorcet's method declares a candidate is a winner if he or she can defeat every other candidate in a one-on-one competition using majority rule.

Example A

Determine if there is a winner using Condorcet's method. If so, who is it?

	Number of voters (11)			
Rank	**2**	**5**	**3**	**1**
First	*A*	*B*	*B*	*C*
Second	*B*	*A*	*C*	*A*
Third	*C*	*C*	*A*	*B*

Solution

You must determine the outcome of three one-on-one competitions. The candidates not considered in each one-on-one competition can be ignored.

A vs *B*

	Number of voters (11)			
Rank	**2**	**5**	**3**	**1**
First	*A*	*B*	*B*	*A*
Second	*B*	*A*	*A*	*B*

A: 2 + 1 = 3; ***B*: 5 + 3 = 8**

A vs *C*

	Number of voters (11)			
Rank	**2**	**5**	**3**	**1**
First	*A*	*A*	*C*	*C*
Second	*C*	*C*	*A*	*A*

***A*: 2 + 5 = 7**; *C*: 3 + 1 = 4

B vs *C*

	Number of voters (11)			
Rank	**2**	**5**	**3**	**1**
First	*B*	*B*	*B*	*C*
Second	*C*	*C*	*C*	*B*

***B*: 2 + 5 + 3 = 10**; *C*: 1

Since *B* can defeat both *A* and *C* in a one-on-one competition, *B* is the winner by the Condorcet's method.

Question 2

Determine if there is a winner using Condorcet's method. If so, who is it?

	Number of voters (15)			
Rank	**5**	**2**	**7**	**1**
First	*A*	*C*	*B*	*B*
Second	*B*	*B*	*C*	*A*
Third	*C*	*A*	*A*	*C*

Answer

B is the winner.

Question 3

In the following table, is there a Condorcet winner? If so, who is it?

	Number of voters (23)			
Rank	**3**	**8**	**7**	**5**
First	*A*	*B*	*C*	*D*
Second	*B*	*C*	*B*	*C*
Third	*C*	*A*	*A*	*B*
Fourth	*D*	*D*	*D*	*A*

Answer

C is the winner.

Key idea

Condorcet's voting paradox *can* occur with three or more candidates in an election where Condorcet's method yields no winners. For example, in a three-candidate race, two-thirds of voters could favor *A* over *B*, two-thirds of voters could favor *B* over *C*, and two-thirds of voters could favor *C* over *A*. This is the example given in the text. With three or more candidates, there are elections in which Condorcet's method yields no winners.

Example B

Does Condorcet's voting paradox occur in the following table?

	Number of voters (9)		
Rank	**3**	**4**	**2**
First	*A*	*B*	*C*
Second	*C*	*A*	*B*
Third	*B*	*C*	*A*

Solution

Yes, the Condorcet's voting paradox occurs. Voters prefer *A* over *C* (7 to 2). Voters prefer *C* over *B* (5 to 4). However, voters prefer *B* over *A* (6 to 3).

Example C

Does Condorcet's voting paradox occur in the following table?

	Number of voters (13)			
Rank	**5**	**2**	**4**	**2**
First	*A*	*B*	*C*	*D*
Second	*B*	*C*	*D*	*B*
Third	*D*	*A*	*A*	*A*
Fourth	*C*	*D*	*B*	*C*

Solution

The answer is no. Voters prefer *A* over *B* (9 to 4). Voters prefer *A* over *C* (7 to 6). Also, voters prefer *A* over *D* (7 to 6). *A* is the Condorcet winner. Therefore, there is no Condorcet paradox.

Question 4

Does Condorcet's voting paradox occur in the following table?

	Number of voters (23)			
Rank	**9**	**4**	**2**	**8**
First	*A*	*B*	*B*	*C*
Second	*B*	*C*	*A*	*A*
Third	*C*	*A*	*C*	*B*

Answer

The answer is yes.

Section 9.2 Other Voting Systems for Three of More Candidates

Key idea

In **plurality voting**, the candidate with the most first-place votes on the preference list ballots is the winner. We do not take into account the voters' preferences for the second, third, etc., places.

Example D

In the following table, who is the winner by plurality voting?

	Number of voters (23)			
Rank	**7**	**8**	**6**	**2**
First	*A*	*C*	*B*	*A*
Second	*B*	*B*	*C*	*C*
Third	*C*	*A*	*A*	*B*

Solution

Since we only need to consider first-place votes, we have the following.

	Number of voters (23)			
Rank	**7**	**8**	**6**	**2**
First	*A*	*C*	*B*	*A*

Thus, *A* has 7 + 2 = 9 first-place votes, *B* has 6 and *C* has 8. Thus, *A* is the winner.

Key idea

A voting system that satisfies the **Condorcet Winner Criterion (CWC)** either has no Condorcet winner or the voting produces exactly the same winner as does Condorcet's method.

Question 5

In the following table, who is the winner by plurality voting? Is there a winner by Condorcet's method? Is there a violation of CWC?

	Number of voters (27)				
Rank	**11**	**2**	**8**	**2**	**4**
First	*A*	*A*	*B*	*C*	*C*
Second	*B*	*C*	*C*	*A*	*B*
Third	*C*	*B*	*A*	*B*	*A*

Answer

The winner is *A*; There is no winner.; There is no violation.

Question 6

In plurality voting, must there always be a Condorcet winner?

Answer

The answer is no.

Key idea

In the **Borda count** method, points are assigned to each position in the set of preference lists. For example, in a 3-person election, first-place votes may be awarded 2 points each, second-place votes receive 1 point each, and third-place votes are given 0 points each. (Other distributions of points may be used to create similar rank methods.)

Example E

In the following table, who is the winner by Borda count?

	Number of voters (27)				
Rank	**11**	**2**	**8**	**2**	**4**
First	*A*	*A*	*B*	*C*	*C*
Second	*B*	*C*	*C*	*A*	*B*
Third	*C*	*B*	*A*	*B*	*A*

Solution

Preference	1^{st} place votes $\times$ 2	2^{nd} place votes $\times$ 1	3^{rd} place votes $\times$ 0	Borda score
A	13×2	2×1	12×0	28
B	8×2	15×1	4×0	31
C	6×2	10×1	11×0	22

The winner is *B.*

🗝 Key idea

Another method of determining Borda scores outlined by the text is to individually replace the candidates below the one you are determining the score for by a box. You then **count up the boxes**, being careful to take note of the number of voters in each column.

Example F

Use the preference list ballots from the previous example to show the Borda scores are 28 for *A*, 31 for *B*, and 22 for *C* using the box-replacement method.

Solution

For *A*:

	Number of voters (27)				
Rank	**11**	**2**	**8**	**2**	**4**
First	*A*	*A*	*B*	*C*	*C*
Second	□	□	*C*	*A*	*B*
Third	□	□	*A*	□	*A*

The Borda score for *A* is $(11\times 2)+(2\times 2)+(2\times 1)=22+4+2=28$.

For *B*:

	Number of voters (27)				
Rank	**11**	**2**	**8**	**2**	**4**
First	*A*	*A*	*B*	*C*	*C*
Second	*B*	*C*	□	*A*	*B*
Third	□	*B*	□	*B*	□

The Borda score for *B* is $(11\times 1)+(8\times 2)+(4\times 1)=11+16+4=31$.

For *C*:

	Number of voters (27)				
Rank	**11**	**2**	**8**	**2**	**4**
First	*A*	*A*	*B*	*C*	*C*
Second	*B*	*C*	*C*	□	□
Third	*C*	□	□	□	□

The Borda score for *C* is $(2\times 1)+(8\times 1)+(2\times 2)+(4\times 2)=2+8+4+8=22$.

🗝 Key idea

In the Borda count method, you can perform a check to make sure your calculations are correct. The number of points to be distributed per preference list ballot times the number of ballots (voters) is equal to the sum of the Borda scores. For example in the previous table, 3 points (2 + 1 + 0 = 3) were to be distributed among 3 candidates. There were 27 voters. The sum of the Borda scores was 28 + 31 + 22 = 81. This is the same as the product of 3 and 27. If there are four candidates, you will be distributing 6 points (3 + 2 + 1 + 0 = 6) using the point distribution of the text.

Question 7

In the following table, who is the winner by Borda count? What is the sum of all the Borda scores?

	Number of voters (37)					
Rank	**8**	**8**	**2**	**12**	**5**	**2**
First	*A*	*A*	*B*	*C*	*C*	*D*
Second	*B*	*D*	*C*	*B*	*D*	*A*
Third	*C*	*B*	*D*	*A*	*A*	*B*
Fourth	*D*	*C*	*A*	*D*	*B*	*C*

Answer

The winner is *A*.; 222

Key idea

A voting system satisfies **independence of irrelevant alternatives (IIA)** if it is impossible for a candidate *B* to move from nonwinner status to winner status unless at least one voter reverses the order in which he or she had *B* and the winning candidate ranked. The Borda count fails to satisfy IIA, as shown in the text.

Key idea

An agenda is the listing (in some order) of the candidates. **Sequential pairwise** voting pits the first candidate against the second in a one-on-one contest. The winner goes on to confront the third candidate on the agenda, while the loser is eliminated. The candidate remaining at the end is the winner. The choice of the agenda can affect the result.

Example G

Who is the winner with sequential pairwise voting with the agenda *B*, *C*, *A*?

	Number of voters (17)			
Rank	**1**	**5**	**4**	**7**
First	*A*	*A*	*B*	*C*
Second	*B*	*C*	*A*	*A*
Third	*C*	*B*	*C*	*B*

Solution

In sequential pairwise voting with the agenda *B*, *C*, *A*, we first pit *B* against *C*. There are 5 voters who prefer *B* to *C* and 12 prefer *C* to *B*. Thus, *C* wins by a score of 12 to 5. *B* is therefore eliminated, and *C* moves on to confront *A*. There are 7 voters who prefer *C* to *A* and 10 prefer *A* to *C*. Thus, *A* wins by a score of 10 to 7. Thus, *A* is the winner by sequential pairwise voting with the agenda *B*, *C*, *A*.

Question 8

Who is the winner with sequential pairwise voting with the agenda *A*, *B*, *C*, *D*? with agenda *B*, *D*, *C*, *A*?

	Number of voters (19)				
Rank	**5**	**3**	**2**	**1**	**8**
First	*A*	*A*	*B*	*C*	*D*
Second	*B*	*D*	*C*	*B*	*B*
Third	*D*	*B*	*A*	*A*	*A*
Fourth	*C*	*C*	*D*	*D*	*C*

Answer

D is the winner.; *A* is the winner.

Key idea

Sequential pairwise voting fails to satisfy the **Pareto condition**, which states that if everyone prefers one candidate, say *A*, to another, say *B*, then *B* cannot be the winner.

Key idea

In the **Hare system**, the winner is determined by repeatedly deleting candidates that are the least preferred, in the sense of being at the top of the fewest preference lists.

Example H

Who is the winner when the Hare system is applied?

	Number of voters (19)			
Rank	**3**	**5**	**4**	**7**
First	*A*	*A*	*B*	*C*
Second	*B*	*C*	*C*	*A*
Third	*C*	*B*	*A*	*B*

Solution

Since *B* has the least number of first-place votes, *B* is eliminated.

	Number of voters (19)			
Rank	**3**	**5**	**4**	**7**
First	*A*	*A*		*C*
Second		*C*	*C*	*A*
Third	*C*		*A*	

Candidates *A* and *C* move up as indicated to form a new table.

	Number of voters (19)			
Rank	**3**	**5**	**4**	**7**
First	*A*	*A*	*C*	*C*
Second	*C*	*C*	*A*	*A*

A now has 3 + 5 = 8 first-place votes. *C* now has 4 + 7 = 11 first-place votes. Thus, *C* is the winner by the Hare system.

Question 9

Who is the winner when the Hare system is applied?

	Number of voters (17)				
Rank	**5**	**2**	**6**	**3**	**1**
First	*A*	*B*	*C*	*D*	*B*
Second	*B*	*A*	*A*	*C*	*D*
Third	*C*	*C*	*B*	*B*	*C*
Fourth	*D*	*D*	*D*	*A*	*A*

Answer

C is the winner.

Question 10

Who is the winner when the Hare system is applied?

	Number of voters (21)				
Rank	**1**	**8**	**7**	**3**	**2**
First	*A*	*B*	*C*	*D*	*E*
Second	*B*	*E*	*A*	*A*	*A*
Third	*C*	*C*	*B*	*C*	*C*
Fourth	*D*	*D*	*D*	*B*	*B*
Fifth	*E*	*A*	*E*	*E*	*D*

Answer

C is the winner.

Key idea

The Hare system does not satisfy monotonicity.

Key idea

Plurality runoff is the voting system in which there is a runoff between the two candidates receiving the most first-place votes. In the case of ties between first or second, three candidates participate in the runoff.

Example I

Who is the winner when the plurality runoff method is applied?

	Number of voters (21)				
Rank	**7**	**3**	**2**	**1**	**8**
First	*A*	*B*	*C*	*D*	*D*
Second	*C*	*A*	*A*	*C*	*B*
Third	*B*	*C*	*B*	*B*	*C*
Fourth	*D*	*D*	*D*	*A*	*A*

Solution

Since *D* and *A* are in first and second-place, respectively, the runoff is between these two candidates.

	Number of voters (21)				
Rank	**7**	**3**	**2**	**1**	**8**
First	*A*	*A*	*A*	*D*	*D*
Second	*D*	*D*	*D*	*A*	*A*

A now has 7 + 3 + 2 = 12 first-place votes, and *D* has 1 + 8 = 9. Thus, *A* is the winner.

Question 11

Who is the winner when the plurality runoff method is applied?

	Number of voters (29)				
Rank	**9**	**6**	**7**	**2**	**5**
First	*A*	*B*	*C*	*D*	*D*
Second	*C*	*D*	*B*	*C*	*B*
Third	*B*	*A*	*A*	*A*	*C*
Fourth	*D*	*C*	*D*	*B*	*A*

Answer

D is the winner.

Key idea

The Plurality runoff method does not satisfy monotonicity.

Section 9.3 Insurmountable Difficulties: Arrow's Impossibility Theorem

Key idea

Arrow's impossibility theorem states that with three or more candidates and any number of voters, there does not exist (and will never exist) a voting system that produces a winner satisfying Pareto and independence of irrelevant alternatives (IIA), and is not a dictatorship.

Key idea

A weak version of Arrow's impossibility theorem states that with three or more candidates and an odd number of voters, there does not exist (and will never exist) a voting system that satisfies both the Condorcet winner criterion (CWC) and independence of irrelevant alternatives (IIA), and that always produces at least one winner every election.

Section 9.4 A Better Approach? Approval Voting

Key idea

In **approval voting**, each voter may vote for as many candidates as he or she chooses. The candidate with the highest number of approval votes wins the election.

Example J

There are 7 voters in a committee. Who is the winner in the following table, where X indicates that the voter approves of that particular candidate? How would they be ranked?

	Voters						
Candidates	**1**	**2**	**3**	**4**	**5**	**6**	**7**
A		X	X			X	
B	X	X			X	X	
C			X				X
D	X	X		X	X		X
E		X		X		X	

Solution

A has 3 approval votes, *B* has 4, *C* has 2, *D* has 5, *E* has 3. Since *D* has the most approval votes, *D* is the winner. Ranking the candidates we have, *D* (5), *B* (4), *A* and *E* (3), and *C* (2).

Example K

There are 25 voters in a committee. Who is the winner in the following table, where X indicates that the voter approves of that particular candidate? How would they be ranked?

	Number of voters (25)						
Nominee	**5**	**8**	**6**	**2**	**1**	**1**	**2**
A	X			X	X		X
B		X		X		X	X
C	X		X			X	X

Solution

A has $5+2+1+2=10$ approval votes. *B* has $8+2+1+2=13$ approval votes. *C* has $5+6+1+2=14$ approval votes. Since *C* has the most approval votes, *C* is the winner. Ranking the candidates we have, *C* (14), *B* (13), and *A* (10).

Question 12

There are 71 voters in a committee. Who is the winner in the following table, where X indicates that the voter approves of that particular candidate? How would they be ranked?

	Number of voters (71)							
Nominee	**8**	**9**	**2**	**12**	**10**	**14**	**12**	**4**
A	X			X	X			X
B		X		X		X		X
C						X		X
D		X		X		X		X
E	X	X	X		X	X		X

Answer

E is the winner.; Ranking is *E*, *B* and *D* (tie), *A*, and *C*.

Homework Help

Exercises 1 – 3
Carefully read Section 9.1 before responding to these exercises. Consider all three desirable properties for each exercise.

Exercise 4
Consider 4 voters (an even number of voters) and 2 candidates. Name one of the voters to distinguish him or her from the others.

Exercise 5
Consider what cannot occur when you have an odd number of voters.

Exercise 6
Consider preference choices like softdrinks, courses, makes of cars, etc.

Exercise 7
(a) Check the one-on-one scores of *D* versus *H*, *D* versus *J*, and *H* versus *J*.
(b) The plurality winner would be the candidate with the highest percentage of votes in this exercise.

Exercises 8, 9, and 12
(a) The plurality winner would be the candidate with the highest number of votes in these exercises.
(b) Use the following table for these exercises unless the "counting box" method is preferred. Be sure to check the total of the Borda scores after you have completed the table.

Preference	1st place votes × 3	2nd place votes × 2	3rd place votes × 1	4th place votes × 0	Borda score
A	×3	×2	×1	×0	
B	×3	×2	×1	×0	
C	×3	×2	×1	×0	
D	×3	×2	×1	×0	

(c) Eliminate the candidate(s) with the least number of first-place votes. Repeat if necessary until there are two candidates. The one with the majority of votes wins.
(d) The exercises have different agendas. Be sure to pay attention to the agenda in determining the three one-on-one competitions.

Exercises 10, 11, and 13
(a) The plurality winner would be the candidate with the highest number of votes in these exercises.
(b) Use the following table for these exercises unless the "counting box" method is preferred. Be sure to check the total of the Borda scores after you have completed the table.

Preference	1st place votes × 4	2nd place votes × 3	3rd place votes × 2	4th place votes × 1	5th place votes × 0	Borda score
A	×4	×3	×2	×1	×0	
B	×4	×3	×2	×1	×0	
C	×4	×3	×2	×1	×0	
D	×4	×3	×2	×1	×0	
E	×4	×3	×2	×1	×0	

(c) Eliminate the candidate(s) with the least number of first-place votes. Repeat if necessary until there are two candidates. The one with the majority of votes wins.
(d) The exercises have different agendas. Be sure to pay attention to the agenda in determining four one-on-one competitions.

Exercise 14

(a) The plurality winner would be the candidate with the highest number of votes in these exercises.

(b) Use the following table for these exercises unless the "counting box" method is preferred. Be sure to check the total of the Borda scores after you have completed the table.

Preference	1st place votes × 4	2nd place votes × 3	3rd place votes × 2	4th place votes × 1	5th place votes × 0	Borda score
A	×4	×3	×2	×1	×0	
B	×4	×3	×2	×1	×0	
C	×4	×3	×2	×1	×0	
D	×4	×3	×2	×1	×0	
E	×4	×3	×2	×1	×0	

(c) This exercise has *A*, *B*, *C*, *D*, *E* as its agenda. Be sure to pay attention to the agenda in determining four one-on-one competitions.

(d) Eliminate the candidate(s) with the least number of first-place votes. Repeat if necessary until there are two candidates. The one with the majority of votes wins.

Exercise 15

(a) You should find the following tables helpful in the elimination process. Make sure you use the Coombs procedure.

	Number of voters (7)						
Rank	**1**	**1**	**1**	**1**	**1**	**1**	**1**
First							
Second							
Third							
Fourth							

	Number of voters (7)						
Rank	**1**	**1**	**1**	**1**	**1**	**1**	**1**
First							
Second							
Third							

	Number of voters (7)						
Rank	**1**	**1**	**1**	**1**	**1**	**1**	**1**
First							
Second							

(b) Consider one possible scenario is having candidates *A*, *B*, and *C*. Fill in the preference list ballots and determine the winner using both the Coombs procedure and the Hare method. Make sure your example fits the criteria in the exercise.

	Number of voters ( )
Rank	
First	
Second	
Third	

Exercise 16

(a) Read carefully the voting procedure known as the Condorcet's method (Section 9.1) and the Pareto condition (Section 9.2).

(b) Read carefully the voting procedure known as the Condorcet's method (Section 9.1) and the monotonicity condition (Section 9.2).

Exercise 17
(a) Read carefully the voting procedure known as plurality voting (Section 9.2) and the Pareto condition (Section 9.2).
(b) Read carefully the voting procedure known as plurality voting (Section 9.2) and the monotonicity condition (Section 9.2).

Exercise 18
(a) Read carefully the voting procedure known as the Borda method (Section 9.2) and the Pareto condition (Section 9.2).
(b) Read carefully the voting procedure known as the Borda method (Section 9.2) and the monotonicity condition (Section 9.2).

Exercise 19
(a) Read carefully the voting procedure known as the sequential pairwise voting method (Section 9.2) and the Condorcet winner criterion condition (Section 9.2).
(b) Read carefully the voting procedure known as the sequential pairwise voting method (Section 9.2) and the monotonicity condition (Section 9.2).

Exercise 20
Read carefully the voting procedure known as the Hare system (Section 9.2) and the Pareto condition (Section 9.2).

Exercise 21
Read carefully the voting procedure known as the plurality runoff method (Section 9.2) and the Pareto condition (Section 9.2).

Exercise 22
Determine the winner using the plurality runoff method. Determine if there is a Condorcet winner. Discuss the outcome.

Exercise 23
Adjust the table in such a way that the change is favorable to the winning candidate. Determine the winner using the plurality runoff method on the adjusted table. Discuss the outcome.

Exercise 24
(a) Determine the winner using plurality voting in the first election, then in the second. Determine the changes that were made from the first to the second election and the effect on the second election. Be sure to have reviewed the independence of irrelevant alternatives (Section 9.2) before making your observations.
(b) Determine the winner using the Hare system in the first election, then in the second. Determine the changes that were made from the first to the second election and the effect on the second election. Be sure to have reviewed the independence of irrelevant alternatives (Section 9.2) before making your observations.

Exercise 25
Try to keep your example simple, say with three candidates and a relatively small number of voters.

	Number of voters ( )
Rank	
First	
Second	
Third	

Determine the winner using the Borda method.

Preference	1st place votes × 2	2nd place votes × 1	3rd place votes × 0	Borda score
A	×2	×1	×0	
B	×2	×1	×0	
C	×2	×1	×0	

Now use Condorcet's method with your preference list ballots. You have shown the Borda count does not satisfy the Condorcet winner criterion if you obtain two different winners.

Exercise 26
Determine the winner using the Hare system. You should find the following table helpful in the elimination process.

	Number of voters (17)			
Rank	**7**	**5**	**4**	**1**
First				
Second				
Third				

Alter the original table to an outcome favorable to the winner.

	Number of voters (17)			
Rank	**7**	**5**	**4**	**1**
First				
Second				
Third				
Fourth				

Using the Hare system again on the altered table.

	Number of voters (17)			
Rank	**7**	**5**	**4**	**1**
First				
Second				
Third				

	Number of voters (17)			
Rank	**7**	**5**	**4**	**1**
First				
Second				

You will have demonstrated nonmonotonicity if the outcome of the altered table is not the same as the original winner, even though the alteration was favorable to the original winner.

Exercise 27

(a) Determine the winner using the Hare system. You should find the following tables helpful in the elimination process.

	Number of voters (21)			
Rank	**7**	**6**	**5**	**3**
First				
Second				
Third				

	Number of voters (21)			
Rank	**7**	**6**	**5**	**3**
First				
Second				

(b) Alter the table as described in the exercise.

	Number of voters (21)			
Rank	**7**	**6**	**5**	**3**
First				
Second				
Third				
Fourth				

Determine the winner again using the Hare system. You should find the following tables helpful in the elimination process.

	Number of voters (21)			
Rank	**7**	**6**	**5**	**3**
First				
Second				
Third				

	Number of voters (21)			
Rank	**7**	**6**	**5**	**3**
First				
Second				

Exercise 28

(a) Read carefully the voting procedure known as sequential pairwise voting (Section 9.2) and consider what cannot occur when you have an odd number of voters.

(b) Read carefully the voting procedure known as the Hare system (Section 9.2) and consider what cannot occur when you have an odd number of voters.

Exercise 29

Consider constructing preference list ballots that rank a candidate (say *C*) last on a majority of ballots (over 50% of the votes are for last-place). Preference list ballots that have three candidates (say *A*, *B*, and *C*) and a relatively small number of voters (say 11) will suffice. Determine what occurs using the plurality runoff method. Discuss in general what would occur if a candidate was to be ranked last on a majority of votes. Note that it is assumed that there are no ties for first or second place in this exercise.

Exercise 30

Consider constructing preference list ballots that rank a candidate with three candidates (say *A*, *B*, and *C*) and a relatively small number of voters. Force one candidate to have the majority of the last-place votes, but is the winner by the plurality voting (i.e. has the highest number of first-place votes).

	Number of voters ( )		
Rank	**3**	**2**	**2**
First			
Second			
Third			

Exercise 31

Follow the argument in the text in Section 9.3. Consider starting with the following preference list ballots for the case involving *B*.

Rank	**Number of voters (3)**		
First	*A*	*B*	*A*
Second	*B*	*C*	*C*
Third	*C*	*A*	*B*

Consider starting with the following preference list ballots for the case involving *C*.

Rank	**Number of voters (3)**		
First	*B*	*B*	*C*
Second	*A*	*C*	*A*
Third	*C*	*A*	*B*

Exercise 32

In the table, the numbers 1 – 10 identify the ten board members. For each candidate (*A* – *H*) count horizontally the number of X marks, then answer the questions, Parts a – d.

Exercise 33

Similar to the preference list ballots, the numbers in the second row correspond to how many voters chose to approve a combination of nominees. If a nominee has an X under a number of voters, you add the number of voters indicated to find the total number of approval votes out of the possible 45. For example, *A* has $7+9+6+1=23$ approval votes. Find the number of approval votes for the other two nominees in order to answer Parts a and b. In Part c, consider what effect the last two columns have on the outcome.

Do You Know the Terms?

Cut out the following 19 flashcards to test yourself on Review Vocabulary. You can also find these flashcards at http://www.whfreeman.com/fapp7e.

Chapter 9 **Social Choice: The Impossible Dream** **Agenda**	**Chapter 9** **Social Choice: The Impossible Dream** **Approval voting**
Chapter 9 **Social Choice: The Impossible Dream** **Arrow's impossibility theorem**	**Chapter 9** **Social Choice: The Impossible Dream** **Borda count**
Chapter 9 **Social Choice: The Impossible Dream** **Condorcet's Method**	**Chapter 9** **Social Choice: The Impossible Dream** **Condorcet winner**
Chapter 9 **Social Choice: The Impossible Dream** **Condorcet winner criterion (CWC)**	**Chapter 9** **Social Choice: The Impossible Dream** **Condorcet's voting paradox**
Chapter 9 **Social Choice: The Impossible Dream** **Hare system**	**Chapter 9** **Social Choice: The Impossible Dream** **Independence of irrelevant alternatives (IIA)**

A method of electing one or more candidates from a field of several in which each voter submits a ballot that indicates which candidates he or she approves of. Winning is determined by the total number of approvals a candidate obtains.	**An ordering of the candidates for consideration. Often used in sequential pairwise voting.**
A voting system for elections with several candidates in which points are assigned to voters' preferences and these points are summed for each candidate to determine a winner.	**Kenneth J. Arrow's discovery that any voting system can give undesirable outcomes.**
A Condorcet winner in an election is a candidate who, based on the ballots, would have defeated every other candidate in a one-on-one contest.	**A voting system for elections with several candidates in which a candidate is a winner precisely when he or she would, on the basis of the ballots, defeat every other candidate on a one-on-one contest.**
There are elections in which Condorcet's method yields no winner.	**A voting system satisfies the Condorcet winner criterion if, for every election in which there is a Condorcet winner, it wins the election when that voting system is used.**
A voting system satisfies independence of irrelevant alternatives if the only way a candidate (call him A) can go from losing one election to being among the winners of a new election (with the same set of candidates and voters) is for at least one voter to reverse his or her ranking	**A voting system for elections with several candidates in which candidates are successively eliminated in an order based on the number of first-place votes.**

Chapter 9 **Social Choice: The Impossible Dream** **Manipulability**	**Chapter 9** **Social Choice: The Impossible Dream** **Majority rule**
Chapter 9 **Social Choice: The Impossible Dream** **May's theorem**	**Chapter 9** **Social Choice: The Impossible Dream** **Monotonicity**
Chapter 9 **Social Choice: The Impossible Dream** **Pareto condition**	**Chapter 9** **Social Choice: The Impossible Dream** **Plurality runoff**
Chapter 9 **Social Choice: The Impossible Dream** **Plurality voting**	**Chapter 9** **Social Choice: The Impossible Dream** **Preference list ballot**
Chapter 9 **Social Choice: The Impossible Dream** **Sequential pairwise voting**	

A voting system for elections with two candidates (and an odd number of voters) in which the candidate preferred by more than half the voters is the winner.	**A voting system is subject to manipulability id there are elections in which it is to a voter's advantage to submit a ballot that misrepresents his or her true preferences.**
A voting system satisfies monotonicity provided that ballot changes favorable to one candidate (and not favorable to any other candidate) can never hurt that candidate.	**Kenneth May's discovery that, for two alternatives and an odd number of voters, majority rule is the only voting system satisfying three natural properties.**
A voting system for elections with several candidates in which, assuming there are no ties, there is a runoff between the two candidates receiving the most first-place votes.	**A voting system satisfies the Pareto condition provided that every voter's ranking of one candidate higher than another precludes the possibility of this latter candidate winning.**
A ballot that ranks the candidates from most preferred to least preferred with no ties	**A voting system for elections with several candidates in which the candidate with the most first-place votes wins.**
	A voting system for elections with several candidates in which one starts with an agenda and pits the candidates against each other in one-on-one contests (based on preference list ballots), with losers being eliminated as one moves along the agenda.

Practice Quiz

1. Majority rule is an effective way to make a choice between

a. two alternatives.

b. a small number of alternatives.

c. any number of alternatives.

2. The first-place votes for each of four candidates are counted, and the candidate with the most votes wins. This voting system is an example of

a. majority rule.

b. approval voting.

c. plurality voting.

3. Each voter ranks the four candidates. The candidate who is ranked above any of the other candidates by a majority of the voters is declared to be the winner. This is an example of

a. Condorcet winner criterion.

b. Borda count.

c. Hare system.

4. 11 committee members need to elect a chair from the candidates *A*, *B*, *C*, and *D*. The preferences of the committee members are given below. Which candidate will be selected if they use majority rule?

	Number of voter members (11)		
Rank	**6**	**2**	**3**
First	*A*	*B*	*C*
Second	*B*	*C*	*D*
Third	*C*	*D*	*B*
Fourth	*D*	*A*	*A*

a. *A*

b. *B*

c. *C*

5. 11 committee members need to elect a chair from the candidates *A*, *B*, *C*, and *D*. The preferences of the committee members are given below. Which candidate will be selected if they use a Borda count?

	Number of voter members (11)		
Rank	**6**	**2**	**3**
First	*A*	*B*	*C*
Second	*B*	*C*	*D*
Third	*C*	*D*	*B*
Fourth	*D*	*A*	*A*

a. *A*

b. *B*

c. *C*

6. 11 committee members need to elect a chair from the candidates *A*, *B*, *C*, and *D*. The preferences of the committee members are given below. Which candidate will be selected if they use the plurality runoff?

	Number of voter members (11)		
Rank	**6**	**2**	**3**
First	*A*	*B*	*C*
Second	*B*	*C*	*D*
Third	*C*	*D*	*B*
Fourth	*D*	*A*	*A*

a. *A*

b. *B*

c. *C*

7. 37 members must elect a club president. Preferences among candidates *A*, *B*, *C*, and *D* are given below. Which candidate wins under the Hare system?

	Number of voter members (37)				
Rank	**14**	**10**	**8**	**4**	**1**
First	*A*	*C*	*D*	*B*	*A*
Second	*B*	*B*	*C*	*D*	*D*
Third	*C*	*D*	*B*	*C*	*B*
Fourth	*D*	*A*	*A*	*A*	*C*

a. *A*

b. *B*

c. *D*

8. 37 members must elect a president of their club. Preferences among candidates *A*, *B*, and *C* are given below. Which candidate is the Condorcet winner?

	Number of voter members (37)		
Rank	**14**	**11**	**12**
First	*A*	*B*	*C*
Second	*B*	*C*	*B*
Third	*C*	*A*	*A*

a. *A*

b. *B*

c. *C*

9. 25 partygoers have enough money together for a one-topping super-size party pizza. They each mark what toppings they find acceptable, as shown below. Which topping will be selected using approval voting?

	Voters				
Toppings	**8**	**6**	**4**	**4**	**3**
pepperoni		X	X	X	
mushrooms				X	X
anchovies	X		X		X

a. pepperoni

b. mushrooms

c. anchovies

10. Which voting method satisfies both the Condorcet winner criterion and the independence of irrelevant alternatives?

I. Condorcet method II. Plurality

a. Only I

b. Only II

c. Neither I nor II

Word Search

Refer to pages 361 – 362 of your text to obtain the Review Vocabulary. There are 17 hidden vocabulary words/expressions in the word search below. *Condorcet's method* and *Manipulability* do not appear. It should be noted that spaces are removed as well as apostrophes. Also, the abbreviations for Condorcet winner criterion and Independence of Irrelevant alternatives were used in the word search.

P L U R A L I T Y R U N O F F A I K X A S T F S S N
P L B E K P F I E G U C R P S E Y N I G X I U C Z C
R R I N M B L S G J M J J O G D C S A E H I D S Q X
A M B N I C S Y S B P H H N O P C M R N X A S J N K
P L A I V Z E P V T F L H X M A H E T D E M I E K A
S N A W Y K A F E X I S S D M N H E O A A N N N T M
G N I T O V L A V O R P P A T N U O C A D R O B A S
O B Q E C N G S N E E E J K H I G S S R M T F P G Y
A N Z C S W R M M O N O T O N I C I T Y F J O T A T
I Q F R E W C E P I R G N I T O V Y T I L A R U L P
M E R O E H T Y T I L I B I S S O P M I S W O R R A
C O N D O R C E T S V O T I N G P A R A D O X Y X R
P L A N R M A Y S T H E O R E M O C I O G E G W N E
T E P O B L R S E T V V S V N D S T L S G I O P R T
Z E G C R U M F B R S B T D R X R G H M M L D H D O
R S H F L A V I T R H D L P M J T M T Z N U E A F C
A L I E D A R C F E E P S T A A B P O S E S U R Q O
T O L L A B T S I L E C N E R E F E R P E K C E N N
G N I T O V E S I W R I A P L A I T N E U Q E S R D
J C Q C X T L N C A Y P O Y C E X F S N V E N Y L I
R X S N S T O K T N I H I O A C N M S N U A I S A T
F M M R C I N E T N P Z E N E C H W F S V P E T F I
F R R A P E X E R B F P O G T D U I I Q H A E E T O
N B S N A O H M E I N T E E O X L F U Q C E R M D N
C K U C N Z P T E P N E N E W H X N Y T P T N X A N
X W I Z N E H L Q A L H F D O O A T S J O Y J A J M

1. _____
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17. _____