Chapter 23
The Economics of Resources

Chapter Outline

Chapter Summary

Geometric growth models for biological populations tend to use the *natural rate of increase* (the difference between the birth and death rates) to represent the growth rate of a population. Unfortunately, birth and death rates fail to remain constant over time. Nonetheless, such models can be used in short-term planning. Since human populations rely on resources to sustain them, it is important to be able to predict future amounts of both renewable and nonrenewable resources. The management of *renewable resources* poses a very interesting problem: determine how much of our resource we can harvest each year and still allow the resource to replenish itself. In other words, what is the maximum harvest our resource can sustain? Finding this value requires knowing the size of next year's population given this year's population. This information is usually provided by a reproduction curve for the resource. This problem is an important one in forest and fishery management. Not surprisingly, economic factors play a role in the management of resources. Harvesting will not take place if it is not profitable. Furthermore, the resource may be eliminated entirely if it is more profitable to invest the proceeds elsewhere than to sustain the resource over time. In dynamical systems the state of a system at any time depends upon its state at previous times. Certain *dynamical systems*, such as weather, are chaotic, in the sense that the evolution of the system is sensitive to initial conditions. That is, a small change in the initial conditions may result in large changes at some future time. Biological population growth can sometimes exhibit chaotic tendencies.

Skill Objectives

1. Determine a country's projected population in a given number of years when its current population and its projected growth rate are given.

2. Explain why a sustainable yield policy is needed for harvestable resources.

3. Given the current generation population size, use its reproduction curve to estimate the next generation's population.

4. Interpret the meaning of the line $y = x$ in relation to the reproduction curve.

5. Approximate from a reproduction curve the projected sustainable yield for a harvestable resource when the current generation population is given.

6. Estimate from the reproduction curve the maximum sustainable yield for a harvestable resource.

Teaching Tips

1. It's somewhat surprising to find that students with an algebra background often don't relate the value of a function to the height of its graph; consequently, the process of interpreting a reproduction curve may require some preparation in terms of discussing functions. Once that has been done, you might consider having students read the next generation population $(y-\text{value})$ for various current generation populations $(x-\text{values})$ before attempting to determine the sustainable yield.

2. When beginning to approach the concept of sustainable yield, some students need a review of the line $y = x$ in terms of identity function. Understanding that it represents an equality between the current population and the next-generation population sets the stage for finding the sustainable yield.

3. The notation $f(x)-x$ can sometimes be understood more clearly in terms of the physical subtraction of lengths of line segments If you use an overhead projector, drawing the vertical segments $f(x)$ and x in different colors on the graph of the reproduction curve may help emphasize this relationship. Drawing their difference in yet a third color and following this changing difference along the curve may help students visualize the maximization problem.

4. This chapter offers many possibilities for extra-credit projects. If your geographic area has a specific renewable resource industry such as lumber in the Pacific Northwest or fishing in the Northeast and the Gulf of Mexico, students can contact local agencies and companies to find out their policy on renewing the natural resources. It would then be interesting to report these finding back to the class.

Research Paper

Students should find researching the topic of fractals very interesting. They may wish to investigate the lives and contributions of mathematicians such as Benoit Mandelbrot (Polish) or Gaston Julia (French). Other students may wish to focus on the complex shapes generated by computers or fractals in nature. In all cases, students can find a wealth of information on the Internet

Spreadsheet Project

To do this project, go to http://www.whfreeman.com/fapp7e.

This spreadsheet project is designed to explore the unpredictable and chaotic aspects of the logistics model, including the effects of rounding.

Collaborative Learning

Industry Awareness

As an icebreaker, ask the students to form groups and determine if they are aware of industries that deal in renewable resources, and how they go about guaranteeing that the resource will not be eliminated. One obvious example is the paper industry, which plants new trees to replace those it cuts down, thereby replenishing its stock. The fishing industry is more difficult to manage, since individual fishermen have no control over the size of what their competitors catch. In this case, government intervention or agreement among the fishermen may be needed to maintain a stable population and regular harvests of fish.

Solutions

Skills Check:

1. b	2. a	3. c	4. b	5. a	6. a	7. c	8. b	9. c	10. c
11. b	12. a	13. c	14. a	15. c	16. b	17. b	18. a	19. a	20. c

Exercises:

1. In late summer 2023

2. population in mid-2025 is as follows.

$$(\text{population in mid-2007})\times(1+\text{growth rate})^{18} = 4.056(1+0.018)^{18} \text{ billion} \approx 5.59 \text{ billion}$$

3. population in mid-2025 is as follows.

$$(\text{population in mid-2007})\times(1+\text{growth rate})^{18} = 4.056(1+0.017)^{18} \text{ billion} \approx 5.49 \text{ billion}$$

4. 4.4 billion

5. The population of Africa would be $925\times(1.024)^{18} = 1418$ million, almost 100% greater than, or twice as large, as Europe's population.

6. (a) $\frac{70}{2.4} \approx 29$ years

(b) $\frac{70}{0.8} \approx 88$ years

7. (a) $\frac{70}{0.6} \approx 117$ years

(b) $\frac{70}{1.3} \approx 54$ years

8. (a) 2016

(b) The demand will be not 2.0 but 2.12 times as much in 2016 as in 1991, of which business will account for almost two-thirds rather than half.

9. (a) population in mid-2025 is as follows.

$$(\text{population in mid-2007})\times(1+\text{growth rate})^{18} = 6.593(1+0.013)^{18} \text{ billion} \approx 8.3 \text{ billion}$$

population in mid-2050 is as follows.

$$(\text{population in mid-2007})\times(1+\text{growth rate})^{43} = 6.593(1+0.013)^{43} \text{ billion} \approx 11.5 \text{ billion}$$

(b) No change in growth rate, no change in death rates, no global catastrophes, etc.

10. (a) For 2025

More-developed countries	$1.216(1+0.001)^{18}$ billion ≈	1.238 billion
Less-developed countries (excluding China)	$4.056(1+0.018)^{18}$ billion ≈	5.592 billion
China	$1.321(1+0.006)^{18}$ billion ≈	1.471 billion
Sum		8.301 billion

For 2050

More-developed countries	$1.216(1+0.001)^{43}$ billion ≈	1.269 billion
Less-developed countries (excluding China)	$4.056(1+0.018)^{43}$ billion ≈	8.735 billion
China	$1.321(1+0.006)^{43}$ billion ≈	1.709 billion
Sum		11.713 billion

There is not much difference.

(b) Answers will vary.

11. (a) The static reserve will be $\frac{2934.8}{77.9} \approx 38$ years.

(b) The exponential reserve will be $\frac{\ln\left[1+\left(\frac{2934.8}{77.9}\right)(0.019)\right]}{\ln[1+0.019]} \approx 29$ years.

(c) Answers will vary.

12. (a) The static reserve will be $\frac{6076.5}{90} \approx 68$ years.

(b) The exponential reserve will be $\frac{\ln\left[1+\left(\frac{6075.5}{90}\right)(0.022)\right]}{\ln[1+0.022]} \approx 42$ years

(c) Answers will vary.

13. (a) The static reserve will be 100 years. We are seeking the exponential reserve. This will be $\frac{\ln\left[1+100(0.025)\right]}{\ln[1+0.025]} \approx 51$ years.

(b) $\frac{\ln\left[1+1000(0.025)\right]}{\ln[1+0.025]} \approx 132$ years

(c) $\frac{\ln\left[1+10,000(0.025)\right]}{\ln[1+0.025]} \approx 224$ years

14. (a) The static reserve will be 100 years. We are seeking the exponential reserve. This will be $\frac{\ln\left[1+100(0.0125)\right]}{\ln[1+0.0125]} \approx 65$ years.

(b) $\frac{\ln\left[1+1000(0.0125)\right]}{\ln[1+0.0125]} \approx 210$ years

(c) $\frac{\ln\left[1+10,000(0.0125)\right]}{\ln[1+0.0125]} \approx 389$ years

15. (a) $\dfrac{\ln\left[1-100(0.005)\right]}{\ln[1-0.005]} \approx 138$ years

(b) $\dfrac{\ln\left[1-100(0.01)\right]}{\ln[1-0.01]} = \dfrac{\ln(1-1)}{\ln(0.99)} = \dfrac{\ln 0}{\ln[0.99]}$

This theoretically would imply forever!

16. (a) The static reserve will be 10,000 years. We are seeking the exponential reserve. This will be $\dfrac{\ln\left[1+10{,}000(0.035)\right]}{\ln[1+0.035]} \approx 170$ years

(b) $\dfrac{\ln\left[1+5{,}000(0.035)\right]}{\ln[1+0.035]} \approx 150$ years

(c) Since $(1.035)^{150} \approx 174.2$ the static reserve will be $\dfrac{5000}{174.2} \approx 29$ years.

(d) Answers will vary.

17. $\dfrac{437 \times 10^9 \text{ tons}}{100 \times 10^6 \text{ plants} \times 800 \text{ years}} \approx$ 5.5 tons/plant/year ≈ 30 lb/plant/day, which is unreasonable.

18. $\dfrac{1}{7}\ln\left(\dfrac{7.211}{6.291}\right) \approx 1.95\%$

19. $\dfrac{1}{100}\ln\left(\dfrac{62.95}{3.93}\right) \approx 2.77\%$

20. $\dfrac{1}{117}\ln\left(\dfrac{301}{62.95}\right) \approx 1.34\%$

21. After the first year, the population stays at 15.

22. 10, 20, 0, 0, ...

23. 7, 18.2, 6.6, 17.6, 8.4, 19.5, 2.0, 7.3, 18.6, 5.3

24. 15

25. We must have $f(x_n) = x_n$, or $4x_n(1-0.05x_n) = x_n$. The only solutions are $x_n = 0$ and $4(1-0.05x_n) = 1$, or $x_n = 15$.

26. Using $x_{n+1} = f(x_n) = 3x_n(1-0.05x_n)$ with $x_1 = 5$ we have the following (rounded).

5, 11.3, 14.8, 11.6, 14.6, 11.8, 14.5, 11.9, 14.4, 12.1

27. Using $x_{n+1} = f(x_n) = 3x_n(1-0.05x_n)$ with $x_1 = 10$ we have the following (rounded).

10, 15.0, 11.3, 14.8, 11.6, 14.6, 11.8, 14.5, 11.9, 14.4

The population is oscillating but slowly converging to $\frac{40}{3} \approx 13.3$.

28. Those are the only values.

29. We must have $f(x_n) = x_n$, or $3x_n(1-0.05x_n) = x_n$. The only solutions are $x_n = 0$ and $3(1-0.05x_n) = 1$, or $x_n = \frac{40}{3} \approx 13.3$.

30. Answers will vary.

31. The red dashed line indicates the same size population next year as this year; where it intersects the blue curve is the equilibrium population size.

32. The graph shows the function $x_{n+1} - x_n = 3x_n(1 - 0.05x_n) - x_n = 2x_n - 0.15x_n^2$, and we want to maximize this quantity. By graphing, symmetry of a parabola, or (for the instructor) by calculus, the maximum occurs at $x_n = \frac{20}{3} \approx 6.7$, for which the yield is $x_{n+1} - x_n \approx 2(6.7) - 0.15(6.7)^2 \approx 6.7$.

33. Using $x_{n+1} = f(x_n) = 1.5x_n(1 - 0.025x_n)$ with $x_1 = 11$ we have the following (rounded).

11, 12.0, 12.6, 12.9, 13.1, 13.2, 13.3, 13.3, 13.3, 13.3

34. Using $x_{n+1} = f(x_n) = 1.5x_n(1 - 0.025x_n)$ with $x_1 = 5$ we have the following (rounded).

5, 6.6, 8.2, 9.8, 11.1, 12.0, 12.6, 13.0, 13.1, 13.2

35. The population sizes are 11, 15.0, 13.7, 14.9, 14.9, 15.0, 14.8, 14.3, 13.0, 9.5 – and the following year the population is wiped out.

36. Using $x_{n+1} = f(x_n) = 1.5x_n(1 - 0.025x_n)$ with $x_1 = 1$ we have the following (rounded).

1.0, 1.5, 2.1, 3.0, 4.2, 5.6, 7.2, 8.9, 10.4, 11.5

Thus, after 10 years harvesting may resume.

37. About 15 million pounds. Maximum sustainable yield is about 35 million pounds for an initial population of 25 million pounds.

38. (a) About 6

(b) MSY $\approx$ 7, for an initial population of approximately 12.

39. (a) The last entry shown for the first sequence is the fourth entry of the second sequence, so the first "joins" the second and they then both end up going through the same cycle (loop) of numbers over and over.

(b) 39, 78, 56, and we have "joined" the second sequence. However, an initial 00 stays 00 forever; and any other initial number ending in 0 "joins" the loop sequence 20, 40, 80, 60, 20, ...

(c) Regardless of the original number, after the second push of the key we have a number divisible by 4, and all subsequent numbers are divisible by 4. There are 25 such numbers between 00 and 99. You can verify that an initial number either joins the self-loop 00 (the only such numbers are 00, 50, and 25); joins the loop 20, 40, 80, 60, 20, . . . (the only such are the multiples of 5 other than 00, 50 and 25); or joins the big loop of the other 20 multiples of 4.

40. Answers will vary.

41. (a) 133, 19, 82, 68, 100, 1, 1, ... The sequence stabilizes at 1.

(b) Answers will vary.

(c) That would trivialize the exercise!

(d) For simplicity, limit consideration to 3-digit numbers. Then the largest value of *f* for any 3-digit number is $9^2 + 9^2 + 9^2 = 243$. For numbers between 1 and 243, the largest value of *f* is $1^2 + 9^2 + 9^2 = 163$. Thus, if we iterate *f* over and over – say 164 times – starting with any number between 1 and 163, we must eventually repeat a number, since there are only 163 potentially different results. And once a number repeats, we have a cycle. Thus, applying *f* to any 3-digit number eventually produces a cycle. How many different cycles are there? That we leave you to work out.

Hints: 1) There aren't very many cycles.

2) There is symmetry in the problem, in that some pairs of numbers give the same result; for example, $f(68) = f(86)$.

42. (a) 1, 4, 2, 1, ...

(b) 13, 40, 20, 10, 5, 16, 8, 4, 2, 1. . . . .

(c) 12, 6, 3, 10, 5, 16, 8, 4, 2, 1, . . .

43. (a) 0.0397, 0.15407173, 0.545072626, 1.288978, 0.171519142, 0.59782012, 1.31911379, 0.0562715776, 0.215586839, **0.722914301**, 1.32384194, 0.0376952973, 0.146518383, 0.521670621, 1.27026177, 0.240352173, 0.78810119, 1.2890943, 0.171084847, **0.596529312**

(b) **0.723**, 1.323813, 0.0378094231, 0.146949035, 0.523014083, 1.27142514, 0.236134903, 0.777260536, 1.29664032, 0.142732915, **0.509813606**

(c) **0.722**, 1.324148, 0.0364882223, 0.141958718, 0.507378039, 1.25721473, 0.287092278, 0.901103183, 1.16845189, **0.577968093**

44. (a) Unless $r = 0$ (which wouldn't be a very dynamic system), the only equilibrium points are $x = 0$ and $x = 1$.

(b) For a logistic model with $\lambda \neq 0$, the only equilibrium points are $x = 0$ and $x = 1$ (carrying capacity).

45. Period 2 begins at $\lambda = 3$, period 4 at $1+\sqrt{6} \approx 3.449$, period 8 at 3.544, period 3 at $1+2\sqrt{2} \approx 3.828$, and chaotic behavior onsets at about 3.57.

See http://www.answers.com/topic/logistic-map .

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