Chapter 4
Linear Programming

Solutions

Exercises:

1. (a) $2x+3y=12$

y-intercept: Substitute $x=0$.

$2(0)+3y=12$

$0+3y=12$

$3y=12 \Rightarrow y=\frac{12}{3}=4$

y-intercept is $(0,4)$.

x-intercept: Substitute $y=0$.

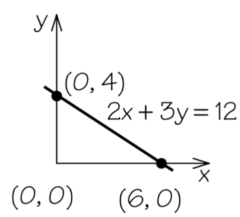
$2x+3(0)=12$

$2x+0=12$

$2x=12 \Rightarrow x=\frac{12}{2}=6$

x-intercept is $(6,0)$.

Graph:



(b) $3x+5y=30$

y-intercept: Substitute $x=0$.

$3(0)+5y=30$

$0+5y=30$

$5y=30 \Rightarrow y=\frac{30}{5}=6$

y-intercept is $(0,6)$.

x-intercept: Substitute $y=0$.

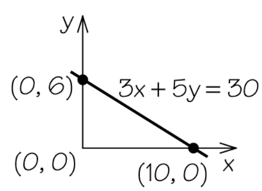
$3x+5(0)=30$

$3x+0=30$

$3x=30 \Rightarrow x=\frac{30}{3}=10$

x-intercept is $(10,0)$.

Graph:



Continued on next page

1. continued

(c) $4x+3y=24$

y-intercept: Substitute $x=0$.

$$4(0)+3y=24$$
$$0+3y=24$$
$$3y=24 \Rightarrow y=\tfrac{24}{3}=8$$

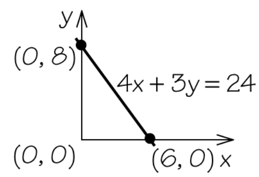
y-intercept is $(0,8)$.

x-intercept: Substitute $y=0$.

$$4x+3(0)=24$$
$$4x+0=24$$
$$4x=24 \Rightarrow x=\tfrac{24}{4}=6$$

x-intercept is $(6,0)$.

Graph:



(d) $7x+4y=42$

y-intercept: Substitute $x=0$.

$$7(0)+4y=42$$
$$0+4y=42$$
$$4y=42 \Rightarrow y=\tfrac{42}{4}=10.5$$

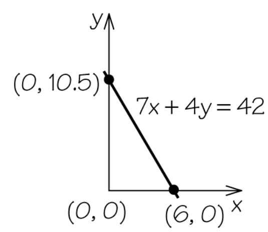
y-intercept is $(0,10.5)$.

x-intercept: Substitute $y=0$.

$$7x+4(0)=42$$
$$7x+0=42$$
$$7x=42 \Rightarrow x=\tfrac{42}{7}=6$$

x-intercept is $(6,0)$.

Graph:



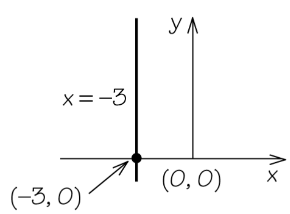
(e) $x=-3$

This form represents a vertical line.

y-intercept: None

x-intercept: $(-3,0)$.

Graph:



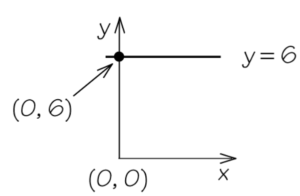
(f) $y=6$

This form represents a horizontal line.

y-intercept is $(0,6)$.

x-intercept: None

Graph:



3. Note: These situations are shown only for the first quadrant.

(a) $x+y=10$ and $x+2y=14$

The *y*-intercept of $x+y=10$ can be found by substituting $x=0$.

$$0+y=10$$
$$y=10$$

The *y*-intercept is $(0,10)$.

The *x*-intercept of $x+y=10$ can be found by substituting $y=0$.

$$x+0=10$$
$$x=10$$

The *x*-intercept is $(10,0)$.

The *y*-intercept of $x+2y=14$ can be found by substituting $x=0$.

$$0+2y=14$$
$$2y=14$$
$$y=\tfrac{14}{2}=7$$

The *y*-intercept is $(0,7)$.

The *x*-intercept of $x+2y=14$ can be found by substituting $y=0$.

$$x+2(0)=14$$
$$x+0=14$$
$$x=14$$

The *x*-intercept is $(14,0)$.

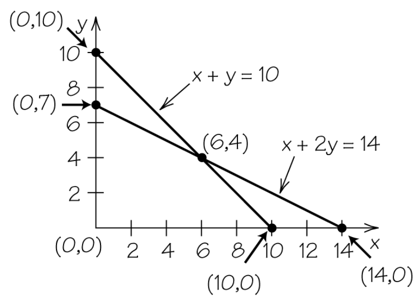
To find the point of intersection, we can multiply both sides of $x+y=10$ by -1, and add the result to $x+2y=14$.

$$\begin{array}{r} -x-y=-10 \\ \underline{x+2y=14} \\ y=4 \end{array}$$

Substitute $y=4$ into $x+y=10$ to solve to *x*.

$$x+4=10 \Rightarrow x=6$$

The point of intersection is therefore $(6,4)$.



Continued on next page

3. continued

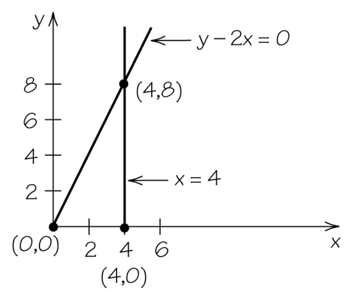
(b) $y-2x=0$ and $x=4$

$x=4$ represents a vertical line, which lies to the right of the *y*-axis.

By substituting either $x=0$ or $y=0$, we see that $y-2x=0$ passes through $(0,0)$, the origin. By substituting an arbitrary value (except 0) for one of the variables, we can find another point that lies on the graph of $y-2x=0$. Since we see the point of intersection with the graph of $x=4$, we could use this value.

$$y-2(4)=0$$
$$y-8=0$$
$$y=8$$

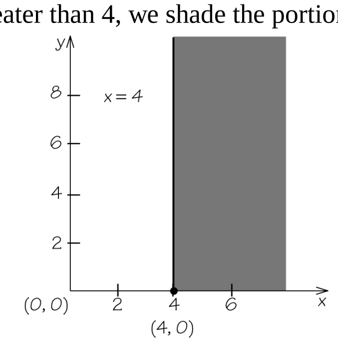
Thus, a second point on the graph of $y-2x=0$ is $(4,8)$. This is also the point of intersection between the two lines.



5. Note: These situations are shown only for the first quadrant.

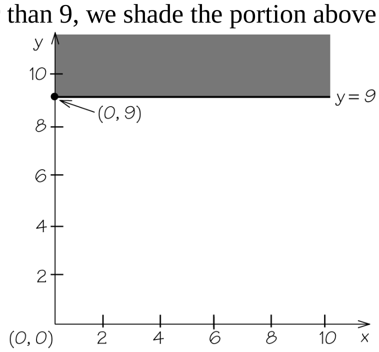
(a) $x \geq 4$

The graph of $x=4$ represents a vertical line with *x*-intercept $(4,0)$. Since any value of *x* to the right of this line is greater than 4, we shade the portion to the right of this vertical line.



(b) $y \geq 9$

The graph of $y=9$ represents a horizontal line with *y*-intercept $(0,9)$. Since any value of *y* above this line is greater than 9, we shade the portion above this horizontal line.



Continued on next page

5. continued

(c) $3x+2y\le 18$

The y-intercept of $3x+2y=18$ can be found by substituting $x=0$.

$$3(0)+2y=18 \Rightarrow 0+2y=18 \Rightarrow 2y=18 \Rightarrow y=\tfrac{18}{2}=9$$

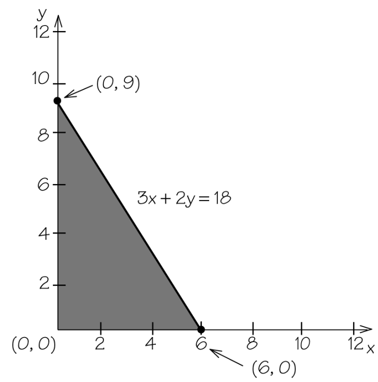
The y-intercept is $(0,9)$.

The x-intercept of $3x+2y=18$ can be found by substituting $y=0$.

$$3x+2(0)=18 \Rightarrow 3x+0=18 \Rightarrow 3x=18 \Rightarrow x=\tfrac{18}{3}=6$$

The x-intercept is $(6,0)$.

We draw a line connecting these points. Testing the point $(0,0)$, we have the statement $3(0)+2(0)\le 18$ or $0\le 18$. This is a true statement, thus we shade the half-plane containing our test point, the down side of the line.



(d) $7x+2y\le 42$

The y-intercept of $7x+2y=42$ can be found by substituting $x=0$.

$$7(0)+2y=42 \Rightarrow 0+2y=42 \Rightarrow 2y=42 \Rightarrow y=\tfrac{42}{2}=21$$

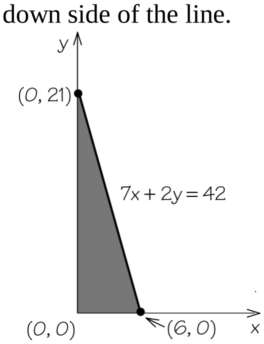
The y-intercept is $(0,21)$.

The x-intercept of $7x+2y=42$ can be found by substituting $y=0$.

$$7x+2(0)=42 \Rightarrow 7x+0=42 \Rightarrow 7x=42 \Rightarrow x=\tfrac{42}{7}=6$$

The x-intercept is $(6,0)$.

We draw a line connecting these points. Testing the point $(0,0)$, we have the statement $7(0)+2(0)\le 42$ or $0\le 42$. This is a true statement, thus we shade the half-plane containing our test point, the down side of the line.



7. (a) $6x+4y\le 300$

(b) $30x+72y\le 420$

9. $x \geq 0$; $y \geq 0$; $x + 2y \leq 12$

The constraints of $x \geq 0$ and $y \geq 0$ indicate that we are restricted to the upper right quadrant created by the *x*-axis and *y*-axis.

The *y*-intercept of $x + 2y = 12$ can be found by substituting $x = 0$.

$$0 + 2y = 12 \Rightarrow 2y = 12 \Rightarrow y = \tfrac{12}{2} = 6$$

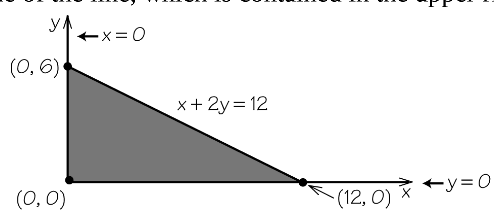
The *y*-intercept is $(0,6)$.

The *x*-intercept of $x + 2y = 12$ can be found by substituting $y = 0$.

$$x + 2(0) = 12 \Rightarrow x + 0 = 12 \Rightarrow x = 12$$

The *x*-intercept is $(12,0)$.

We draw a line connecting these points. Testing the point $(0,0)$, we have the statement $0 + 2(0) \leq 12$ or $0 \leq 12$. This is a true statement, thus we shade the half-plane containing our test point, the down side of the line, which is contained in the upper right quadrant.



11. $x \geq 0$; $y \geq 0$; $2x + 5y \leq 60$

The constraints of $x \geq 0$ and $y \geq 0$ indicate that we are restricted to the upper right quadrant created by the *x*-axis and *y*-axis.

The *y*-intercept of $2x + 5y = 60$ can be found by substituting $x = 0$.

$$2(0) + 5y = 60 \Rightarrow 0 + 5y = 60 \Rightarrow y = \tfrac{60}{5} = 12$$

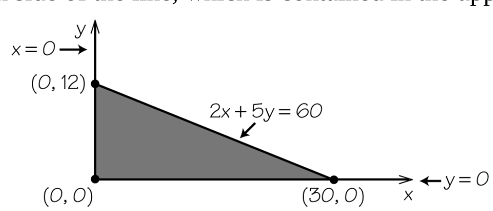
The *y*-intercept is $(0,12)$.

The *x*-intercept of $2x + 5y = 60$ can be found by substituting $y = 0$.

$$2x + 5(0) = 60 \Rightarrow 2x + 0 = 60 \Rightarrow 2x = 60 \Rightarrow x = \tfrac{60}{2} = 30$$

The *x*-intercept is $(30,0)$.

We draw a line connecting these points. Testing the point $(0,0)$, we have the statement $2(0) + 5(0) \leq 60$ or $0 \leq 60$. This is a true statement, thus we shade the half-plane containing our test point, the down side of the line, which is contained in the upper right quadrant.



13. $x \geq 0$; $y \geq 4$; $x + y \leq 20$

The constraints of $x \geq 0$ and $y \geq 4$ indicate that we are restricted to the upper right quadrant, above the horizontal line $y = 4$.

The point of intersection between $y = 4$ and $x + y = 20$ can be found by substituting $y = 4$ into $x + y = 20$.

$$x + 4 = 20 \Rightarrow x = 16$$

Thus, the point of intersection is $(16, 4)$.

The y-intercept of $x + y = 20$ can be found by substituting $x = 0$.

$$0 + y = 20 \Rightarrow y = 20$$

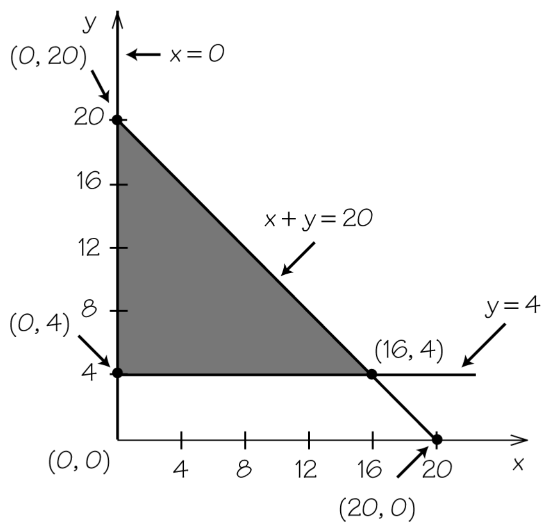
The y-intercept is $(0, 20)$.

The x-intercept of $x + y = 20$ can be found by substituting $y = 0$.

$$x + 0 = 20 \Rightarrow x = 20$$

The x-intercept is $(20, 0)$.

We draw a line connecting these points. Testing the point $(0, 0)$, we have the statement $0 + 0 \leq 20$ or $0 \leq 20$. This is a true statement, thus we shade the half-plane containing our test point, the down side of the line, which is contained in the upper right quadrant above the horizontal line $y = 4$.



15. For Exercise 9: $x \geq 0$; $y \geq 0$; $x + 2y \leq 12$

$(2,4)$: Since $2 \geq 0$, the constraint $x \geq 0$ is satisfied.
Since $4 \geq 0$, the constraint $y \geq 0$ is satisfied.
Since $2 + 2(4) = 2 + 8 = 10 \leq 12$, the condition $x + 2y \leq 12$ is satisfied.
Thus, $(2,4)$ is feasible.

$(10,6)$: Since $10 \geq 0$, the constraint $x \geq 0$ is satisfied.
Since $6 \geq 0$, the constraint $y \geq 0$ is satisfied.
Since $10 + 2(6) = 10 + 12 = 22 > 12$, the condition $x + 2y \leq 12$ is not satisfied.
Thus, $(10,6)$ is not feasible.

For Exercise 11: $x \geq 0$; $y \geq 0$; $2x + 5y \leq 60$

$(2,4)$: Since $2 \geq 0$, the constraint $x \geq 0$ is satisfied.
Since $4 \geq 0$, the constraint $y \geq 0$ is satisfied.
Since $2(2) + 5(4) = 4 + 20 = 24 \leq 60$, the condition $x + 2y \leq 12$ is satisfied.
Thus, $(2,4)$ is feasible.

$(10,6)$: Since $10 \geq 0$, the constraint $x \geq 0$ is satisfied.
Since $6 \geq 0$, the constraint $y \geq 0$ is satisfied.
Since $2(10) + 5(6) = 20 + 30 = 50 \leq 60$, the condition $x + 2y \leq 12$ is satisfied.
Thus, $(10,6)$ is feasible.

For Exercise 13: $x \geq 0$; $y \geq 4$; $x + y \leq 20$

$(2,4)$: Since $2 \geq 0$, the constraint $x \geq 0$ is satisfied.
Since $4 \geq 4$, the constraint $y \geq 4$ is satisfied.
Since $2 + 4 = 6 \leq 20$, the condition $x + y \leq 20$ is satisfied.
Thus, $(2,4)$ is feasible. Note: It is on the boundary.

$(10,6)$: Since $10 \geq 0$, the constraint $x \geq 0$ is satisfied.
Since $6 \geq 4$, the constraint $y \geq 4$ is satisfied.
Since $10 + 6 = 16 \leq 20$, the condition $x + y \leq 20$ is satisfied.
Thus, $(10,6)$ is feasible.

17. We wish to maximize $\$2.30x + \$3.70y$.

Corner Point	Value of the Profit Formula: $\$2.30x + \$3.70y$							
$(0,0)$	$\$2.30(0)$	+	$\$3.70(0)$	=	\$0.00	+	\$0.00	= \$0.00
$(0,30)$	$\$2.30(0)$	+	$\$3.70(30)$	=	\$0.00	+	\$111.00	= \$111.00*
$(12,0)$	$\$2.30(12)$	+	$\$3.70(0)$	=	\$27.60	+	\$0.00	= \$27.60

Optimal production policy: Make 0 skateboards and 30 dolls for a profit of \$111.

19. Note: These situations are shown only for the first quadrant.

(a) $5x+4y=22$ and $5x+10y=40$

The *y*-intercept of $5x+4y=22$ can be found by substituting $x=0$.

$$5(0)+4y=22$$
$$0+4y=22$$
$$4y=22$$
$$y=\tfrac{22}{4}=5.5$$

The *y*-intercept is $(0,5.5)$.

The *x*-intercept of $5x+4y=22$ can be found by substituting $y=0$.

$$5x+4(0)=22$$
$$5x+0=22$$
$$5x=22$$
$$x=\tfrac{22}{5}=4.4$$

The *x*-intercept is $(4.4,0)$.

The *y*-intercept of $5x+10y=40$ can be found by substituting $x=0$.

$$5(0)+10y=40$$
$$0+10y=40$$
$$y=\tfrac{40}{10}=4$$

The *y*-intercept is $(0,4)$.

The *x*-intercept of $5x+10y=40$ can be found by substituting $y=0$.

$$5x+10(0)=40$$
$$5x+0=40$$
$$5x=40$$
$$x=\tfrac{40}{4}=8$$

The *x*-intercept is $(8,0)$.

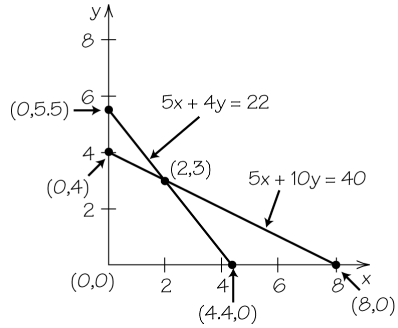
To find the point of intersection, we can multiply both sides of $5x+4y=22$ by -1, and adding the result to $5x+10y=40$.

$$\begin{array}{r} -5x-4y=-22 \\ \underline{5x+10y=40} \\ 6y=18 \Rightarrow y=\tfrac{18}{6}=3 \end{array}$$

Substitute $y=3$ into $5x+10y=40$ and solve for *x*.

$$5x+10(3)=40 \Rightarrow 5x+30=40 \Rightarrow 5x=10 \Rightarrow x=\tfrac{10}{5}=2$$

The point of intersection is therefore $(2,3)$.



Continued on next page

19. continued

(b) $x + y = 7$ and $3x + 4y = 24$

The *y*-intercept of $x + y = 7$ can be found by substituting $x = 0$.

$$0 + y = 7$$
$$y = 7$$

The *y*-intercept is $(0, 7)$.

The *x*-intercept of $x + y = 7$ can be found by substituting $y = 0$.

$$x + 0 = 7$$
$$x = 7$$

The *x*-intercept is $(7, 0)$.

The *y*-intercept of $3x + 4y = 24$ can be found by substituting $x = 0$.

$$3(0) + 4y = 24$$
$$0 + 4y = 24$$
$$y = \tfrac{24}{4} = 6$$

The *y*-intercept is $(0, 6)$.

The *x*-intercept of $3x + 4y = 24$ can be found by substituting $y = 0$.

$$3x + 4(0) = 24$$
$$3x + 0 = 24$$
$$3x = 24$$
$$x = \tfrac{24}{3} = 8$$

The *x*-intercept is $(8, 0)$.

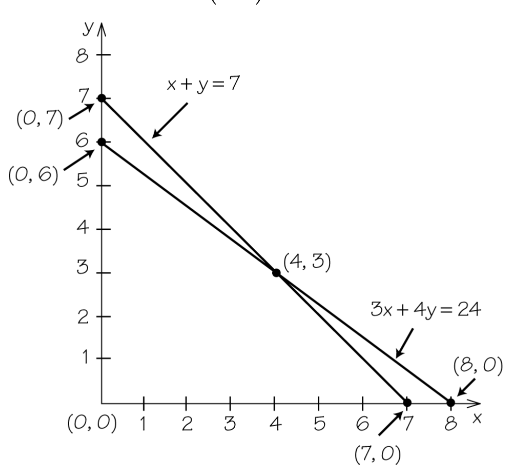
To find the point of intersection, we can multiply both sides of $x + y = 7$ by -3, and adding the result to $3x + 4y = 24$

$$\begin{array}{r} -3x - 3y = -21 \\ \underline{3x + 4y = 24} \\ y = 3 \end{array}$$

Substitute $y = 3$ into $3x + 4y = 24$ and solve for *x*.

$$3x + 4(3) = 24 \Rightarrow 3x + 12 = 24 \Rightarrow 3x = 12 \Rightarrow x = \tfrac{12}{3} = 4$$

The point of intersection is therefore $(4, 3)$.



21. $x \ge 0$; $y \ge 0$; $2x + y \le 4; 4x + 4y \le 12$

The constraints of $x \ge 0$ and $y \ge 0$ indicate that we are restricted to the upper right quadrant created by the *x*-axis and *y*-axis.

The *y*-intercept of $2x + y = 4$ can be found by substituting $x = 0$.

$$2(0) + y = 4 \Rightarrow 0 + y = 4 \Rightarrow y = 4$$

The *y*-intercept is $(0,4)$.

The *x*-intercept of $2x + y = 4$ can be found by substituting $y = 0$.

$$2x + 0 = 4 \Rightarrow 2x = 4 \Rightarrow x = \tfrac{4}{2} = 2$$

The *x*-intercept is $(2,0)$.

We draw a line connecting these points. Testing the point $(0,0)$, we have the statement $2(0) + 0 \le 4$ or $0 \le 4$. This is a true statement.

The *y*-intercept of $4x + 4y = 12$ can be found by substituting $x = 0$.

$$4(0) + 4y = 12 \Rightarrow 0 + 4y = 12 \Rightarrow y = \tfrac{12}{4} = 3$$

The *y*-intercept is $(0,3)$.

The *x*-intercept of $4x + 4y = 12$ can be found by substituting $y = 0$.

$$4x + 4(0) = 12 \Rightarrow 4x + 0 = 12 \Rightarrow 4x = 12 \Rightarrow x = \tfrac{12}{4} = 3$$

The *x*-intercept is $(3,0)$.

We draw a line connecting these points. Testing the point $(0,0)$, we have the statement $4(0) + 4(0) \le 12$ or $0 \le 12$. This is a true statement.

Thus, we shade the part of the plane in the upper right quadrant which is on the down side of both the lines $2x + y = 4$ and $4x + 4y = 12$.

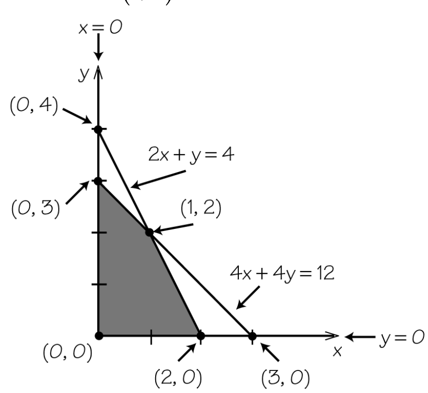
Three of the corner points, $(0,0)$, $(0,3)$, and $(2,0)$ lie on the coordinate axes. The fourth corner point is the point of intersection between the lines $2x + y = 4$ and $4x + 4y = 12$. We can find this by multiplying both sides of $2x + y = 4$ by -4, and adding the result to $4x + 4y = 12$.

$$\begin{array}{l} -8x - 4y = -16 \\ \underline{4x + 4y = 12} \\ -4x \qquad\; = -4 \Rightarrow x = \tfrac{-4}{-4} = 1 \end{array}$$

Substitute $x = 1$ into $2x + y = 4$ and solve for *y*.

$$2(1) + y = 4 \Rightarrow 2 + y = 4 \Rightarrow y = 2$$

The point of intersection is therefore $(1,2)$.



23. $x \geq 4;\ y \geq 0;\ 5x + 4y \leq 60;\ x + y \leq 13$

The constraints of $x \geq 4$ and $y \geq 0$ indicate that we are restricted to the upper right quadrant, to the right of the vertical line $x = 4$.

The y-intercept of $5x + 4y = 60$ can be found by substituting $x = 0$.

$$5(0) + 4y = 60 \Rightarrow 0 + 4y = 60 \Rightarrow y = \tfrac{60}{4} = 15$$

The y-intercept is $(0,15)$.

The x-intercept of $5x + 4y = 60$ can be found by substituting $y = 0$.

$$5x + 4(0) = 60 \Rightarrow 5x + 0 = 60 \Rightarrow 5x = 60 \Rightarrow x = \tfrac{60}{5} = 12$$

The x-intercept is $(12,0)$.

We draw a line connecting these points. Testing the point $(0,0)$, we have the statement $5(0) + 4(0) \leq 60$ or $0 \leq 60$. This is a true statement.

The y-intercept of $x + y = 13$ can be found by substituting $x = 0$.

$$0 + y = 13 \Rightarrow y = 13$$

The y-intercept is $(0,13)$.

The x-intercept of $x + y = 13$ can be found by substituting $y = 0$.

$$x + 0 = 13 \Rightarrow x = 13$$

The x-intercept is $(13,0)$.

We draw a line connecting these points. Testing the point $(0,0)$, we have the statement $0 + 0 \leq 13$ or $0 \leq 13$. This is a true statement.

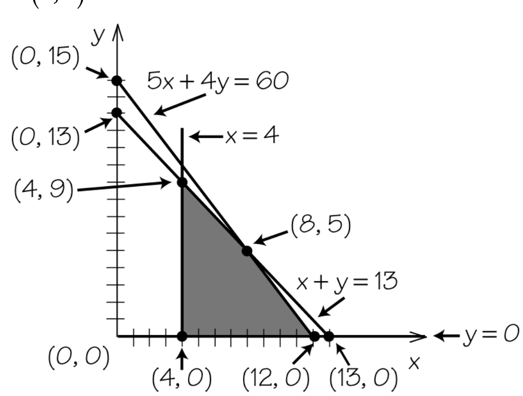
Thus, we shade the part of the plane in the upper right quadrant which is on the down side of both the lines $5x + 4y = 60$ and $x + y = 13$, which is also to the right of the vertical line $x = 4$.

Two of the corner points, $(4,0)$ and $(12,0)$, lie on the coordinate axes. The third corner point is the point of intersection between the lines $x + y = 13$ and $x = 4$. We can find this by substituting $x = 4$ into $x + y = 13$ to solve to y. We have, $4 + y = 13 \Rightarrow y = 9$. The point of intersection is therefore $(4,9)$.

The fourth corner point is the point of intersection between the lines $5x + 4y = 60$ and $x + y = 13$. We can find this by multiplying both sides of $x + y = 13$ by -4, and adding the result to $5x + 4y = 60$.

$$\begin{array}{r} -4x - 4y = -52 \\ 5x + 4y = 60 \\ \hline x \qquad = 8 \end{array}$$

Substitute $x = 8$ into $x + y = 13$ and solve for y. We have $8 + y = 13 \Rightarrow y = 5$. The point of intersection is therefore $(8,5)$.



25. Maximize $P = 3x + 2y$ subject to $x \ge 3$; $y \ge 2$; $x + y \le 10$; $2x + 3y \le 24$

We need to first graph the feasible region while finding the corner points.

The constraints of $x \ge 3$ and $y \ge 2$ indicate that we are restricted to the upper right quadrant, to the right of the vertical line $x = 3$ and above the horizontal line $y = 2$.

The point of intersection between $x = 3$ and $y = 2$ is $(3, 2)$.

The *y*-intercept of $x + y = 10$ can be found by substituting $x = 0$.

$$0 + y = 10 \Rightarrow y = 10$$

The *y*-intercept is $(0, 10)$.

The *x*-intercept of $x + y = 10$ can be found by substituting $y = 0$.

$$x + 0 = 10 \Rightarrow x = 10$$

The *x*-intercept is $(10, 0)$.

The *y*-intercept of $2x + 3y = 24$ can be found by substituting $x = 0$.

$$2(0) + 3y = 24 \Rightarrow 0 + 3y = 24 \Rightarrow 3y = 24 \Rightarrow y = \tfrac{24}{3} = 8$$

The *y*-intercept is $(0, 8)$.

The *x*-intercept of $2x + 3y = 24$ can be found by substituting $y = 0$.

$$2x + 3(0) = 24 \Rightarrow 2x + 0 = 24 \Rightarrow 2x = 24 \Rightarrow x = \tfrac{24}{2} = 12$$

The *x*-intercept is $(12, 0)$.

Testing the point $(0, 0)$ in both $x + y \le 10$ and $2x + 3y \le 24$, we have the following.

$$0 + 0 = 0 \le 10 \text{ and } 2(0) + 3(0) = 0 \le 24$$

Since these are both true statements, we shade the down side of the line of both lines which is contained in the upper right quadrant to the right of the vertical line $x = 3$ and above the horizontal line $y = 2$.

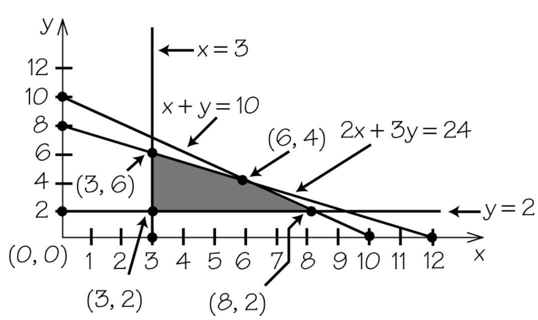
The point of intersection between $x = 3$ and $2x + 3y = 24$ can be found by substituting $x = 3$ into $2x + 3y = 24$. We have $2(3) + 3y = 24 \Rightarrow 6 + 3y = 24 \Rightarrow 3y = 18 \Rightarrow y = \tfrac{18}{3} = 6$. Thus, the point of intersection is $(3, 6)$.

The point of intersection between $y = 2$ and $x + y = 10$ can be found by substituting $y = 2$ into $x + y = 10$. We have, $x + 2 = 10 \Rightarrow x = 8$. Thus, the point of intersection is $(8, 2)$.

The final corner point is the point of intersection between $x + y = 10$ and $2x + 3y = 24$. We can find this by multiplying both sides of $x + y = 10$ by -2, and adding the result to $2x + 3y = 24$.

$$\begin{array}{r} -2x - 2y = -20 \\ 2x + 3y = 24 \\ \hline y = 4 \end{array}$$

Substitute $y = 4$ into $x + y = 10$ and solve for *x*. We have, $x + 4 = 10 \Rightarrow x = 6$. The point of intersection is therefore $(6, 4)$.



Continued on next page

25. continued

We wish to maximize $P = 3x + 2y$.

Corner Point	Value of the Profit Formula: $3x + 2y$
$(3,6)$	$3(3) + 2(6) = 9 + 12 = 21$
$(3,2)$	$3(3) + 2(2) = 9 + 4 = 13$
$(8,2)$	$3(8) + 2(2) = 24 + 4 = 28*$
$(6,4)$	$3(6) + 2(4) = 18 + 8 = 26$

The maximum value occurs at the corner point $(8,2)$, where P is equal to 28.

27. Maximize $P = 5x + 2y$ subject to $x \geq 2$; $y \geq 4$; $x + y \leq 10$

We need to first graph the feasible region while finding the corner points.

The constraints of $x \geq 2$ and $y \geq 4$ indicate that we are restricted to the upper right quadrant, to the right of the vertical line $x = 2$ and above the horizontal line $y = 4$.

The point of intersection between $x = 2$ and $y = 4$ is $(2,4)$.

The y-intercept of $x + y = 10$ can be found by substituting $x = 0$.

$$0 + y = 10 \Rightarrow y = 10$$

The y-intercept is $(0,10)$.

The x-intercept of $x + y = 10$ can be found by substituting $y = 0$.

$$x + 0 = 10 \Rightarrow x = 10$$

The x-intercept is $(10,0)$.

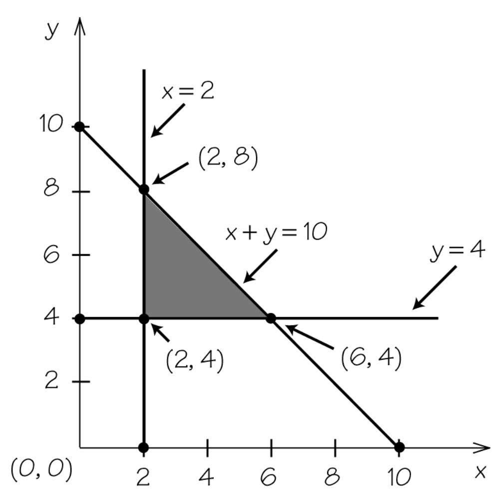
Testing the point $(0,0)$ in $x + y \leq 10$, we have the following.

$$0 + 0 = 0 \leq 10$$

Since this is a true statement, we shade the down side of this line which is contained in the upper right quadrant to the right of the vertical line $x = 2$ and above the horizontal line $y = 4$.

The point of intersection between $x = 2$ and $x + y = 10$ can be found by substituting $x = 2$ into $x + y = 10$. We have $2 + y = 10 \Rightarrow y = 8$. Thus, the point of intersection is $(2,8)$.

The point of intersection between $y = 4$ and $x + y = 10$ can be found by substituting $y = 4$ into $x + y = 10$. We have $x + 4 = 10 \Rightarrow x = 6$. Thus, the point of intersection is $(6,4)$.



Continued on next page

27. continued

We wish to maximize $P = 5x + 2y$.

Corner Point	Value of the Profit Formula: $5x+2y$
$(2,4)$	$5(2) + 2(4) = 10 + 8 = 18$
$(2,8)$	$5(2) + 2(8) = 10 + 16 = 26$
$(6,4)$	$5(6) + 2(4) = 30 + 8 = 38$*

The maximum value occurs at the corner point $(6,4)$, where P is equal to 38.

29. (a) The optimal corner point for Exercise 28 being $\left(\frac{13}{7},0\right)$, the coordinates of $Q = (2,0)$.

(b) $(2,0)$ is not a feasible point.

(c) The profit at $(2,0)$ is greater than the profit at $\left(\frac{13}{7},0\right)$.

(d) Since $0 \ge 0$ and $3 \ge 0$ the constraints $x \ge 0$ and $y \ge 0$ are satisfied, respectively. Also, since $7(0)+4(3) = 0+12 = 12 \le 13$, the constraint $7x+4y \le 13$ is satisfied. Thus, $R = (0,3)$ is feasible. The profit at R is $21(0)+11(3) = 33$. This is less than the profit at Q.

(e) Solving maximization problems involving linear constraints but where the variables are required to take on integer values cannot be solved by first solving the associated linear programming problem and rounding the answer to the nearest integers. This example shows the rounded value used to obtain an integer solution may not be feasible. Even if the rounded value is feasible it may not be optimal. "Integer programming" is unfortunately a much harder problem to solve than linear programming.

For Exercises 31 – 41, part (e) (using a simplex algorithm program) will not be addressed in the solutions.

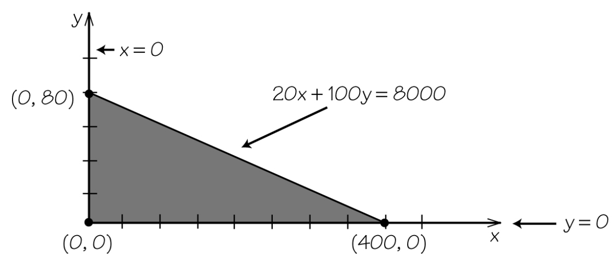
31. (a) Let *x* be the number of oil changes and *y* be the number tune-ups.

	Time (8,000 min)	Minimums	Profit
Oil changes, *x*	20	0	\$15
Tune-ups, *y*	100	0	\$65

(b) Profit formula: $P = \$15x + \$65y$

Constraints: $x \ge 0$ and $y \ge 0$ (minimums); $20x + 100y \le 8{,}000$ (time)

(c) Feasible region:



Corner points: The corner points are intercepts on the axes. These are $(0,0)$, $(0,80)$, and $(400,0)$.

Continued on next page

31. continued

(d) We wish to maximize $\$15x+\$65y$.

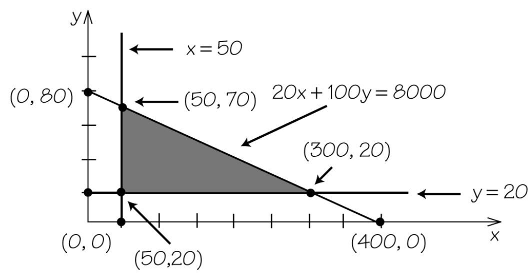
Corner Point	Value of the Profit Formula: $\$15x+\$65y$							
$(0,0)$	$\$15(0)$	+	$\$65(0)$	=	\$0	+	\$0	=	\$0
$(0,80)$	$\$15(0)$	+	$\$65(80)$	=	\$0	+	\$5200	=	\$5200
$(400,0)$	$\$15(400)$	+	$\$65(0)$	=	\$6000	+	\$0	=	\$6000*

Optimal production policy: Schedule 400 oil changes and no tune-ups.

With non-zero minimums, the constraints are as follows.

$$x \geq 50 \text{ and } y \geq 20 \ (\text{minimums}); 20x+100y \leq 8,000 \ (\text{time})$$

The feasible region looks like the following.



Corner points: One corner point is the point of intersection between $x=50$ and $y=20$, namely $(50,20)$. Another is the point of intersection between $x=50$ and $20x+100y=8000$. Substituting $x=50$ into $20x+100y=8000$, we have the following.

$$20(50)+100y=8000 \Rightarrow 1000+100y=8000 \Rightarrow 100y=7000 \Rightarrow y=70$$

Thus, $(50,70)$ is the second corner point. The third corner point is the point of intersection between $y=20$ and $20x+100y=8000$. Substituting $y=20$ into $20x+100y=8000$, we have the following.

$$20x+100(20)=8000 \Rightarrow 20x+2000=8000 \Rightarrow 20x=6000 \Rightarrow x=300$$

Thus, $(300,20)$ is the third corner point.

We wish to maximize $\$15x+\$65y$.

Corner Point	Value of the Profit Formula: $\$15x+\$65y$								
$(50,20)$	$\$15(50)$	+	$\$65(20)$	=	\$750	+	\$1300	=	\$2050
$(50,70)$	$\$15(50)$	+	$\$65(70)$	=	\$750	+	\$4550	=	\$5300
$(300,20)$	$\$15(300)$	+	$\$65(20)$	=	\$4500	+	\$1300	=	\$5800*

Optimal production policy: Schedule 300 oil changes and 20 tune-ups.

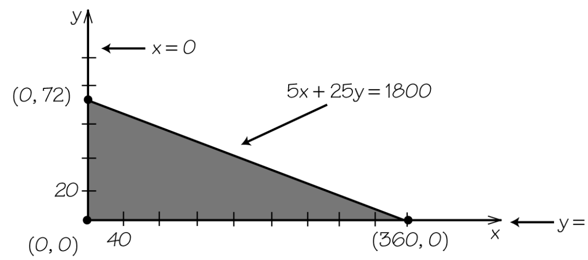
33. (a) Let x be the number of routine visits and y be the number of comprehensive visits.

	Doctor Time (1800 min)	Minimums	Profit
Routine, x visits	5	0	\$30
Comprehensive, y visits	25	0	\$50

(b) Profit formula: $P = \$30x + \$50y$

Constraints: $x \geq 0$ and $y \geq 0$ (minimums); $5x + 25y \leq 1800$ (time)

(c) Feasible region:



Corner points: The corner points are intercepts on the axes. These are $(0,0)$, $(0,72)$, and $(360,0)$.

(d) We wish to maximize $\$30x + \$50y$.

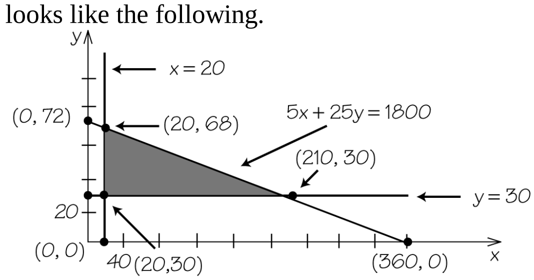
Corner Point	Value of the Profit Formula: $\$30x + \$50y$								
$(0,0)$	$\$30(0)$	+	$\$50(0)$	=	\$0	+	\$0	=	\$0
$(0,72)$	$\$30(0)$	+	$\$50(72)$	=	\$0	+	\$3600	=	\$3600
$(360,0)$	$\$30(360)$	+	$\$50(0)$	=	\$10,800	+	\$0	=	\$10,800*

Optimal production policy: Schedule 360 routine visits and no comprehensive visits.

With non-zero minimums, the constraints are as follows.

$$x \geq 20 \text{ and } y \geq 30 \text{ (minimums)}; 5x + 25y \leq 1800 \text{ (time)}$$

The feasible region looks like the following.



Corner points: One corner point is the point of intersection between $x = 20$ and $y = 30$, namely $(20,30)$. Another is the point of intersection between $x = 20$ and $5x + 25y = 1800$. Substituting $x = 20$ into $5x + 25y = 1800$, we have the following.

$$5(20) + 25y = 1800 \Rightarrow 100 + 25y = 1800 \Rightarrow 25y = 1700 \Rightarrow y = 68$$

Thus, $(20,68)$ is the second corner point. The third corner point is the point of intersection between $y = 30$ and $5x + 25y = 1800$. Substituting $y = 30$ into $5x + 25y = 1800$, we have the following.

$$5x + 25(30) = 1800 \Rightarrow 5x + 750 = 1800 \Rightarrow 5x = 1050 \Rightarrow x = 210$$

Thus, $(210,30)$ is the third corner point.

Continued on next page

33. continued

We wish to maximize $\$30x + \$50y$.

Corner Point	Value of the Profit Formula: $\$30x + \$50y$
$(20, 30)$	$\$30(20) + \$50(30) = \$600 + \$1500 = \$2100$
$(20, 68)$	$\$30(20) + \$50(68) = \$600 + \$3400 = \$4000$
$(210, 30)$	$\$30(210) + \$50(30) = \$6300 + \$1500 = \$7800*$

Optimal production policy: Schedule 210 routine visits and 30 comprehensive visits.

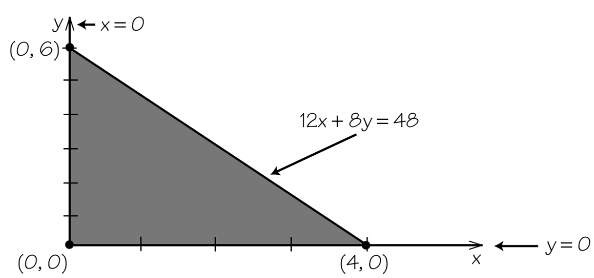
35. (a) Let x be the number of hours spent on math courses and y be the number of hours spent on other courses.

	Time (48 hr)	Minimums	Value Points
Math, x courses	12	0	2
Other, y courses	8	0	1

(b) Value Point formula: $V = 2x + y$

Constraints: $x \geq 0$ and $y \geq 0$ (minimums); $12x + 8y \leq 48$ (time)

(c) Feasible region:



Corner points: The corner points are intercepts on the axes. These are $(0,0)$, $(0,6)$, and $(4,0)$.

(d) We wish to maximize $2x + y$.

Corner Point	Value of the Value Point Formula: $2x + y$
$(0,0)$	$2(0) + 0 = 0 + 0 = 0$
$(0,6)$	$2(0) + 6 = 0 + 6 = 6$
$(4,0)$	$2(4) + 0 = 8 + 0 = 8*$

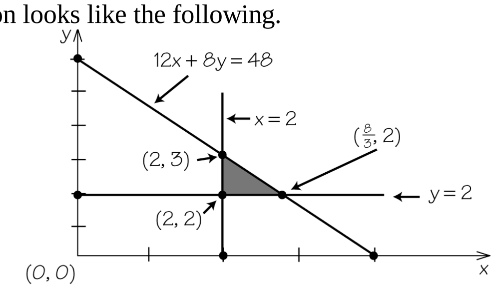
Optimal production policy: Take four math courses and no other courses.

Continued on next page

35. continued

With non-zero minimums, the constraints are $x \geq 2$ and $y \geq 2$ (minimums); $12x + 8y \leq 48$ (time).

The feasible region looks like the following.



Corner points: One corner point is the point of intersection between $x = 2$ and $y = 2$, namely $(2,2)$. Another is the point of intersection between $x = 2$ and $12x + 8y = 48$. Substituting $x = 2$ into $12x + 8y = 48$, we have the following.

$$12(2) + 8y = 48 \Rightarrow 24 + 8y = 48 \Rightarrow 8y = 24 \Rightarrow y = 3$$

Thus, $(2,3)$ is the second corner point.

The third corner point is the point of intersection between $y = 2$ and $12x + 8y = 48$. Substituting $y = 2$ into $12x + 8y = 48$, we have the following.

$$12x + 8(2) = 48 \Rightarrow 12x + 16 = 48 \Rightarrow 12x = 32 \Rightarrow x = \tfrac{32}{12} = \tfrac{8}{3} = 2\tfrac{2}{3}$$

Thus, $\left(\frac{8}{3}, 2\right)$ is the third corner point.

We wish to maximize $2x + y$.

Corner Point	Value of the Profit Formula: $2x + y$
$(2,2)$	$2(2) + 2 = 4 + 2 = 6$
$(2,3)$	$2(2) + 3 = 4 + 3 = 7$
$\left(\frac{8}{3}, 2\right)$	$2\left(\frac{8}{3}\right) + 2 = 5\frac{1}{3} + 2 = 7\frac{1}{3}$*

Optimal production policy: Take $2\frac{2}{3}$ math courses and 2 other courses. However, one cannot take a fractional part of a course. So given the constraint on study time, the student should take two math courses and 2 other courses.

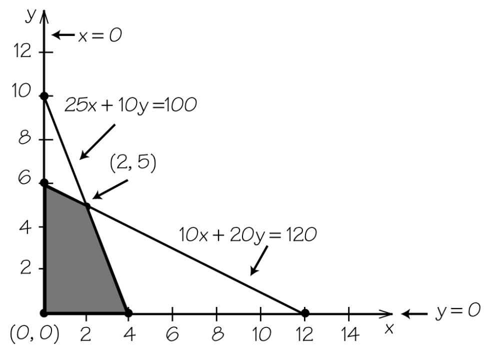
37. (a) Let x be the number of Grade A batches and y be the number of Grade B batches.

	Scrap cloth (100 lb)	Scrap Paper (120 lb)	Minimums	Profit
Grade A, x batches	25	10	0	\$500
Grade B, y batches	10	20	0	\$250

(b) Profit formula: $P = \$500x + \$250y$

Constraints: $x \geq 0$ and $y \geq 0$ (minimums); $25x + 10y \leq 100$ (cloth); $10x + 20y \leq 120$ (paper)

(c) Feasible region:



Corner points: Three of the corner points are intercepts on the axes. These are $(0,0)$, $(0,6)$, and $(4,0)$. The final corner point is the point of intersection between $25x + 10y = 100$ and $10x + 20y = 120$. We can find this by multiplying both sides of $25x + 10y = 100$ by -2, and adding the result to $10x + 20y = 120$.

$$\begin{array}{rl} -50x - 20y &= -200 \\ \underline{10x + 20y} &\underline{= 120} \\ -40x \qquad &= -80 \Rightarrow x = \frac{-80}{-40} = 2 \end{array}$$

Substitute $x = 2$ into $25x + 10y = 100$ and solve for y. We have the following.

$$25(2) + 10y = 100 \Rightarrow 50 + 10y = 100 \Rightarrow 10y = 50 \Rightarrow y = 5$$

The point of intersection is therefore $(2,5)$.

(d) We wish to maximize $\$500x + \$250y$.

Corner Point	Value of the Profit Formula: $\$500x + \$250y$								
$(0,0)$	\$500(0)	+	\$250(0)	=	\$0	+	\$0	=	\$0
$(0,6)$	\$500(0)	+	\$250(6)	=	\$0	+	\$1500	=	\$1500
$(4,0)$	\$500(4)	+	\$250(0)	=	\$2000	+	\$0	=	\$2000
$(2,5)$	\$500(2)	+	\$250(5)	=	\$1000	+	\$1250	=	\$2250*

Optimal production policy: Make 2 grade A and 5 grade B batches.

With non-zero minimums $x \geq 1$ and $y \geq 1$, there will be no change because the optimal production policy already obeys these non-zero minimums.

39. (a) Let x be the number of cartons of regular soda and y be the number of cartons of diet soda.

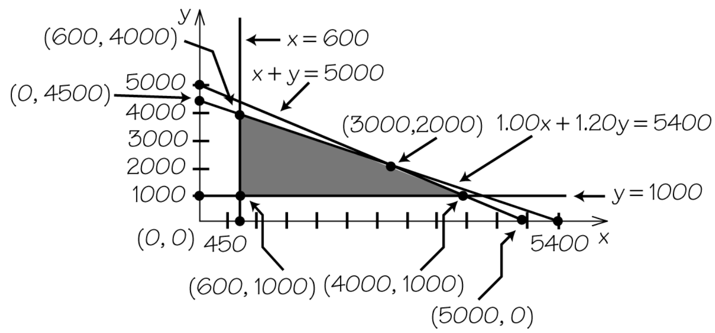
	Cartons (5000)	Money ($5400)	Minimums	Profit
Regular, x cartons	1	$1.00	600	$0.10
Diet, y cartons	1	$1.20	1000	$0.11

(b) Profit formula: $P = \$0.10x + \$0.11y$

Constraints: $x \geq 600$ and $y \geq 1000$ (minimums)

$$x + y \leq 5000 \text{ (cartons)}; 1.00x + 1.20y \leq 5400 \text{ (money)}$$

(c) Feasible region:



Corner points: One corner point is the point of intersection between $x = 600$ and $y = 1000$, namely $(600,1000)$. Another is the point of intersection between $y = 1000$ and $x + y = 5000$. Substituting $y = 1000$ into $x + y = 5000$, we have $x + 1000 = 5000 \Rightarrow x = 4000$. Thus, $(4000,1000)$ is the second corner point. The third corner point is the point of intersection between $x = 600$ and $1.00x + 1.20y = 5400$. Substituting $x = 600$ into $1.00x + 1.20y = 5400$, we have the following.

$$1.00(600) + 1.20y = 5400 \Rightarrow 600 + 1.20y = 5400 \Rightarrow 1.20y = 4800 \Rightarrow y = 4000$$

Thus, $(600, 4000)$ is the third corner point. The fourth corner point is the point of intersection between $x + y = 5000$ and $1.00x + 1.20y = 5400$. We can find this by multiplying both sides of $x + y = 5000$ by -1, and adding the result to $1.00x + 1.20y = 5400$.

$$\begin{array}{r} -x - \quad y = -5000 \\ 1.00x + 1.20y = 5400 \\ \hline 0.20y = 400 \Rightarrow y = 2000 \end{array}$$

Substitute $y = 2000$ into $x + y = 5000$ and solve for y. We have $x + 2000 = 5000$ or $x = 3000$. The point of intersection is therefore $(3000, 2000)$.

(d) We wish to maximize $\$0.10x + \$0.11y$.

Corner Point	Value of the Profit Formula: $\$0.10x + \$0.11y$								
$(600,1000)$	$\$0.10(600)$	+	$\$0.11(1000)$	=	$60	+	$110	=	$170
$(4000,1000)$	$\$0.10(4000)$	+	$\$0.11(1000)$	=	$400	+	$110	=	$510
$(600, 4000)$	$\$0.10(600)$	+	$\$0.11(4000)$	=	$60	+	$440	=	$500
$(3000, 2000)$	$\$0.10(3000)$	+	$\$0.11(2000)$	=	$300	+	$220	=	$520*

Optimal production policy: Make 3000 cartons of regular and 2000 cartons of diet.

With zero minimums there is no change in the optimal production policy because the corresponding corner point does not touch either line from a minimum constraint.

41. (a) Let x be the number of desk lamps and y be the number of floor lamps.

	Labor (1200 hr)	Money ($4200)	Minimums	Profit
Desk, x lamps	0.8	$4	0	$2.65
Floor, y lamps	1.0	$3	0	$4.67

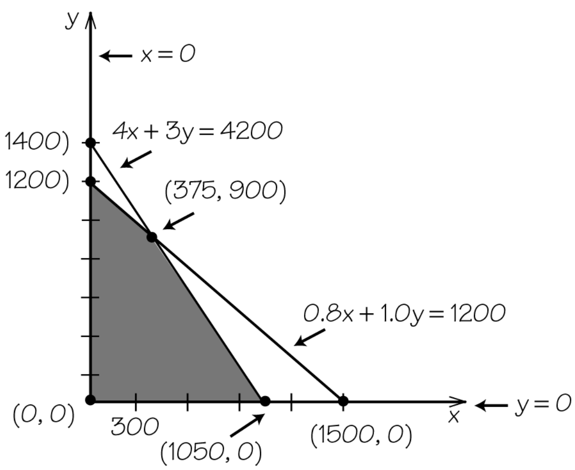
(b) Profit formula: $P = \$2.65x + \$4.67y$

Constraints: $x \geq 0$ and $y \geq 0$ (minimums)

$0.8x + 1.0y \leq 1200$ (labor)

$4x + 3y \leq 4200$ (money)

(c) Feasible region:



Corner points: Three of the corner points are intercepts on the axes. These are $(0,0)$, $(0,1200)$, and $(1050,0)$. The final corner point is the point of intersection between $0.8x + 1.0y = 1200$ and $4x + 3y = 4200$. We can find this by multiplying both sides of $0.8x + 1.0y = 1200$ by -3, and adding the result to $4x + 3y = 4200$.

$$\begin{array}{r} -2.4x - 3y = -3600 \\ 4x + 3y = 4200 \\ \hline 1.6x \qquad = 600 \Rightarrow x = 375 \end{array}$$

Substitute $x = 375$ into $0.8x + 1.0y = 1200$ and solve for y. We have the following.

$$0.8(375) + y = 1200 \Rightarrow 300 + y = 1200 \Rightarrow y = 900$$

The point of intersection is therefore $(375, 900)$.

(d) We wish to maximize $\$2.65x + \$4.67y$.

Corner Point	Value of the Profit Formula: $\$2.65x + \$4.67y$							
$(0,0)$	$2.65(0)	+	$4.67(0)	=	$0.00	+	$0.00	= $0.00
$(0,1200)$	$2.65(0)	+	$4.67(1200)	=	$0.00	+	$5604.00	= $5604.00*
$(1050,0)$	$2.65(1050)	+	$4.67(0)	=	$2782.50	+	$0.00	= $2782.50
$(375,900)$	$2.65(375)	+	$4.67(900)	=	$993.75	+	$4203.00	= $5196.75

Optimal production policy: Make no desk lamps and 1200 floor lamps.

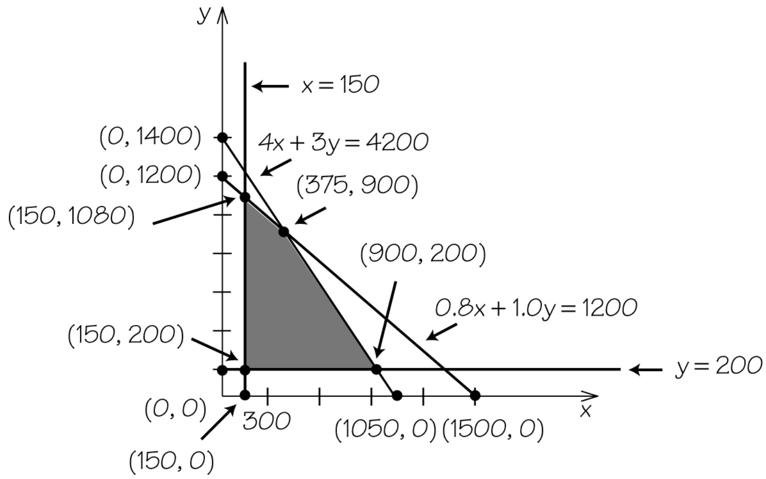
Continued on next page

41. continued

With non-zero minimums, the constraints are as follows.

$x \geq 150$ and $y \geq 200$ (minimums); $0.8x + 1.0y \leq 1200$ (labor); $4x + 3y \leq 4200$ (money)

The feasible region looks like the following.



Corner points: One corner point is the point of intersection between $x = 150$ and $y = 200$, namely $(150, 200)$. Another is the point of intersection between $y = 200$ and $4x + 3y = 4200$. Substituting $y = 200$ into $4x + 3y = 4200$, we have the following.

$$4x + 3(200) = 4200 \Rightarrow 4x + 600 = 4200 \Rightarrow 4x = 3600 \Rightarrow x = 900$$

Thus, $(900, 200)$ is the second corner point. The third corner point is the point of intersection between $x = 150$ and $0.8x + 1.0y = 1200$. Substituting $x = 150$ into $0.8x + 1.0y = 1200$, we have the following.

$$0.8(150) + y = 1200 \Rightarrow 120 + y = 1200 \Rightarrow y = 1080$$

Thus, $(150, 1080)$ is the third corner point. The fourth corner point is the point of intersection between $0.8x + 1.0y = 1200$ and $4x + 3y = 4200$. In part (a) this was found to be $(375, 900)$.

We wish to maximize $\$2.65x + \$4.67y$.

Corner Point	Value of the Profit Formula: $\$2.65x + \$4.67y$ (in thousands)									
$(150, 200)$	$\$2.65(150)$	+	$\$4.67(200)$	=	\$397.50	+	\$934.00	=	\$1331.50	
$(150, 1080)$	$\$2.65(150)$	+	$\$4.67(1080)$	=	\$397.50	+	\$5043.60	=	\$5441.10	*
$(375, 900)$	$\$2.65(375)$	+	$\$4.67(900)$	=	\$993.75	+	\$4203.00	=	\$5196.75	
$(900, 200)$	$\$2.65(900)$	+	$\$4.67(200)$	=	\$2385.00	+	\$934.00	=	\$3319.00	

Optimal production policy: Make 150 desk lamps and 1080 floor lamps.

For Exercises 43 & 45, part (c) (using a simplex algorithm program) will not be addressed in the solutions.

43. (a) Let x be the number of chairs, y be the number of tables, and z be the number of beds.

	Chis (80 hr)	Sue (200 hr)	Juan (200)	Minimums	Profit
Chairs, x	1	3	2	0	\$100
Tables, y	3	5	4	0	\$250
Beds, z	5	4	8	0	\$350

(b) Profit formula: $P = \$100x + \$250y + \$350z$

Constraints: $x \geq 0,\ y \geq 0,$ and $z \geq 0$ (minimums)

$$x + 3y + 5z \leq 80\ (\text{Chris}); 3x + 5y + 4z \leq 200(\text{Sue}); 2x + 4y + 8z \leq 200(\text{Juan})$$

(c) Optimal product policy: Make 50 chairs, 10 tables, and no beds each month for a profit of 7500 in one month.

45. (a) Let w be the number of pounds of Excellent coffee, x be the number of pounds of Southern coffee, y be the number of pounds of World coffee, and z be the number of pounds of Special coffee.

	African (17,600 oz)	Brazilian (21,120 oz)	Columbian (12,320 oz)	Minimums	Profit
Excellent, w pounds	0	0	16	0	\$1.80
Southern, x pounds	0	12	4	0	\$1.40
World, y pounds	6	8	2	0	\$1.20
Special, z pounds	10	6	0	0	\$1.00

(b) Profit formula: $P = \$1.80w + \$1.40x + \$1.20y + \$1.00z$

Constraints: $w \geq 0, x \geq 0,\ y \geq 0,$ and $z \geq 0$ (minimums)

$$0w + 0x + 6y + 10z \leq 17,600\ (\text{African})$$

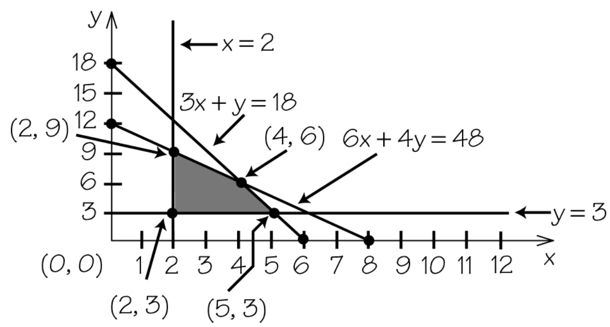
$$0w + 12x + 8y + 6z \leq 12,120\ (\text{Brazilian})$$

$$16w + 4x + 2y + 6z \leq 12,320\ (\text{Columbian})$$

(c) Optimal product policy: Make 470 pounds of Excellent, none of Southern, 2400 pounds of World, and 320 pounds for a profit of \$4046.

47. Minimize $C = 3x + 11y$ subject to $x \geq 2$; $y \geq 3$; $3x + y \leq 18$; $6x + 4y \leq 48$

The corner points are $(2,3), (5,3), (4,6)$, and $(2,9)$.



We wish to minimize $C = 3x + 11y$.

Corner Point	Value of the Profit Formula: $3x + 11y$								
$(2,3)$	$3(2)$	+	$11(3)$	=	6	+	33	=	39*
$(5,3)$	$3(5)$	+	$11(3)$	=	15	+	33	=	48
$(4,6)$	$3(4)$	+	$11(6)$	=	12	+	66	=	78
$(2,9)$	$3(2)$	+	$11(9)$	=	6	+	99	=	105

The minimum value occurs at the corner point $(2,3)$, where C is equal to 39.

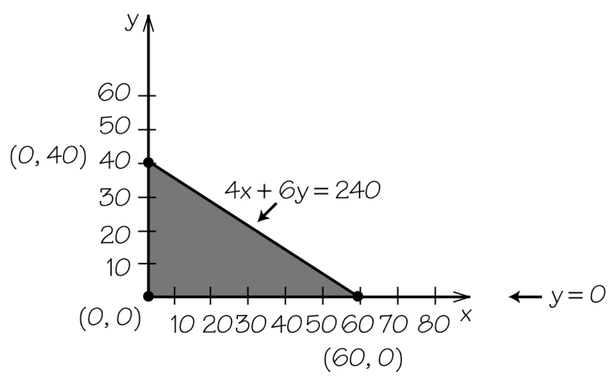
49. (a) Let x be the number of business calls and y be the charity calls.

	Time (240 min)	Minimums	Profit
Business, x calls	4	0	\$0.50
Charity, y calls	6	0	\$0.40

(b) Profit formula: $P = \$0.50x + \$0.40y$

Constraints: $x \geq 0$ and $y \geq 0$ (minimums); $4x + 6y \leq 240$ (time)

(c) Feasible region:



Corner points: The corner points are intercepts on the axes. These are $(0,0)$, $(0,40)$, and $(60,0)$.

Continued on next page

49. continued

(d) We wish to maximize $\$0.50x + \$0.40y$.

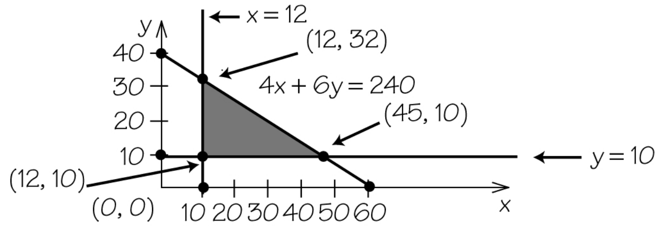
Corner Point	Value of the Profit Formula: $\$0.50x+\$0.40y$								
$(0,0)$	$\$0.50(0)$	$+$	$\$0.40(0)$	$=$	$\$0.00$	$+$	$\$0.00$	$=$	$\$0.00$
$(0,40)$	$\$0.50(0)$	$+$	$\$0.40(40)$	$=$	$\$0.00$	$+$	$\$16.00$	$=$	$\$16.00$
$(60,0)$	$\$0.50(60)$	$+$	$\$0.40(0)$	$=$	$\$30.00$	$+$	$\$0.00$	$=$	$\$30.00$*

Optimal production policy: Make 60 business and no charity calls.

With non-zero minimums, the constraints are as follows.

$$x \geq 12 \text{ and } y \geq 10 \ (\text{minimums}); 4x + 6y \leq 240 \ (\text{time})$$

The feasible region looks like the following.



Corner points: One corner point is the point of intersection between $x = 12$ and $y = 10$, namely $(12,10)$. Another is the point of intersection between $x = 12$ and $4x + 6y = 240$. Substituting $x = 12$ into $4x + 6y = 240$, we have the following.

$$4(12) + 6y = 240 \Rightarrow 48 + 6y = 240 \Rightarrow 6y = 192 \Rightarrow y = 32$$

Thus, $(12,32)$ is the second corner point. The third corner point is the point of intersection between $y = 10$ and $4x + 6y = 240$. Substituting $y = 10$ into $4x + 6y = 240$ we have the following.

$$4x + 6(10) = 240 \Rightarrow 4x + 60 = 240 \Rightarrow 4x = 180 \Rightarrow x = 45$$

Thus, $(45,10)$ is the third corner point.

We wish to maximize $\$0.50x + \$0.40y$.

Corner Point	Value of the Profit Formula: $\$0.50x+\$0.40y$								
$(12,10)$	$\$0.50(12)$	$+$	$\$0.40(10)$	$=$	$\$6.00$	$+$	$\$4.00$	$=$	$\$10.00$
$(12,32)$	$\$0.50(12)$	$+$	$\$0.40(32)$	$=$	$\$6.00$	$+$	$\$12.80$	$=$	$\$18.80$
$(45,10)$	$\$0.50(45)$	$+$	$\$0.40(10)$	$=$	$\$22.50$	$+$	$\$4.00$	$=$	$\$26.50$*

Optimal production policy: Make 45 business and 10 charity calls.

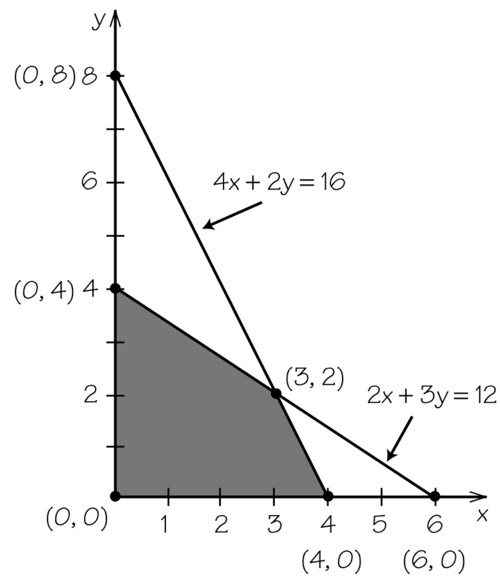
51. (a) Let x be the number of bikes and y be the number of wagons.

	Machine (12 hr)	Paint(16 hr.)	Minimums	Profit
Bikes, x	2	4	0	\$12
Wagons, y	3	2	0	\$10

(b) Profit formula: $P = \$12x + \$10y$

Constraints: $x \geq 0$ and $y \geq 0$ (minimums); $2x + 3y \leq 12$ (machine); $4x + 2y \leq 16$ (paint)

(c) Feasible region:



Corner points: Three of the corner points are intercepts on the axes. These are $(0,0)$, $(0,4)$, and $(4,0)$. The final corner point is the point of intersection between $2x + 3y = 12$ and $4x + 2y = 16$. We can find this by multiplying both sides of $2x + 3y = 12$ by -2, and adding the result to $4x + 2y = 16$.

$$\begin{array}{r} -4x - 6y = -24 \\ \underline{4x + 2y = 16} \\ -4y = -8 \Rightarrow y = 2 \end{array}$$

Substitute $y = 2$ into $4x + 2y = 16$ and solve for x. We have the following.

$$4x + 2(2) = 16 \Rightarrow 4x + 4 = 16 \Rightarrow 4x = 12 \Rightarrow x = 3$$

The point of intersection is therefore $(3,2)$.

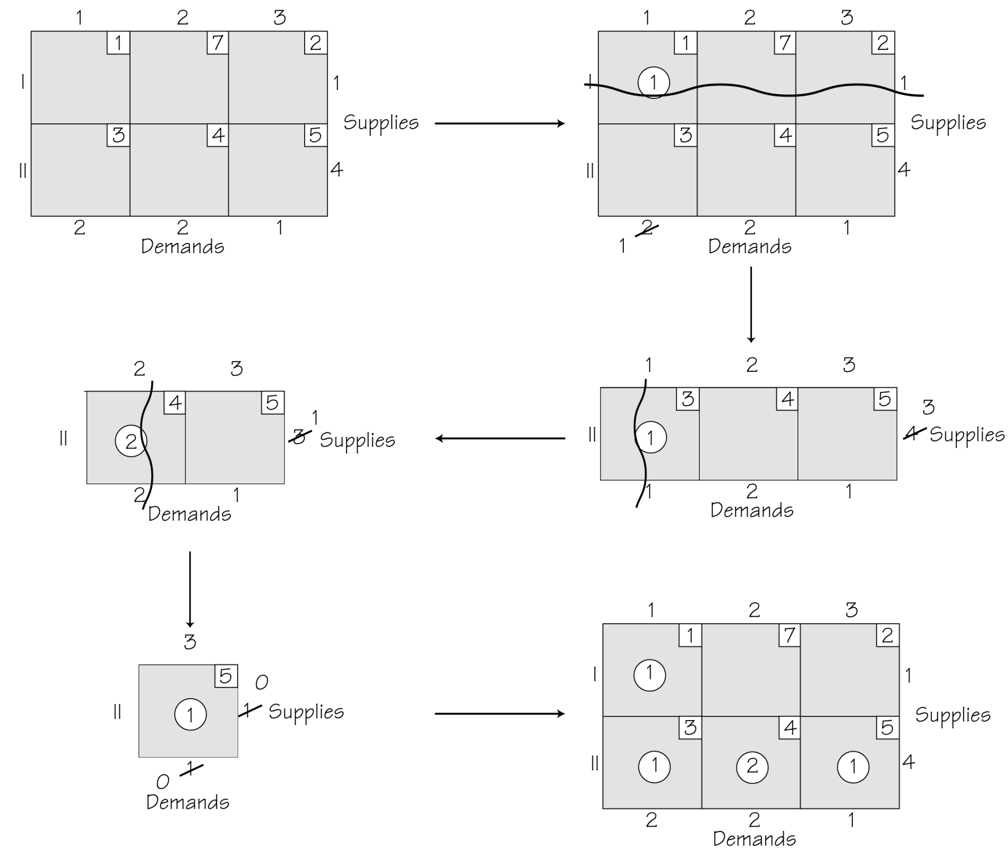
(d) We wish to maximize $\$12x + \$10y$.

Corner Point	Value of the Profit Formula: $\$12x + \$10y$								
$(0,0)$	$\$12(0)$	+	$\$10(0)$	=	\$0	+	\$0	=	\$0
$(0,4)$	$\$12(0)$	+	$\$10(4)$	=	\$0	+	\$40	=	\$40
$(4,0)$	$\$12(4)$	+	$\$10(0)$	=	\$48	+	\$0	=	\$48
$(3,2)$	$\$12(3)$	+	$\$10(2)$	=	\$36	+	\$20	=	\$56*

Optimal production policy: Make 3 bikes and 2 wagons.

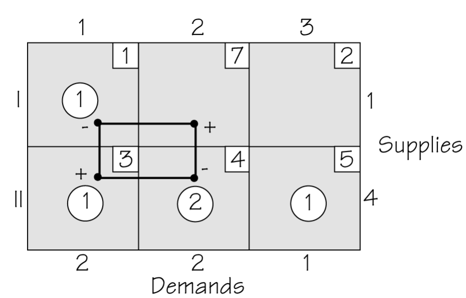
With non-zero minimums $x \geq 2$ and $y \geq 2$, there will be no change because the optimal production policy already obeys these non-zero minimums.

53. (a)

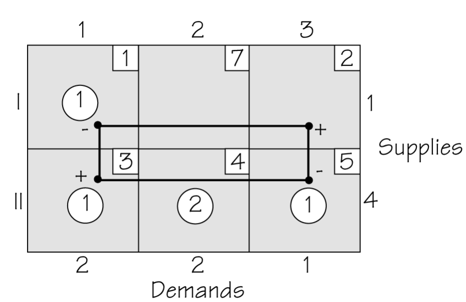


(b) The cost for this solution is $1(1)+1(3)+2(4)+1(5)=1+3+8+5=17$.

(c) The indicator value for cell $(\mathrm{I},2)$ is $7-1+3-4=5$.

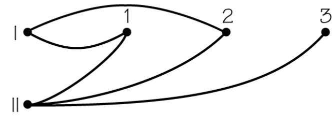


The indicator value for cell $(\mathrm{I},3)$ is $2-1+3-5=-1$.

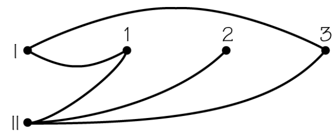


55. (a) The graph is a tree because it is connected and has no circuit.

(b) If we add the edge joining Vertex I to Vertex 2 we get the circuit 2, I, 1, II, 2.



If we add the edge from Vertex I to Vertex 3 we get the circuit 3, I, 1, II, 3.



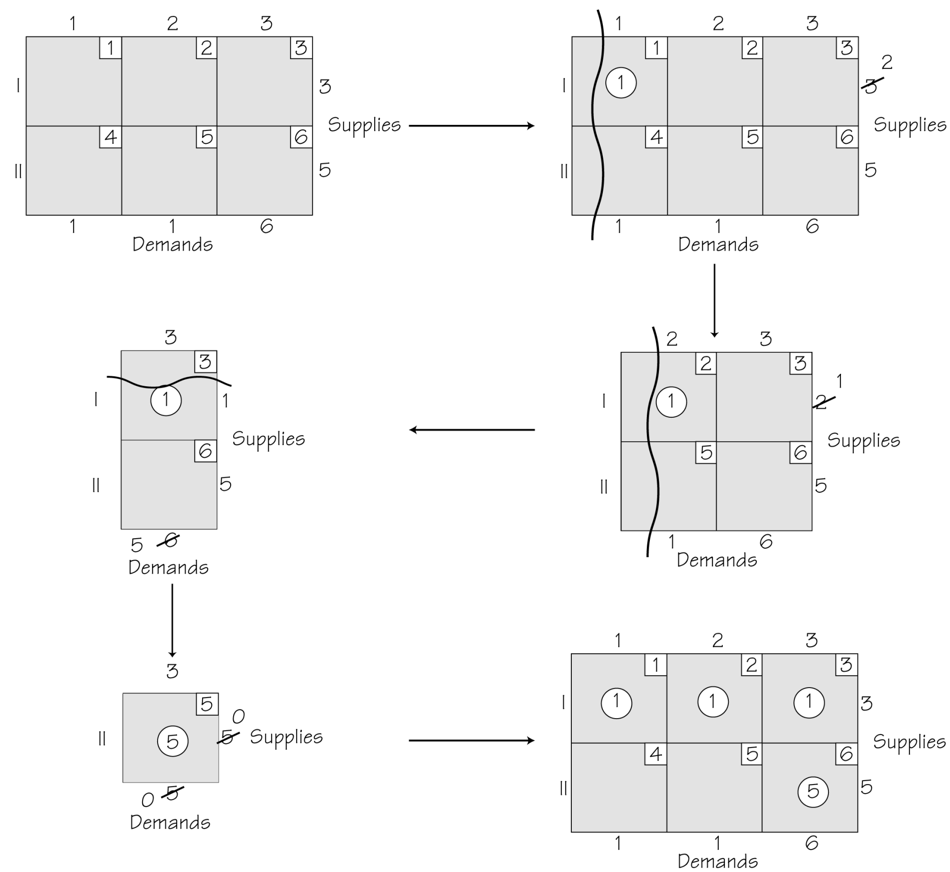
(c) For the circuit 2, I, 1, II, 2 it corresponds to the following circuit of cells.

$$(\mathrm{I},2),(\mathrm{I},1),(\mathrm{II},1),(\mathrm{II},2),(\mathrm{I},2)$$

For the circuit 3, I, 1, II, 3 it corresponds to the following circuit of cells.

$$(\mathrm{I},3),(\mathrm{I},1),(\mathrm{II},1),(\mathrm{II},3),(\mathrm{I},3)$$

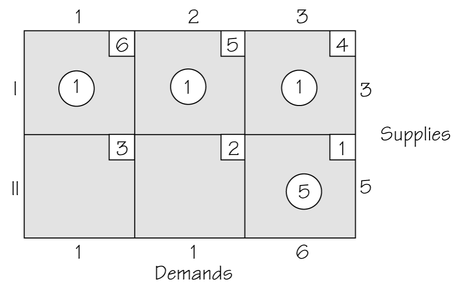
57. (a) (i)



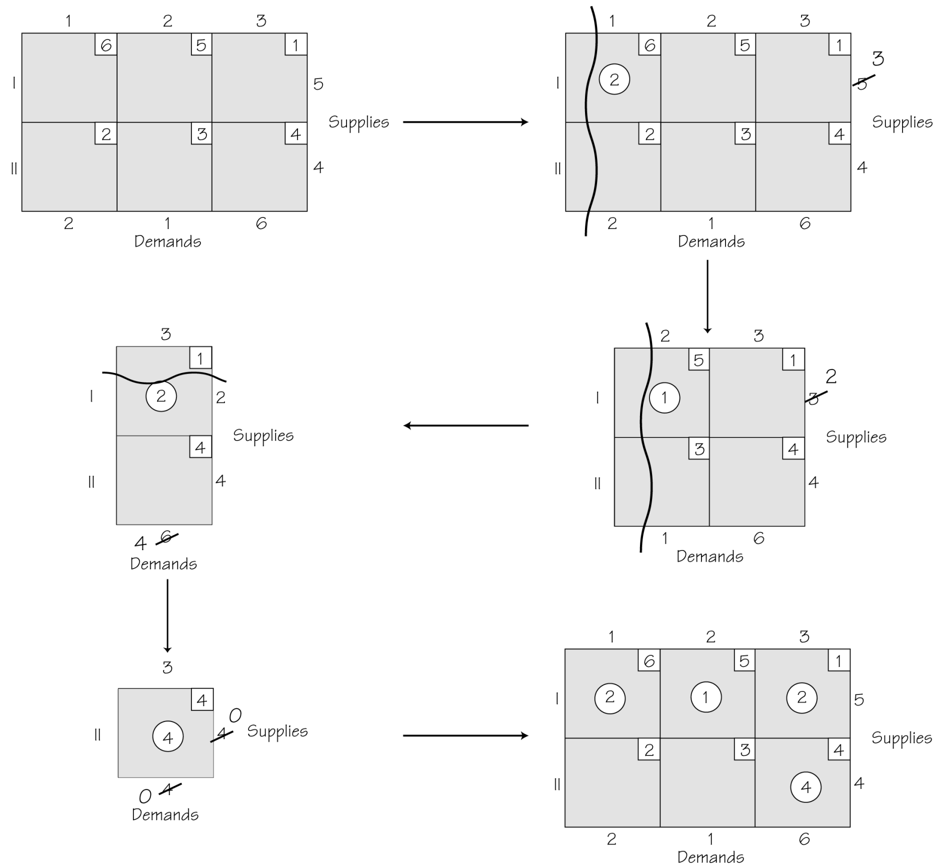
Continued on next page

57. (a) continued

(ii) Since the rim values are the same, the end result will be the same (relative to the new table)



(iii)

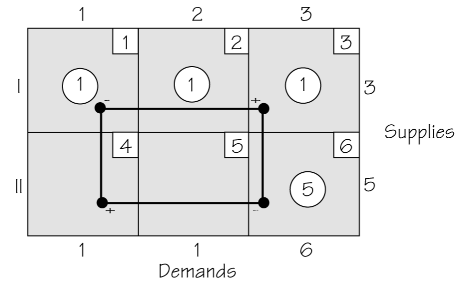


Continued on next page

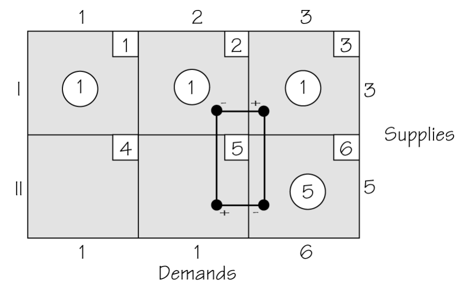
57. continued

(b) For (i) the indicator value for each non-circled cell is calculated as follows.

cell $(\text{II},1): 4-6+3-1=0$



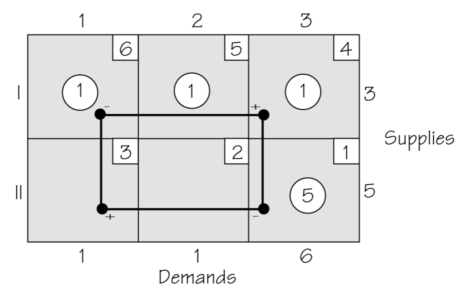
cell $(\text{II},2): 5-6+3-2=0$



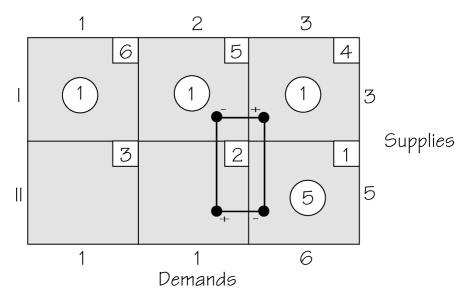
The tableau shown is optimal. However, there are also other optimal tableaux, as can be seen from the fact that the indicator values for each of the cells that have no circled entries are 0.

For (ii) the indicator value for each non-circled cell is calculated as follows.

cell $(\text{II},1): 3-1+4-6=0$



cell $(\text{II},2): 2-1+4-5=0$



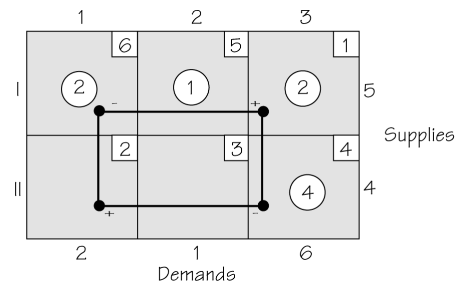
The tableau shown is optimal, although there are also other optimal tableaux.

Continued on next page

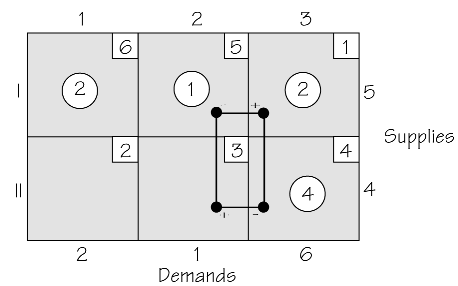
57. (b) continued

For (iii) the indicator value for each non-circled cell is calculated as follows.

cell $(\text{II},1): 2-4+1-6=-3$



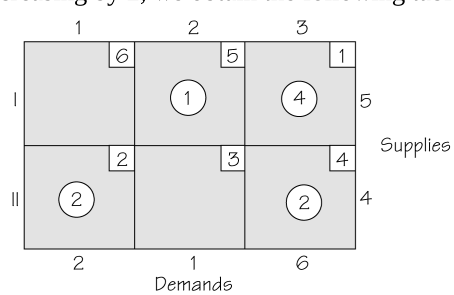
cell $(\text{II},2): 3-4+1-5=-5$



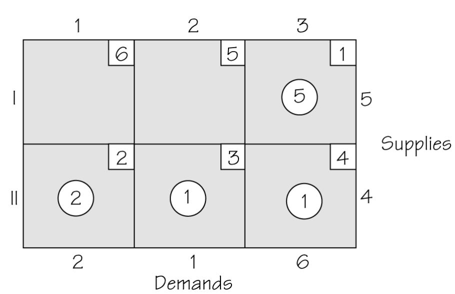
The tableau shown is not optimal. The current cost is as follows.

$$2(6)+1(5)+2(1)+4(4)=12+5+2+16=35$$

Since cell $(\text{II},1)$ has a more negative indicator value, we can reduce the cost more by using that cell. Increasing by 2, we obtain the following tableau.



The cost is now $1(5)+4(1)+2(4)+2(2)=5+4+8+4=21$. We can reduce the cost more by using cell $(\text{II},2)$. Increasing by 1, we obtain the following tableau.



The cost is now $2(2)+1(3)+1(4)+5(1)=4+3+4+5=16$.