Chapter 2
Business Efficiency

Solutions

Exercises:

1. (a) $X_5X_6X_1X_3X_4X_2X_5$

(b) $X_5X_4X_3X_2X_1X_6X_7X_8X_9X_{10}X_{11}X_{12}X_5$

(c) $X_5X_4X_3X_1X_2X_7X_6X_9X_8X_5$

(d) $X_5X_8X_3X_4X_7X_6X_1X_2X_5$

(e) $X_5X_4X_3X_2X_8X_1X_{10}X_7X_6X_9X_5$

3. (a) A Hamiltonian circuit will remain for (a) and (b), but there will be no Hamiltonian circuit for (c), (d), and (e),

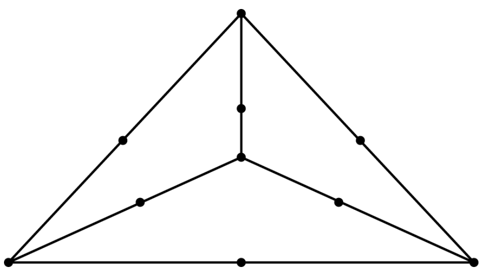
(b) The removal of a vertex might correspond to the failure of the equipment at that site.

5. Other Hamiltonian circuits include *ABIGDCEFHA* and *ABDCEFGIHA*.

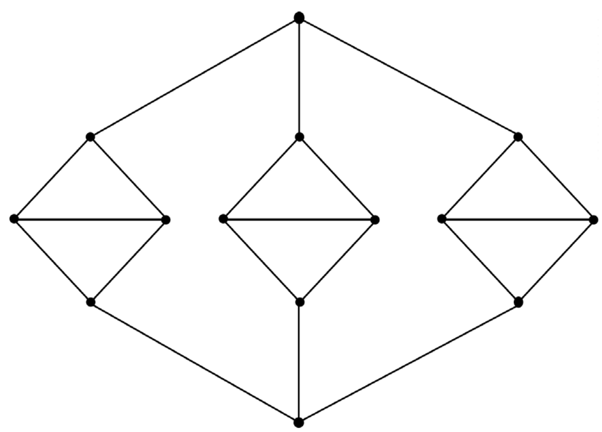
7. (a) a. Add edge *AB*.

b. Add edge X_1X_3.

(b)



9. The graph below has no Hamiltonian circuit and every vertex of the graph has valence 3.



11. (a) Any Hamiltonian circuit would have to use both edges at the vertices X_5, X_4, and X_2. This would cause a problem in the way a Hamiltonian circuit could visit vertex X_1. Thus, no Hamiltonian circuit exists.

(b) If there were a Hamiltonian circuit, it would have to use the edges X_4 and X_5 and X_6 and X_7. This would make it impossible for the Hamiltonian circuit to visit X_8 and X_9. Thus, no Hamiltonian circuit exists.

13. (a) No Hamilton circuit.

(b) No Hamilton circuit.

(c) No Hamilton circuit.

15. (a) There is a Hamiltonian path from X_3 to X_4.

(b) No. There is a Hamiltonian path from X_1 to X_8 in graph (b).

(c) Here are two examples. A worker who inspects sewers may start at one garage at the start of the work day but may have to report to a different garage for the afternoon shift. A school bus may start at a bus garage and then pick up students to take them to school, where the bus sits until the end of the school day.

17. (a) Hamiltonian circuit, yes. One example is: $X_1X_3X_7X_5X_6X_8X_2X_4X_1$; Euler circuit, no.

(b) Hamiltonian circuit, yes. Euler circuit, yes.

(c) Hamiltonian circuit, yes. Euler circuit, no.

(d) Hamiltonian circuit, no. Euler circuit, yes. One example is as follows.

$$U_1U_2U_5U_6U_{16}U_{15}U_{11}U_4U_5U_{12}U_{11}U_{10}U_{14}U_{13}U_7U_3U_8U_{10}U_9U_3U_1.$$

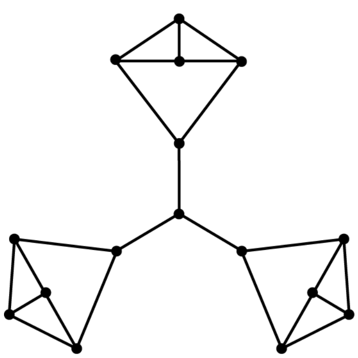
19. (a) Hamiltonian circuit, yes; Euler circuit, no.

(b) Hamiltonian circuit, yes; Euler circuit, no.

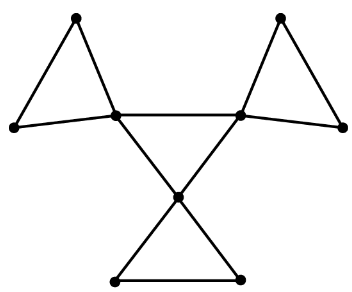
(c) Hamiltonian circuit, yes; Euler circuit, no.

(d) Hamiltonian circuit, no; Euler circuit, no.

21. (a)



(b)



(c) A graph has an Eulerian path if for two different vertices *u* and *v* of the graph there is a path from *u* to *v* that uses each edge of the graph once and only once.

23. The new system is an improvement since it codes 676 locations compared with 504 for the old system. This is 172 more locations.

25. (a) $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 15{,}120$

(b) $(26)(26)(26) = 17{,}576$

27. (a) $7 \times 6 \times 5 \times 4 \times 3 = 2520$

(b) $7 \times 7 \times 7 \times 7 \times 7 = 16{,}807$

(c) $7^5 - 7 = 16{,}800$

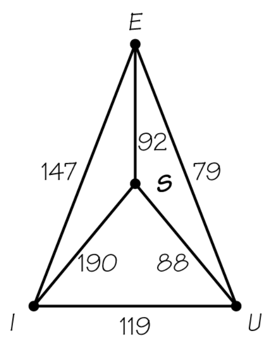
29. (a) $(26)(26)(26)(10)(10)(10)-(26)(26)(26)=26^3\left(10^3-1\right)=17{,}558{,}424$

(b) Answers will vary.

31. With no other restrictions, $10^7=10{,}000{,}000$. With no other restrictions, $9\times10^2=900$.

33. These graphs have 6, 10, and 15 edges, respectively. The *n*-vertex complete graph has $\frac{n(n-1)}{2}$ edges. The number of TSP tours is 3, 12, and 60, respectively.

35. (a)



(b) (1) *UISEU;* mileage $=119+190+92+79=480$
(2) *USIEU;* mileage $=88+190+147+79=504$
(3) *UIESU;* mileage $=119+147+92+88=446$

(c) *UIESU* (Tour 3)

(d) No.

(e) Starting from U, one gets *UESIU* Tour 1. From *S* one gets *SUEIS* Tour 2; from *E* one gets *EUSIE* Tour 2; and from *I* one gets *IUESI* Tour 1.

(f) *EUSIE* Tour 2. No.

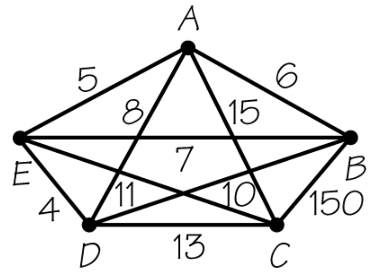
37. *FMCRF* gets her home in 36 minutes.

39. *MACBM* takes 344 minutes to traverse.

41. A traveling salesman problem.

43. A sewer drain inspection route at corners involves finding a Hamiltonian circuit, and there is such a circuit. If the drains are along the blocks, a route in this case involves solving a Chinese postman problem. Since there are 18 odd-valent vertices, an optimal route would require at least 9 reuses of edges. There are many such routes that achieve 9 reuses.

45. The complete graph shown has a different nearest-neighbor tour that starts at *A* (*AEDBCA*), a sorted-edges tour (*AEDCB*A), and a cheaper tour (*ADBECA*).



47. The optimal tour is the same but its cost is now $4200 + 10(30) = 4500$.

49. (a) a. Not a tree because there is a circuit. Also, the wiggled edges do not include all vertices of the graph.
b. The circuit does not include all the vertices of the graph.

(b) a. The tree does not include all vertices of the graph.
b. Not a circuit.

(c) a. Not a tree.
b. Not a circuit.

(d) a. Not a tree.
b. Not a circuit.

51. (a) 1, 2, 3, 4, 5, 8

(b) 1, 1, 1, 2, 2, 3, 3, 4, 5, 6, 6

(c) 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 5, 5, 6, 7

(d) 1, 2, 2, 3, 3, 3, 4, 5, 5, 5, 6, 6

The cost is found by adding the numbers given.

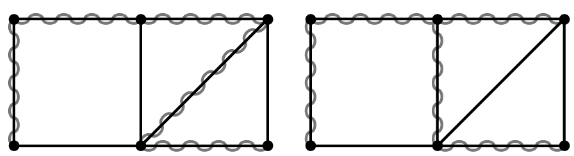
53. The spanning tree will have 26 vertices. *H* also will have 26 vertices. The exact number of edges in *H* cannot be determined, but *H* has at least 25 edges and no more edges than the complete graph having 26 vertices.

55. Yes.

57. Yes. Change all the weights to negative numbers and apply Kruskal's algorithm. The resulting tree works, and the maximum cost is the negative of the answer you get. If the numbers on the edges represent subsidies for using the edges, one might be interested in finding a maximum-cost spanning tree.

59. A negative weight on an edge is conceivable, perhaps a subsidization payment. Kruskal's algorithm would still apply.

61. Two different trees with the same cost are shown:



63. (a) True

(b) False (unless all the edges of the graph have the same weight)

(c) True

(d) False

(e) False

65. (a) Answers will vary for each edge, but the reason it is possible to find such trees is that each edge is an edge of some circuit.

(b) The number of edges in every spanning tree is five, one less than the number of vertices in the graph.

(c) Every spanning tree must include the edge joining vertices *C* and *D*, since this edge does not belong to any (simple) circuit in the graph.

67.

	A	B	C	D
A	0	14	13	5
B	14	0	17	9
C	13	17	0	8
D	5	9	8	0

69. (a) The earliest completion time is 22 since the longest path, the unique critical path $T_3T_2T_5$, has length 22.

(b) The earliest completion time is 30 since the longest path, the unique critical path $T_3T_5T_7$, has length 30.

71. The only tasks which if shortened will reduce the earliest completion time are those on the critical path, so in this case, these are the tasks T_1, T_5, and T_7. If T_5 is shortened to 7, then the longest path will have length 28, and this becomes the earliest completion time. The tasks on this critical path are T_1, T_4, and T_7.

73. Different contractors will have different times and order-requirement digraphs. However, in any sensible order requirement digraph, the laying of the foundation will come before the erection of the side walls and the roof. The fastest time for completing all the tasks will be the length of the longest path in the order-requirement digraph.

75. One example is given below.