Chapter 16
Identification Numbers

Solutions

Exercises:

1. Since $3+9+5+3+8+1+6+4+0=48=9\times5+3$, the check digit is 3.

3. Since $873345672=7\times124763667+3$, the check digit is 3.

5. Since $30860422052=7\times4408631721+5$, the check digit is 5.

7. Since $3\cdot3+8+3\cdot1+3+3\cdot7+0+3\cdot0+9+3\cdot2+1+3\cdot3=69$, the check digit is 1.

9. Since $10\cdot0+9\cdot6+8\cdot6+7\cdot9+6\cdot1+5\cdot9+4\cdot4+3\cdot9+2\cdot3=265=11\times24+1$, the check digit is X.

11. Since $7\cdot0+3\cdot9+9\cdot1+7\cdot9+3\cdot0+9\cdot2+7\cdot0+3\cdot4=129$, the check digit is 9.

13. Since $4+6+1+2+1+2+0+2+3=21$, the check digit is 6.

15. Since $7\cdot3+8+7\cdot1+3+7\cdot7+0+7\cdot0+9+7\cdot2+1+7\cdot3=133$, the check digit is 7. This check-digit scheme will detect all single-digit errors.

17. In the odd-numbered positions, if a digit a is replaced by the digit b where $a-b$ is even, the error is not detected.

19. We begin with $(3+0+2+6+0+9+4+1)\times2=50$. Adding 2, we obtain 52 and have the following.

$$52+0+1+5+0+1+6+3=68$$

So, the check digit is 2.

21. (a) Since $1\cdot0+1\cdot1+3\cdot2+3\cdot1+1\cdot6+3\cdot9+1\cdot0=43$, the check digit is 7.

(b) Since $1\cdot0+1\cdot2+3\cdot7+3\cdot4+1\cdot5+3\cdot5+1\cdot1=56$, the check digit is 4.

(c) Since $1\cdot0+1\cdot7+3\cdot6+3\cdot0+1\cdot0+3\cdot2+1\cdot2=33$, the check digit is 7.

(d) Since $1\cdot0+1\cdot4+3\cdot9+3\cdot6+1\cdot5+3\cdot8+1\cdot0=78$, the check digit is 2.

23. First observe that the given number 0669039254 results in a weighted sum that has a remainder of 5 after division by 11. So all we need to do is check for successive pairs of digits of this number that results in a contribution to the weighted sum of 5 less or 6 more, since either of these will make the weighted sum divisible by 11. Checking each pair of consecutive digits, we see that 39 contributes $5\cdot3+4\cdot9=51$ whereas 93 contributes $5\cdot9+4\cdot3=57$. So, the correct number is 0669093254.

25. Notice that when we add the weighted sum used for the actual check digit:

$$7a_1 + 3a_2 + 9a_3 + 7a_4 + 3a_5 + 9a_6 + 7a_7 + 3a_8$$

and the weighted sum

$$3a_1 + 7a_2 + a_3 + 3a_4 + 7a_5 + a_6 + 3a_7 + 7a_8,$$

we obtain

$$10a_1 + 10a_2 + 10a_3 + 10a_4 + 10a_5 + 10a_6 + 10a_7 + 10a_8,$$

which always ends with 0. So, the actual check digit and the check digit calculated with the weighted sum $3a_1 + 7a_2 + a_3 + 3a_4 + 7a_5 + a_6 + 3a_7 + 7a_8$ are both 0 or their sum is 10.

27. Replacing Z by 9 or vice versa is not detected.

29. Since the value of the weighted sum determines whether or not a number is valid, the position of the check digit is not relevant.

31. Since the remainder after dividing by 9 is between 0 and 8, 9 cannot be a check digit.

33. If the remainder of the sum of the noncheck digits after dividing by 7 is *k*, then the check digit is $7-k$ if $k \neq 0$ and 0 if $k = 0$. So, 7,8, and 9 can never be a check digit.

35. Yes. The ISBN scheme detects all transposition errors.

37. For the transposition to go undetected, it must be the case that the difference of the correct number and the incorrect number is evenly divisible by 11. That is,

$$(10a_1 + 9a_2 + 8a_3 + \cdots + a_{10}) - (10a_3 + 9a_2 + 8a_1 + \cdots + a_{10})$$

is divisible by 11. This reduces to $2a_1 - 2a_3 = 2(a_1 - a_3)$ is divisible by 11. But $2(a_1 - a_3)$ is divisible by 11 only when $a_1 - a_3$ is divisible by 11 and this only happens when $a_1 - a_3 = 0$. In this case, there is no error. The same argument works for the fourth and sixth digits.

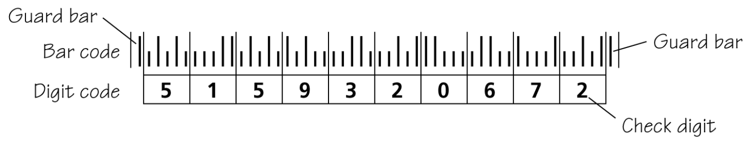
39. The combination 72 contributes $7 \cdot 1 + 2 \cdot 3 = 13$ or $7 \cdot 3 + 2 \cdot 1 = 23$ (depending on the location of the combination) towards the total sum, while the combination 27 contributes $2 \cdot 1 + 7 \cdot 3 = 23$ or $2 \cdot 3 + 7 \cdot 1 = 13$. So, the total sum resulting from the number with the transposition is still divisible by 10. Therefore, the error is not detected. When the combination 26 contributes $2 \cdot 1 + 6 \cdot 3 = 20$ towards the total sum, the combination 62 contributes $6 \cdot 1 + 2 \cdot 3 = 12$ toward the total sum; so the new sum will not be divisible by 10. Similarly, when the combination 26 contributes $2 \cdot 3 + 6 \cdot 1 = 12$ to the total, the combination 62 contributes $6 \cdot 3 + 2 \cdot 1 = 20$ to the total. So, the total for the number resulting from the transposition will not be divisible by 10 and the error is detected. In general, an error that occurs by transposing *ab* to *ba* is undetected if and only if $a - b = \pm 5$

41. The error $\cdots abc \cdots \to \cdots cba \cdots$ is undetectable if and only if $a - c = \pm 5$. To see this in the case that the weights for *abc* are 7, 3, 9, notice that *a* and *c* contribute $7a + 9c$ toward the weighted sum, whereas in the case of *cba*, the c and a contribute $7c + 9a$. Thus, the error is undetectable if and only if $7a + 9c$ and $7c + 9a$ contribute equal amounts to the last digit of the weighted sum. This means that they differ by a multiple of 10. That is, $-2a + 2c = 2(c - a)$ is a multiple of 10. This occurs when $c - a = 0$ or $c - a = \pm 5$. When $c - a = 0$, there is no error.

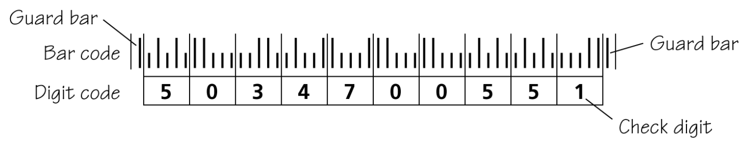
43. Since any error in the position with weight 10 does not change the last digit of the weighted sum, no error in that position is detected. In the position with weight 5, replacing an even digit by any other even digit is not detected. In positions with weights 12, 8, 6, 4, or 2, replacing a by b is undetectable if $a - b = \pm 5$. In positions 11, 9, 7, 3, or 1, all errors are detected.

45. Since both numbers are valid the difference of the weighted sums is divisible by 10. That is, $(7w+3+2w+1+5w+6+7w+4)-(7w+3+2w+1+5w+6+6w+1)$ is divisible by 10. The difference simplifies to $w+3$. So, $w=7$.

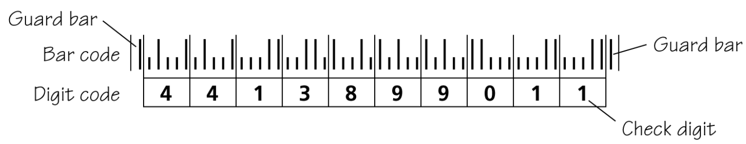
47. (a) The code is 51593-2067; since $5+1+5+9+3+2+0+6+7=38$, the check digit is 2.



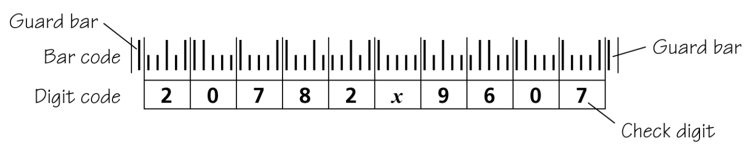
(b) The code is 50347-0055; since $5+0+3+4+7+0+0+5+5=29$, the check digit is 1.



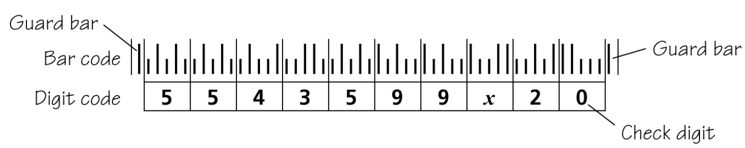
(c) The code is 44138-9901; since $4+4+1+3+8+9+9+0+1=39$, the check digit is 1.



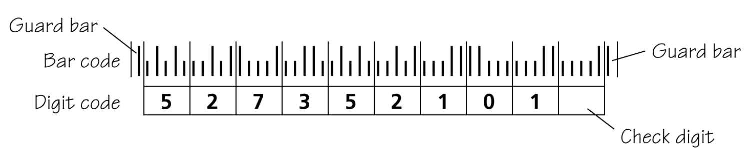
49. (a) Since the sixth block of five bars (ignoring the first bar) has one long bar and four short bars, that block is incorrect. Call the digit corresponding to that block *x*. Then the code is 20782*x*960. Since the sum of the digits is $x+41$, $x=9$. Finally, we write 20782-9960.



(b) Since the eighth block of five bars (ignoring the first bar) has three long bars and two short bars, that block is incorrect. Call the digit corresponding to that block *x*. Then the code is 5543599*x*2. Since the sum of the digits is $x+42$, $x=8$. Finally, we write 55435-9982.



(c) Since the tenth block of five bars (ignoring the first bar) has one long bar and four short bars, that block is incorrect. Since this is the check digit, the nine-digit ZIP code is 52735-2101.



51. If a double error in a block results in a new block that does not contain exactly two long bars, we know this block has been misread. If a double error in a block of five results in a new block with exactly two long bars, the block now gives a different digit from the original one. If no other digit is in error, the check digit catches the error, since the sum of the 10 digits will not end in 0. So, in every case an error has been detected. Errors of the first type can be corrected just as in the case of a single error. When a double error results in a legitimate code number, there is no way to determine which digit is incorrect.

53. The strings are *aaabb*, *aabab*, *aabba*, *abaab*, *ababa*, *abbaa*, *baaab*, *baaba*, *babaa*, and *bbaaa* (in alphabetical order). If you replace each short bar in the bar code table (page 603) by an a and replace each long bar in the bar code table by a b, the resulting strings are listed in alphabetical order.

55. Since there is an even number of 1's in 1000100, the scanner is reading from right to left.

57. Wyoming, Nevada, and Alaska.

59. The size of the population.

61. The Canadian scheme detects any transposition error involving adjacent characters. Also, there are $26^3 \times 10^3 = 17,576,000$ possible Canadian codes but only $10^5 = 100,000$ U.S. five-digit ZIP codes. Hence the Canadian scheme can target a location more precisely.

63. Skow $\to$ Sko $\to$ 220 $\to$ 20 $\to$ 0 $\to$ all numbers gone $\to$ S-000
Sachs $\to$ Sacs $\to$ 2022 $\to$ 202 $\to$ 02 $\to$ 2 $\to$ S-200
Lennon $\to$ Lennon $\to$ 405505 $\to$ 0505 $\to$ 55 $\to$ L-550
Lloyd $\to$ Lloyd $\to$ 44003 $\to$ 403 $\to$ 03 $\to$ 3 $\to$ L-300
Ehrheart $\to$ Ereart $\to$ 060063 $\to$ 06063 $\to$ 6063 $\to$ 663 $\to$ E-663
Ollenburger $\to$ Ollenburger $\to$ 04405106206 $\to$ 0405106206 $\to$ 405106206 $\to$ 451626 $\to$ O-451

65. A person born in 1999 is too young for a driver's license.

67. For a woman born in November or December the formula $40(m-1)+b+600$ gives a number requiring four digits.

69. The 58 indicates that the year of birth is 1958. Since 818 is larger than $12 \cdot 31 = 372$, we subtract 600 from 818 to obtain 218. Then $218 = 7 \cdot 31 + 1$ tells us that the person was born on the first day of the eighth month. So, the birth date is August 1, 1958.

71. Since $248 = 63(3) + 58 + 1$, the number 248 corresponds to a female born on March 29; since $601 = 63(9) + 34$ the number corresponds to a male born on September 17.

73. Likely circumstances could be twins; sons named after their fathers (such as John L. Smith, Jr.); common names such as John Smith and Mary Johnson; and states that do not include year of birth in the code.

75. Because of the short names and large population there would be a significant percentage of people whose names would be coded the same.