Chapter 22
Borrowing Models

Solutions

Exercises:

1. (a) For the 11.25%, a cost of \$15,529 has an annual yield of $\frac{0.0551(\$10{,}000)}{\$9802} = 5.62\%$.

For the 5.51%, a cost of \$9802 has an annual yield of $\frac{0.1125(\$10{,}000)}{\$15{,}529} = 7.24\%$.

(b) Answers will vary but should remark that the first bond locks in the interest rate much farther into the future.

3. We add on interest of $4 \times 0.059 \times \$5000 = \$1180$.

The monthly payment is $\frac{\$5000 + \$1180}{48} = \$128.75$.

5. To realize the \$3000 that you need, you need to borrow $\frac{\$3000}{1-0.09\times 4} = \$4687.50$, for which the monthly payment is $\frac{\$4687.50}{48} = \$97.66$. If you do just the \$3000 discounted loan, the lender gives you just $\$3000(1-0.09\times 4) = \$1920$, on which the monthly payment is $\frac{\$1920}{48} = \$40.00$.

7. Answers will vary but should conclude that the add-on loan always has a lower payment.

9. All but a tiny amount after 91 months

11. After 203 months (more than 16 years!), the balance is \$500.16.

13. 294 months (= 24.5 years!), plus a few cents in the 295th month

15. $\frac{\$24{,}995 - \$2000}{60} = \frac{\$22{,}995}{60} = \$383.25$

17. The amortization formula gives \$19.28 per month for each \$1000 financed. Either I miscopied, or the difference is probably due to what is considered a month (30 days?) and what method is used to calculate the monthly interest rate.

19. Use the amortization formula $d = \frac{Ai}{1-(1+i)^{-n}}$, with A = \$100,000, $i = \frac{0.065}{12}$, and $n = 30\times 12 = 360$.

$$d = \frac{Ai}{1-(1+i)^{-n}} = \frac{\$100{,}000\times \frac{0.065}{12}}{1-\left(1+\frac{0.065}{12}\right)^{-360}} = \$632.07$$

21. Use the amortization formula $d=\dfrac{Ai}{1-(1+i)^{-n}}$, with A = \$100,000, $i=\frac{0.06125}{12}$, and $n=15\times12=180.$

$$d=\frac{Ai}{1-(1+i)^{-n}}=\frac{\$100,000\times\frac{0.06125}{12}}{1-\left(1+\frac{0.06125}{12}\right)^{-180}}=\$850.62$$

23. Use the amortization formula $A=d\left[\dfrac{1-(1+i)^{-n}}{i}\right]$, with d = \$632.07, $i=\frac{0.065}{12}$, and $n=360-5\times12=360-60=300.$

$$A=d\left[\frac{1-(1+i)^{-n}}{i}\right]=\$632.07\left[\frac{1-\left(1+\frac{0.065}{12}\right)^{-300}}{\frac{0.065}{12}}\right]=\$93,611.27$$

Thus, the amount of equity is $\$100,000-\$93,611.27=\$6388.73.$

25. Use the amortization formula $A=d\left[\dfrac{1-(1+i)^{-n}}{i}\right]$, with d = \$850.62, $i=\frac{0.06125}{12}$, and $n=180-5\times12=180-60=120.$

$$A=d\left[\frac{1-(1+i)^{-n}}{i}\right]=\$850.62\left[\frac{1-\left(1+\frac{0.06125}{12}\right)^{-120}}{\frac{0.06125}{12}}\right]=\$76,186.80$$

Thus, the amount of equity is $\$100,000-\$76,186.80=\$23,813.20.$

27. Using Solver in Excel or otherwise, we get an annual rate of 3.68%.

29. Use the amortization formula $d=\dfrac{Ai}{1-(1+i)^{-n}}$, with A = \$180,000, $i=\frac{0.0675}{12}$, and $n=30\times12=360.$

The monthly payment would be $d=\dfrac{Ai}{1-(1+i)^{-n}}=\dfrac{\$180,000\times\frac{0.0675}{12}}{1-\left(1+\frac{0.0675}{12}\right)^{-360}}=\$1167.48.$

31. We must first solve $3000=146.25\left[\dfrac{1-(1+i)^{-24}}{i}\right]$. Using a spreadsheet or calculator, we have $i=0.01296119.$ Thus, APR is approximately $12\times0.01296119=15.55\%.$ The EAR is $(1+0.01296119)^{12}-1=16.71\%.$

33. We must first solve $3000=86.96\left[\dfrac{1-(1+i)^{-60}}{i}\right]$. Using a spreadsheet or calculator, we have $i=0.02031339.$ Thus, APR is approximately $12\times0.02031339=24.37\%.$ The EAR is $(1+0.02031339)^{12}-1=27.29\%.$

35. The principal is \$300 − \$54 = \$246 and the (simple) interest over the two weeks is \$54, so the interest rate is $100\% \times \frac{\$54}{\$246} = 21.95\%$ for 2 weeks, for an annual rate of $\frac{52}{2} \times 21.95\% = 571\%$. For a 365-day year, we get a daily rate of $\frac{21.95\%}{14} = 1.57\%$ per day and an annual percentage rate of 365 × 1.57% = 572%.

37. The principal is \$1500 − \$88 = \$1412 and the (simple) interest over the week is \$88, so the interest rate is $100\% \times \frac{\$88}{\$1412} = 6.23\%$ for 1 week, for an annual rate of 52 × 6.23% = 324%. For a 365-day year, we get a daily rate of $\frac{6.23\%}{7} = 0.89\%$ per day and an annual percentage rate of 365 × 0.89% = 325%.

39. (a) Use the amortization formula $d = \frac{Ai}{1-(1+i)^{-n}}$, with A = \$100,000, $i = \frac{0.08375}{12}$, and $n = 30 \times 12 = 360$.

$$d = \frac{Ai}{1-(1+i)^{-n}} = \frac{\$100{,}000 \times \frac{0.08375}{12}}{1-\left(1+\frac{0.08375}{12}\right)^{-360}} = \$760.07$$

(b) Use the amortization formula $A = d\left[\frac{1-(1+i)^{-n}}{i}\right]$, with d = \$760.07, $i = \frac{0.08375}{12}$, and $n = 360 - 5 \times 12 = 360 - 60 = 300$.

$$A = d\left[\frac{1-(1+i)^{-n}}{i}\right] = \$760.07\left[\frac{1-\left(1+\frac{0.08375}{12}\right)^{-300}}{\frac{0.08375}{12}}\right] = \$95{,}387.80$$

Thus, the amount of equity is $\$100{,}000 - \$95{,}387.80 = \$4612.20$.

(c) Use the amortization formula $d = \frac{Ai}{1-(1+i)^{-n}}$, with A = \$95,387.80, $i = \frac{0.070}{12}$, and $n = 30 \times 12 = 360$.

$$d = \frac{Ai}{1-(1+i)^{-n}} = \frac{\$95{,}387.80 \times \frac{0.070}{12}}{1-\left(1+\frac{0.070}{12}\right)^{-360}} = \$634.62$$

(d) Since the difference between the two payments is $\$760.07 - \$634.62 = \$125.45$, it would take $\frac{\$2000}{\$125.45} \approx 15.9426$ or 16 months.

41. (a) \$100,250

(b) The payment is \$648.60.
The interest for one-twelfth of the year is $\$100{,}000 \times \frac{0.07}{12} = \$583.33$, so the payment on the principal is just \$65.27.

(c) \$583.33 − 250.00 = \$333.33.

(d) The inflation adjusted cost is $\frac{\$333.33}{1 + \frac{0.03}{12}} = \$332.50$; the interest rate for the month is $\frac{\$332.50}{\$100{,}000} = 0.0033250$, which is an annual rate of $12(0.0033250) = 3.99\%$. We have left out any costs (such as realtor's fee) involved in the sale.

43. There would be $\frac{\$81.6 \text{ million}}{30} = \$2.72$ million paid annually (including immediately).

$$2.72 + 2.72\left(\frac{1}{1.07}\right) + 2.72\left(\frac{1}{1.07}\right)^2 + \ldots + 2.72\left(\frac{1}{1.07}\right)^{29}$$

This is a geometric series. From Chapter 21, the sum will be $2.72\frac{\left(\frac{1}{1.07}\right)^{30} - 1}{\frac{1}{1.07} - 1} \approx 36.1.$

Thus, the payoff is approximately \$36.1 million.

45. Because her payments increase each year exactly as much as inflation, she will receive \$50,000 in 2005 dollars every year. So she needs to accumulate 45 × \$50,000 = \$2.25 million in 2005 dollars. Her effective yield on investment each year, taking into account inflation and using Fisher's formula, is $\frac{1+r}{1+a} - 1 = \frac{1.072}{1.04} - 1 = 3.076923077\%$. Use the savings formula with A = 2,250,000, i = 0.03076923077, and n = 35 × 4 = 140 quarters.

$$A = d\left[\frac{(1+i)^n - 1}{i}\right] \Rightarrow 2,250,000 = d\left[\frac{\left(1+\frac{0.03076923077}{4}\right)^{140} - 1}{\frac{0.03076923077}{4}}\right]$$

Thus, d = \$8997.74 per quarter.

47. The expected Social Security monthly income of \$1628 in 2001 dollars is $\frac{\$1628 \times 195}{177.1} = \$1792.55$ in 2005 dollars, using the Consumer Price Index values from Table 21.5. Reduction of the benefit by 32.5% makes it \$1792.55×(1− 0.325) = \$1209.97 per month. Her desired income in 2005 dollars is \$50,000/year = \$4166.67/month, or \$4166.67 − \$1209.97 = \$2956.70 more than Social Security will provide. To determine how much she must accumulate to have this much monthly for 32 years, we apply the amortization formula with payment d = \$2956.70, n = 32×12= 384 months, and $i = \frac{0.06}{12} = 0.005$.

$$A = d\left[\frac{1-(1+i)^{-n}}{i}\right] = \$2956.70\left[\frac{1-(1+0.005)^{-384}}{0.005}\right] = \$504{,}229.69$$

This is how much her savings must be worth in 2005 dollars; in 2053 dollars, she must have $\$504{,}229.69 \times (1.03)^{48} = \$2{,}083{,}604.08$. We use the savings formula to save up this amount A over n = 48×12 = 576 months with monthly interest rate $i = \frac{0.06}{12} = 0.005$.

$$A = d\left[\frac{(1+i)^n - 1}{i}\right] \Rightarrow \$2{,}083{,}604.08 = d\left[\frac{(1+0.005)^{576} - 1}{0.005}\right]$$

$$\$2{,}083{,}604.08 = d(3337.37879) \Rightarrow d = \frac{\$2{,}083{,}604.08}{3337.37879} = \$624.32$$

The monthly amount to save is \$624.32.

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47. continued

If instead of the CPI from Ch. 21 we use 3% inflation per year (CPI is from Ch. 21), the expected Social Security monthly income of \$1628 in 2001 dollars is $\$1628 \times (1.03)^4 = \$1832.33$ in 2005 dollars. Reduction of the benefit by 32.5% makes it \$1832.33×(1− 0.325) = \$1236.82. Her desired income is \$50,000/year = \$4166.67/month, or \$4166.67 − \$1236.82 = \$2929.85 more than Social Security will provide.

To determine how much she must accumulate to have this much monthly for 32 years, we apply the amortization formula with payment d = \$2929.85, n = 32×12 = 384 months, and monthly interest rate $i = \frac{0.06}{12} = 0.005$.

$$A = d\left[\frac{1-(1+i)^{-n}}{i}\right] = \$2929.85\left[\frac{1-(1+0.005)^{-384}}{0.005}\right] = \$499{,}650.75$$

This is how much her savings must be worth in 2005 dollars; in 2053 dollars, she must have $\$499{,}650.75 \times (1.03)^{48} = \$2{,}064{,}682.75$.

We use the savings formula to save up this amount A over $n = 48 \times 12 = 576$ months with monthly interest rate $i = \frac{0.06}{12} = 0.005$.

$$A = d\left[\frac{(1+i)^n - 1}{i}\right] \Rightarrow \$2{,}064{,}682.75 = d\left[\frac{(1+0.005)^{576} - 1}{0.005}\right]$$

$$\$2{,}064{,}682.75 = d\,(3337.37879) \Rightarrow d = \frac{\$2{,}064{,}682.75}{3337.37879} = \$618.65$$

The monthly amount to save is \$618.65.

If the 6% APR is interpreted as 6% APY (a topic from Ch. 21), then the monthly interest rate is $1.06^{1/12} - 1 = 0.00486755$. The corresponding divisor is 3162.54921 (instead of 3337.37879), and she must put away \$658.84 (using CPI for inflation 2001–2005) or \$652.85 (using 3% annual inflation 2001–2005).