Chapter 23
The Economics of Resources

Solutions

Exercises:

1. In late summer 2023

3. population in mid-2025 is as follows.

$$\left(\text{population in mid-2007}\right)\times\left(1+\text{growth rate}\right)^{18}=4.056\left(1+0.017\right)^{18}\text{ billion}\approx\ 5.49\text{ billion}$$

5. The population of Africa would be $925\times\left(1.024\right)^{18}=1418$ million, almost 100% greater than, or twice as large, as Europe's population.

7. (a) $\frac{70}{0.6}\approx 117$ years

(b) $\frac{70}{1.3}\approx 54$ years

9. (a) population in mid-2025 is as follows.

$$\left(\text{population in mid-2007}\right)\times\left(1+\text{growth rate}\right)^{18}=6.593\left(1+0.013\right)^{18}\text{ billion}\approx\ 8.3\text{ billion}$$

population in mid-2050 is as follows.

$$\left(\text{population in mid-2007}\right)\times\left(1+\text{growth rate}\right)^{43}=6.593\left(1+0.013\right)^{43}\text{ billion}\approx\ 11.5\text{ billion}$$

(b) No change in growth rate, no change in death rates, no global catastrophes, etc.

11. (a) The static reserve will be $\frac{2934.8}{77.9}\approx 38$ years.

(b) The exponential reserve will be $\frac{\ln\left[1+\left(\frac{2934.8}{77.9}\right)\left(0.019\right)\right]}{\ln\left[1+0.019\right]}\approx 29$ years.

(c) Answers will vary.

13. (a) The static reserve will be 100 years. We are seeking the exponential reserve. This will be $\frac{\ln\left[1+100\left(0.025\right)\right]}{\ln\left[1+0.025\right]}\approx 51$ years.

(b) $\frac{\ln\left[1+1000\left(0.025\right)\right]}{\ln\left[1+0.025\right]}\approx 132$ years

(c) $\frac{\ln\left[1+10,000\left(0.025\right)\right]}{\ln\left[1+0.025\right]}\approx 224$ years

15. (a) $\dfrac{\ln\left[1-100(0.005)\right]}{\ln[1-0.005]} \approx 138$ years

(b) $\dfrac{\ln\left[1-100(0.01)\right]}{\ln[1-0.01]} = \dfrac{\ln(1-1)}{\ln(0.99)} = \dfrac{\ln 0}{\ln[0.99]}$

This theoretically would imply forever!

17. $\dfrac{437 \times 10^9 \text{ tons}}{100 \times 10^6 \text{ plants} \times 800 \text{ years}} \approx$ 5.5 tons/plant/year ≈ 30 lb/plant/day, which is unreasonable.

19. $\dfrac{1}{100}\ln\left(\dfrac{62.95}{3.93}\right) \approx 2.77\%$

21. After the first year, the population stays at 15.

23. 7, 18.2, 6.6, 17.6, 8.4, 19.5, 2.0, 7.3, 18.6, 5.3

25. We must have $f(x_n) = x_n$, or $4x_n(1-0.05x_n) = x_n$. The only solutions are $x_n = 0$ and $4(1-0.05x_n) = 1$, or $x_n = 15$.

27. Using $x_{n+1} = f(x_n) = 3x_n(1-0.05x_n)$ with $x_1 = 10$ we have the following (rounded).

10, 15.0, 11.3, 14.8, 11.6, 14.6, 11.8, 14.5, 11.9, 14.4

The population is oscillating but slowly converging to $\frac{40}{3} \approx 13.3$.

29. We must have $f(x_n) = x_n$, or $3x_n(1-0.05x_n) = x_n$. The only solutions are $x_n = 0$ and $3(1-0.05x_n) = 1$, or $x_n = \frac{40}{3} \approx 13.3$.

31. The red dashed line indicates the same size population next year as this year; where it intersects the blue curve is the equilibrium population size.

33. Using $x_{n+1} = f(x_n) = 1.5x_n(1-0.025x_n)$ with $x_1 = 11$ we have the following (rounded).

11, 12.0, 12.6, 12.9, 13.1, 13.2, 13.3, 13.3, 13.3, 13.3

35. The population sizes are 11, 15.0, 13.7, 14.9, 14.9, 15.0, 14.8, 14.3, 13.0, 9.5 – and the following year the population is wiped out.

37. About 15 million pounds. Maximum sustainable yield is about 35 million pounds for an initial population of 25 million pounds.

39. (a) The last entry shown for the first sequence is the fourth entry of the second sequence, so the first “joins” the second and they then both end up going through the same cycle (loop) of numbers over and over.

(b) 39, 78, 56, and we have “joined” the second sequence. However, an initial 00 stays 00 forever; and any other initial number ending in 0 “joins” the loop sequence 20, 40, 80, 60, 20, ...

(c) Regardless of the original number, after the second push of the key we have a number divisible by 4, and all subsequent numbers are divisible by 4. There are 25 such numbers between 00 and 99. You can verify that an initial number either joins the self-loop 00 (the only such numbers are 00, 50, and 25); joins the loop 20, 40, 80, 60, 20, . . . (the only such are the multiples of 5 other than 00, 50 and 25); or joins the big loop of the other 20 multiples of 4.

41. (a) 133, 19, 82, 68, 100, 1, 1, ... The sequence stabilizes at 1.

(b) Answers will vary.

(c) That would trivialize the exercise!

(d) For simplicity, limit consideration to 3-digit numbers. Then the largest value of *f* for any 3-digit number is $9^2 + 9^2 + 9^2 = 243$. For numbers between 1 and 243, the largest value of *f* is $1^2 + 9^2 + 9^2 = 163$. Thus, if we iterate *f* over and over $-$ say 164 times $-$ starting with any number between 1 and 163, we must eventually repeat a number, since there are only 163 potentially different results. And once a number repeats, we have a cycle. Thus, applying *f* to any 3-digit number eventually produces a cycle. How many different cycles are there? That we leave you to work out.

Hints: 1) There aren't very many cycles.

2) There is symmetry in the problem, in that some pairs of numbers give the same result; for example, $f(68) = f(86)$.

43. (a) 0.0397, 0.15407173, 0.545072626, 1.288978, 0.171519142, 0.59782012, 1.31911379, 0.0562715776, 0.215586839, **0.722914301**, 1.32384194, 0.0376952973, 0.146518383, 0.521670621, 1.27026177, 0.240352173, 0.78810119, 1.2890943, 0.171084847, **0.596529312**

(b) **0.723**, 1.323813, 0.0378094231, 0.146949035, 0.523014083, 1.27142514, 0.236134903, 0.777260536, 1.29664032, 0.142732915, **0.509813606**

(c) **0.722**, 1.324148, 0.0364882223, 0.141958718, 0.507378039, 1.25721473, 0.287092278, 0.901103183, 1.16845189, **0.577968093**

45. Period 2 begins at $\lambda = 3$, period 4 at $1 + \sqrt{6} \approx 3.449$, period 8 at 3.544, period 3 at $1 + 2\sqrt{2} \approx 3.828$, and chaotic behavior onsets at about 3.57.

See http://www.answers.com/topic/logistic-map .