Chapter 1
Urban Services

Solutions

Exercises:

1. *E* has valence 0; *A* has valence 1; *H*, *D*, and *G* have valence 2; *B* and *F* have valence 3; *C* has valence 5. *E* is "isolated." *E* might have valence 0 because it is on an island with no road access.

3. (a) This diagram fails to be a graph because a line segment joins a single vertex to itself. The definition being used does not allow this.

(b) The edge *EC* crosses edges *AD* and *BD* at points which are not vertices; edge *AC* crosses *BD* at a point that is not a vertex.

(c) This graph has 5 vertices and 5 edges.

5. (a) *FDCBF*

(b) (i) *BD*; *BCD*.

(ii) *CBF*; *CDF*; *CDBF*.

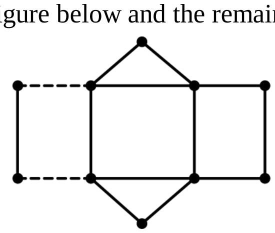
(c) *CDFBC*

7. (a) 4 vertices; 4 edges.

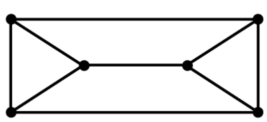
(b) 7 vertices; 6 edges.

(c) 10 vertices; 14 edges.

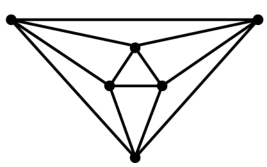
9. Remove the edges dotted in the figure below and the remaining graph will be disconnected.



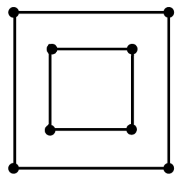
11. (a)



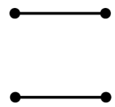
(b)



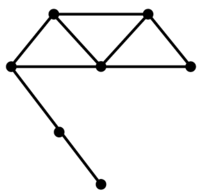
13. Yes, a disconnected graph can arise. One, possible example is shown below:



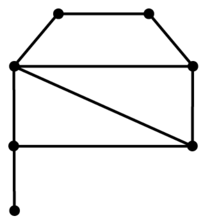
which gives rise to the disconnected graph:



15. (a)



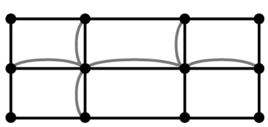
(b)



17. The supervisor is not satisfied because all of the edges are not traveled upon by the postal worker. The worker is unhappy because the end of the worker's route wasn't the same point as where the worker began. The original job description is unrealistic because there is no Euler circuit in the graph.

19. There is such an efficient route. The appropriate graph model has an additional edge joining the same pair of vertices for each of the edges shown in the graph of Exercise 17. Since this graph is connected and even-valent, it has an Euler circuit, any one of which will provide a route for the snowplow. Routes without 180-degree turns are better choices.

21.



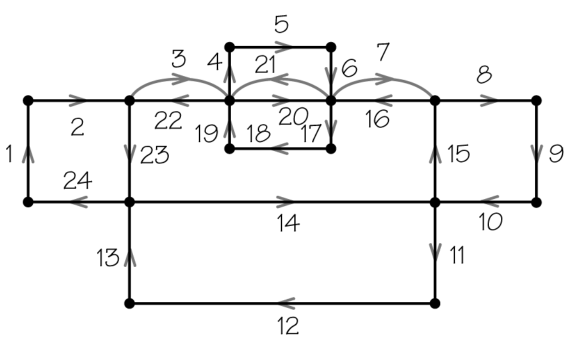
23. (a) The largest number of such paths is 3. One set of such paths is *AF*, *ABEF*, and *ACF*.

(b) This task is simplified by noticing there are many symmetries in this graph.

(c) In a communication system such a graph offers redundant ways to get messages between pairs of points even when the failure of some of the communication links (edges removed) occurs.

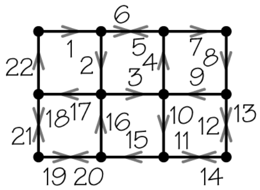
25. Do not choose edge 2, but edges 1 or 10 could be chosen.

27.

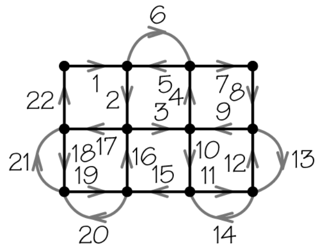


29. Two edges need to be dropped to produce a graph with an Euler circuit. Persons who parked along these stretches of sidewalk without putting coins in the meters would not need to fear that they would get tickets.

31. (a)



(b)



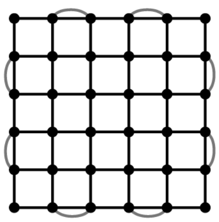
33. (a) There are four 3-valent vertices. By properly removing two edges adjacent to these four vertices, (edge between left two 3-valent vertices and edge between right two 3-valent vertices) one can make the graph even-valent.

(b) Yes, because the resulting graph is connected and even-valent.

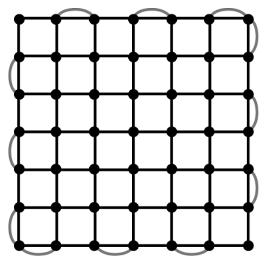
(c) It is possible to remove two edges and have the resulting graph be even-valent.

(d) No, because the resulting graph is not connected, even though it is even-valent.

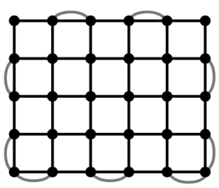
35. (a)



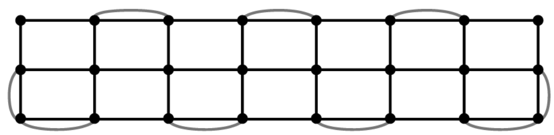
(c)



(b)

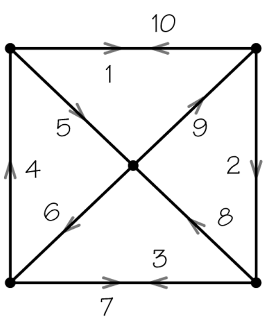


(d)



Nine is the best one can do.

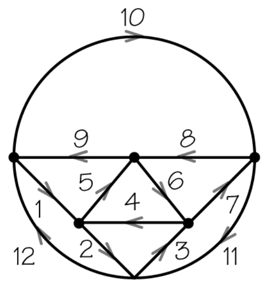
37.

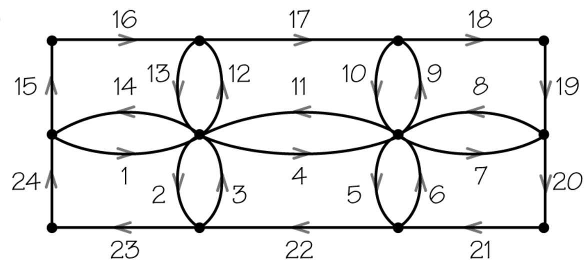


39. There are many different circuits which will involve three reuses of edges. These are the edges which join up the six 3-valent vertices in pairs.

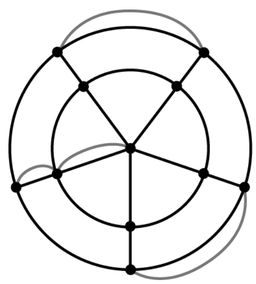
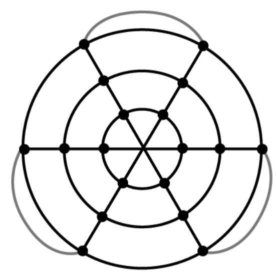
41. There are many circuits that achieve a length of 44,000 feet. The number of edges reused is eight because a shorter length tour can be found by repeating more shorter edges than fewer longer edges.

43. Both graphs (b) and (c) have Euler circuits. The valences of all of the vertices in (a) are odd, which makes it impossible to have an Euler circuit there.

(b) 

(c) 

45. If the graph *G* is connected, the newly constructed graph will be even-valent and, thus, will have an Euler circuit. If *G* is not connected, the new graph will not have an Euler circuit because it, too, will not be connected.

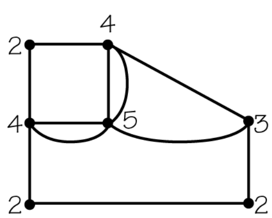
47. (a)  

(b) The best eulerization for the four-circle, four-ray case adds two edges.

(c) Hint: Consider the cases where *r* is even and odd separately.

49. A graph with six vertices where each vertex is joined to every other vertex will have valence 5 for each vertex.

51. When you attach a new edge to an existing graph, it gets attached at two ends. At each of its ends, it makes the valence of the existing vertex go up by one. Thus the increase in the sum of the valences is two. Therefore, if the graph had an even sum of the valences before, it still does, and if its valence sum was odd before, it still is.

53. 

55. In chemistry when we say, for example, that hydrogen has valence 1, we mean that it forms one chemical bond with other elements. This usage has similarities with the graph theory concept of valence.

57. A tour that begins and ends at vertex *A* and which respects the traffic directions would be: *ABDEFBEBFEDBACDCA*. The cutting machine has to make "sharp turns" at some intersections.