Chapter 8
Probability: The Mathematics of Chance

Solutions

Exercises:

1. Results will vary, but the probability of a head is usually greater than 0.5 when spinning pennies. One possible explanation is the "bottle cap effect." The rim on a penny is slightly wider on the head side, so just as spinning bottle caps almost always fall with the open side up, pennies fall more often with the head side up.

3. The first five lines contain 200 digits, of which 21 are zeros. The proportion of zeros is $\frac{21}{200} = 0.105.$

TABLE 7.1 Random Digits

101	19223	95034	05756	28713	96409	12531	42544	82853
102	73676	47150	99400	01927	27754	42648	82425	36290
103	45467	71709	77558	00095	32863	29485	82226	90056
104	52711	38889	93074	60227	40011	85848	48767	52573
105	95592	94007	69971	91481	60779	53791	17297	59335
106	68417	35013	15529	72765	85089	57067	50211	47487

5. (a) S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

(b) S = {0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100}.

(c) S = {Yes, No}.

7. (a) S = {HHHH, HHHM, HHMH, HMHH, MHHH, HHMM, HMMH, MMHH, HMHM, MHHM, MHMH, HMMM, MMMH, MHMM, MMHM, MMMM}.

(b) S = {0, 1, 2, 3, 4}.

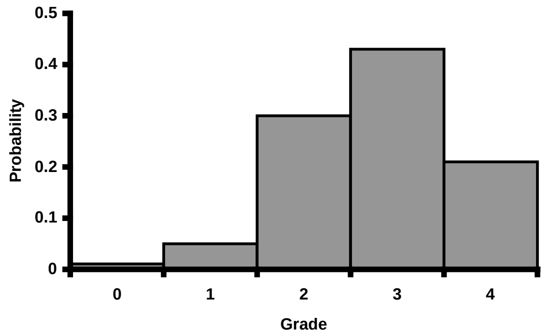
9. (a) The given probabilities have sum 0.81, so the probability of any other topic is $1-0.81=0.19$.

(b) The probability of adult or scam is $0.145+0.142=0.287$.

11. Answers will vary. Any two events that can occur together will do.

A = a student is female and B = a student is taking a mathematics course.

13. (a) Here is the probability histogram:



(b) $0.43 + 0.21 = 0.64$.

15. (a) Yes: the probabilities are between 0 and 1, inclusively, and have sum 1.

$$0 + \frac{1}{6} + \frac{1}{3} + \frac{1}{3} + \frac{1}{6} + 0 = \frac{1}{6} + \frac{2}{6} + \frac{2}{6} + \frac{1}{6} = \frac{6}{6} = 1$$

(Think of a die with no 1 or 6 face and two 3 and 4 faces.)

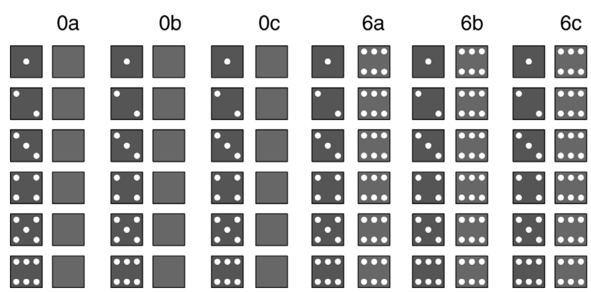
(b) No: the probabilities are between 0 and 1, but the sum is greater than 1.

$$0.56 + 0.24 + 0.44 + 0.17 = 1.41$$

(c) Yes: the probabilities are between 0 and 1, inclusively, and have sum 1.

$$\frac{12}{52} + \frac{12}{52} + \frac{12}{52} + \frac{16}{52} = \frac{52}{52} = 1$$

17. Each count between 1 and 12 occurs 3 times in the 36 possible outcomes. For example, 1 and 7 can only occur when the first die shows a 1.



Outcome	1	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

19. All 90 guests are equally likely to get the prize, so $P(\text{woman}) = \frac{42}{90} = \frac{7}{15}$.

21. (a) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{10} = 1024$.

(b) $\frac{2}{1024} = \frac{1}{512}$.

23. There are $36\times36\times36=36^3=46,656$ different codes. The probability of no *x* is as follows.

$$\frac{35\times35\times35}{46,656}=\frac{42,875}{46,656}\approx0.919$$

The probability of no digits is $\frac{26\times26\times26}{46,656}=\frac{17,576}{46,656}=\frac{2197}{5832}\approx0.377.$

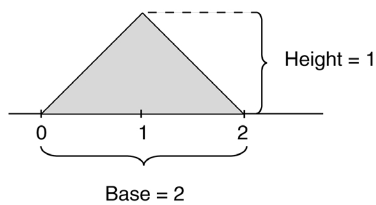
25. The possibilities are *ags*, *asg*, *gas*, *gsa*, *sag*, *sga*, of which "*gas*" and "*sag*" are English words. The probability is $\frac{2}{6}=\frac{1}{3}\approx0.333.$

27. There are $26\times36^2+26\times36^3+26\times36^4=44,916,768$ possible IDs. The number of IDs with no numbers is the sum of the numbers of 3-, 4-, and 5-character IDs, or the following.

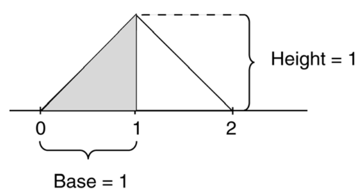
$$26^3+26^4+26^5=15,576+456,976+11,881,376=12,355,928.$$

The probability is therefore $\frac{12,355,928}{44,916,768}\approx0.275.$

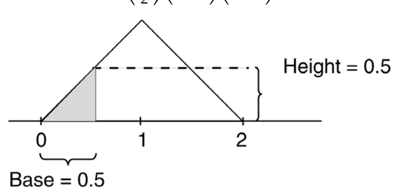
29. (a) The area is $\frac{1}{2}\times\text{base}\times\text{height}=\frac{1}{2}(2)(1)=1.$



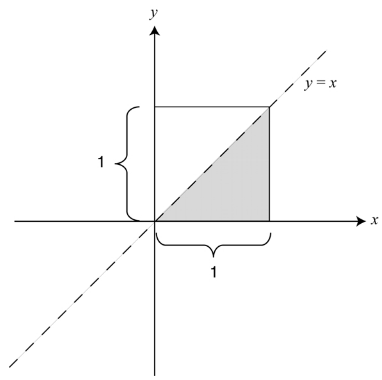
(b) Probability $\frac{1}{2}$ by symmetry or finding the area, $\frac{1}{2}\times\text{base}\times\text{height}=\frac{1}{2}(1)(1)=\frac{1}{2}.$.



(c) The area representing this event is $\left(\frac{1}{2}\right)(0.5)(0.5)=0.125.$



31. The area is the half of the square below the $y = x$ line. The probability is the area, $\left(\frac{1}{2}\right)(1)(1) = \frac{1}{2}$.



The area is half the square.

33. The mean is as follows.

$$\begin{aligned}\mu &= (0)(0.01)+(1)(0.05)+(2)(0.30)+(3)(0.43)+(4)(0.21)\\ &= 0+0.05+0.60+1.29+0.84 = 2.78\end{aligned}$$

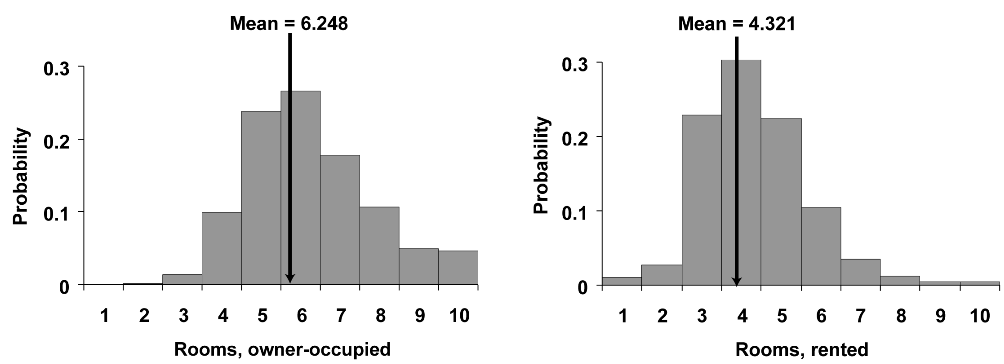
The variance is as follows.

$$\begin{aligned}\sigma^2 &= (0-2.78)^2(0.01)+(1-2.78)^2(0.05)+(2-2.78)^2(0.30)+(3-2.78)^2(0.43)+(4-2.78)^2(0.21)\\ &= (-2.78)^2(0.01)+(-1.78)^2(0.05)+(-0.78)^2(0.30)+(0.22)^2(0.43)+(1.22)^2(0.21)\\ &= (7.7284)(0.01)+(3.1684)(0.05)+(0.6084)(0.30)+(0.0484)(0.43)+(1.4884)(0.21)\\ &= 0.077284+0.15842+0.18252+0.020812+0.312564\\ &= 0.7516\end{aligned}$$

Thus, the standard deviation is $\sigma = \sqrt{0.7516} \approx 0.8669$.

35. The mean for owner-occupied units is $\mu = (1)(0.000)+(2)(0.001)+...+(10)(0.047) = 6.248$.

For rented units, $\mu = (1)(0.011)+(2)(0.027)+...+(10)(0.005) = 4.321$.



37. Both models have mean 1, because both density curves are symmetric about 1.

39. (a) $\mu = (1)\left(\frac{1}{6}\right)+(2)\left(\frac{1}{6}\right)+(3)\left(\frac{1}{6}\right)+(4)\left(\frac{1}{6}\right)+(5)\left(\frac{1}{6}\right)+(6)\left(\frac{1}{6}\right) = \frac{1}{6}+\frac{2}{6}+\frac{3}{6}+\frac{4}{6}+\frac{5}{6}+\frac{6}{6} = \frac{21}{6} = 3.5.$

(b)

Outcome	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{12}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

The mean is as follows.

$$\mu = (2)\left(\tfrac{1}{36}\right)+(3)\left(\tfrac{2}{36}\right)+(4)\left(\tfrac{3}{36}\right)+(5)\left(\tfrac{4}{36}\right)+(6)\left(\tfrac{5}{36}\right)+(7)\left(\tfrac{6}{36}\right)$$
$$+(8)\left(\tfrac{5}{36}\right)+(9)\left(\tfrac{4}{36}\right)+(10)\left(\tfrac{3}{36}\right)+(11)\left(\tfrac{2}{36}\right)+(12)\left(\tfrac{1}{36}\right)$$
$$= \tfrac{2}{36}+\tfrac{6}{36}+\tfrac{12}{36}+\tfrac{20}{36}+\tfrac{30}{36}+\tfrac{42}{36}+\tfrac{40}{36}+\tfrac{36}{36}+\tfrac{30}{36}+\tfrac{22}{36}+\tfrac{12}{36} = \tfrac{252}{36} = 7$$

(c) Answers will vary.

We could roll two dice separately and add the spots later. We expect the average outcome for two dice to be twice the average for one die. Remember that expected values are averages, so they behave like averages.

41. (a) $\mu = (1)\left(\frac{1}{6}\right)+(3)\left(\frac{1}{6}\right)+(4)\left(\frac{1}{6}\right)+(5)\left(\frac{1}{6}\right)+(6)\left(\frac{1}{6}\right)+(8)\left(\frac{1}{6}\right) = \frac{1}{6}+\frac{3}{6}+\frac{4}{6}+\frac{5}{6}+\frac{6}{6}+\frac{8}{6} = \frac{27}{6} = 4.5.$

(b) $\mu = (1)\left(\frac{1}{6}\right)+(2)\left(\frac{2}{6}\right)+(3)\left(\frac{2}{6}\right)+(4)\left(\frac{1}{6}\right) = \frac{1}{6}+\frac{4}{6}+\frac{6}{6}+\frac{4}{6} = \frac{15}{6} = 2.5.$

(c) The mean count for the two dice is 7. This is the same as for rolling two standard dice, with mean 3.5 for each. See the answer to Exercise 39.

43. (a) Since $0.00039+0.00044+0.00051+0.00057+0.00060 = 0.00251$, the probability is therefore $1-0.00251 = 0.99749$.

(b) The probability model for the company's cash intake is as follows.

Probability	Outcome
0.00039	$175-100,000 = -99,825$
0.00044	$2(175)-100,000 = -99,650$
0.00051	$3(175)-100,000 = -99,475$
0.00057	$4(175)-100,000 = -99,300$
0.00060	$5(175)-100,000 = -99,125$
0.99749	875

From this table, the mean is as follows.

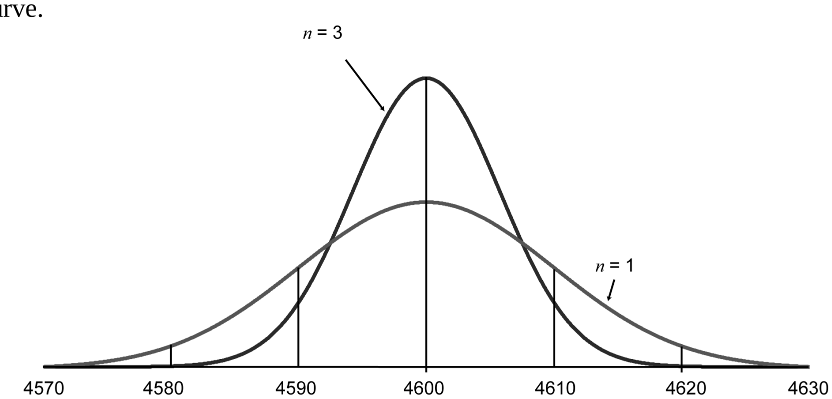
$$(-99,825)(0.00039)+(-99,650)(0.00044)+(-99,475)(0.00051)$$
$$+(-99,300)(0.00057)+(-99,125)(0.00060)+(875)(0.99749)$$
$$=(-38.93175)+(-43.846)+(-50.73225)+(-56.601)+(-59.475)+872.80375$$
$$=623.21775 \approx 623.218$$

45. Sample means $\bar{x}$ have a sampling distribution close to normal with mean $\mu = 0.15$ and standard deviation $\dfrac{\sigma}{\sqrt{n}} = \dfrac{0.4}{\sqrt{400}} = \dfrac{0.4}{20} = 0.02.$ Therefore, 95% of all samples have an $\bar{x}$ between $0.15-2(0.02) = 0.15-0.04 = 0.11$ and $0.15+2(0.02) = 0.15+0.04 = 0.19.$

47. (a) The standard deviation of the average measurement is $\frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{3}} \approx 5.77$ mg.

(b) To cut the standard deviation in half (from 10 mg to 5 mg), we need $n = 4$ measurements because $\frac{\sigma}{\sqrt{n}}$ is then $\frac{\sigma}{\sqrt{4}} = \frac{\sigma}{2}$. Averages of several measurements are less variable than individual measurements, so an average is more likely to give about the same result each time.

49. (a) Sketch a normal curve and mark the center at 4600 and the change-of-curvature points at 4590 and 4610. The curve will extend from about 4570 to 4630. This is the curve for one measurement. The mean of n = 3 measurements has mean μ = 4600 mg and standard deviation 5.77 mg. Mark points about 5.77 above and below 4600 and sketch a second curve.



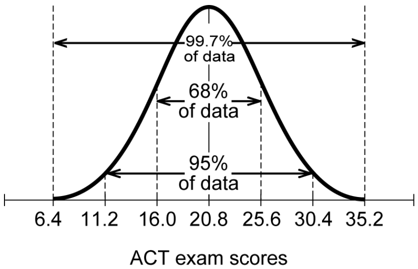
(b) Use the 95 part of the 68-95-99.7 rule with $\sigma = 10$.

$$4600 - 2(10) = 4600 - 20 = 4580 \text{ to } 4600 + 2(5.77) = 4600 + 20 = 4620$$

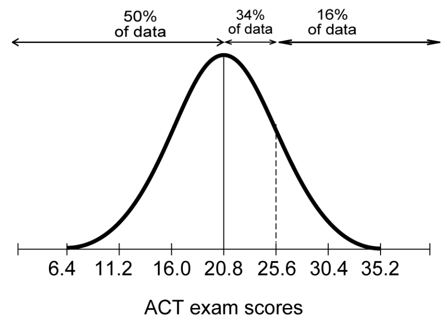
(c) Now the standard deviation is 5.77, so we have the following.

$$4600 - 2(5.77) = 4600 - 11.54 = 4588.46 \quad \text{to} \quad 4600 + 2(10) = 4600 + 11.54 = 4611.54$$

51.



(a) Because 25.6 is one standard deviation above the mean, the probability is about 0.16.



(b) The mean remains $\mu = 20.8$. The standard deviation is $\frac{\sigma}{\sqrt{9}} = \frac{4.8}{3} = 1.6.$

(c) Because $25.6 = 20.8 + 4.8 = 20.8 + 3(1.6)$ is three standard deviations above the mean, the probability is about 0.0015. (This is half of the 0.003 probability for outcomes more than three standard deviations from the mean, using the 99.7 part of the 68-95-99.7 rule.)

53. (a) There are $26 \times 10 \times 10 \times 26 \times 26 \times 26 = 45,697,600$ different license plates of this form.

(b) There are $26 \times 10 \times 10 = 2600$ plates ending in AAA, because that leaves only the first three characters free.

(c) The probability is $\frac{2600}{45,697,600} \approx 0.0000569.$

55. (a) The probability is $0.07 + 0.08 = 0.15.$

(b) The complement to the event of working out at least one day is working out no days. Thus, using the complement rule, the desired probability is $1 - 0.68 = 0.32.$

57. (a) The variance is as follows.

$$\sigma^2 = (0-1.03)^2(0.68)+(1-1.03)^2(0.05)+(2-1.03)^2(0.07)+(3-1.03)^2(0.08)$$
$$+(4-1.03)^2(0.05)+(5-1.03)^2(0.04)+(6-1.03)^2(0.01)+(7-1.03)^2(0.02)$$
$$=(-1.03)^2(0.68)+(-0.03)^2(0.05)+(0.97)^2(0.07)+(1.97)^2(0.08)$$
$$+(2.97)^2(0.05)+(3.97)^2(0.04)+(4.97)^2(0.01)+(5.97)^2(0.02)$$
$$=(1.0609)(0.68)+(0.0009)(0.05)+(0.9409)(0.07)+(3.8809)(0.08)$$
$$+(8.8209)(0.05)+(15.7609)(0.04)+(24.7009)(0.01)+(35.6409)(0.02)$$
$$=0.721412+0.000045+0.065863+0.310472+0.441045+0.630436+0.247009+0.712818$$
$$=3.1291$$

Thus, the standard deviation is $\sigma=\sqrt{3.1291}\approx 1.7689$ days.

(b) The mean, $\bar{x}$, of n = 100 observations has mean $\mu=1.03$ and standard deviation

$$\frac{\sigma}{\sqrt{n}}=\frac{1.7689}{\sqrt{100}}=\frac{1.7689}{10}\approx 0.177.$$

The central limit theorem says that $\bar{x}$ is approximately normal with this mean and standard deviation. The 95 part of the 68-95-99.7 rule says that with probability 0.95, values of $\bar{x}$ lie between $1.03-2(0.177)=1.03-0.354=0.676$ days and $1.03+2(0.177)=1.03+0.354=1.384$ days.