Chapter 21
Savings Models

Chapter Outline

Chapter Summary

Interest paid on money provides simple illustrations of both geometric and arithmetic growth. Suppose we deposit P dollars in a bank, with an interest rate of r% per year. With *simple interest*, only the principal earns interest and each year we get $.01Pr$ dollars in interest. In this case, the money in our account grows arithmetically. If, however, the interest is compounded annually, then over a number of years, we receive interest on our principal *and* on the accumulated interest. In this case, our money grows *geometrically*. Typically, banks compound more than once a year and the *effective annual yield* on our money is somewhat more than r%. However, there is a limit to the yield obtained through more and more frequent compounding, and this limit is determined by the number e. Mortgages, auto loans, and college loans require the repayment of the principal borrowed on a monthly basis over a period of years. The calculation of the monthly payment is made by summing a geometric series. The same technique can be used to find the final accumulation in systematic savings programs, in which equal payments are made periodically over the course of time.

Systematic savings programs are ones in which equal payments are made periodically over the course of time, with a constant interest rate. The final accumulation in such a program can be made by summing a geometric series.

During periods of inflation, prices increase in a geometric manner. As prices rise, the value of the dollar decreases, undergoing exponential decay, which is geometric growth with a negative rate of growth. The *Consumer Price Index* computes price increases over many years.

The price of a stock depends upon future potential dividends, and also takes into account the return possible from safer investments, such as government bonds, which carry no risk. Stocks that are predicted to have growing dividends will command higher prices than ones with stable dividend rates.

The importance of the pricing of options and other financial derivatives became known to the public when Robert Merton and Myron Scholes received the 1997 Nobel Prize in economics for work they had done in the 1970s with the late Fischer Black. Probabilistic notions, especially that of expected value, play a key role in these valuations.

Skill Objectives

1. Describe the difference between arithmetic and geometric growth.

2. List three applications of geometric growth.

3. Apply the formula for geometric growth to calculate compound interest.

4. Calculate the depreciation of an item by applying the formula for geometric growth.

5. Calculate the number of years necessary to double an investment value by applying the formula for geometric growth to given information.

6. Use the savings formula, $A = d\left[\frac{(1+i)^n - 1}{i}\right]$ to determine A or d.

7. Solve applications involving inflation and depreciation.

8. Solve applications using Consumer Price Index.

Teaching Tips

1. Many of the concepts in this chapter can be found in advertisements in newspapers. Among others, ads appear for certificates of deposit at banks, money market funds, 401k plans, and annuities for retirement. If you can get the students to look for such items regularly, the practical importance of this topic will be driven home.

2. During the course of the semester, ask the students to monitor the monthly announcements of the changes in the Consumer Price Index, which can be found on radio and TV, as well as in newspapers. Ask them to compare the actual CPI with the predicted value given in the text for 2006.

Research Paper

Have students research the origin of banking. It is believed that the very first banks were religious temples in the ancient world. There are records of loans, which date to the Babylonians around the 18th century BC. Ancient Greece and ancient Rome also has evidence of banking.

Another research paper could focus on more modern ways of banking. It may be of interest to compare and contrast the different types of banks such as central banks, investment banks, merchant banks, and universal banks.

Spreadsheet Project

To do this project, go to http://www.whfreeman.com/fapp7e.

This projects uses spreadsheets to compare prices using the Consumer Price Index as well as other financial and physical applications.

Collaborative Learning

Rate and Yield

1. Ask your students to bring in ads from banks for certificates of deposit. Such ads generally contain two numbers, one called the *rate*, the other the *yield*. Ask the students if they know what these numbers represent. This can be a good lead-in to the notion of compound interest. (The small print at the bottom of some of the ads may give partial explanations of how the yield is obtained from the rate. Also, some banks may use continuous compounding.)

2. If you had the students do the previous exercise, take one of the ads that they brought in (or bring in one of your own), which states a rate and a yield, but without explaining how they obtain the yield. Ask the students to try to discover the frequency of compounding. Since this involves solving a transcendental equation, the only method of solution available to the students is trial and error. Moreover, since the figures are usually given to just two decimal places, there will not be a unique answer to this question. For example, if the rate is 6% and the yield is stated to be 6.18%, then the compounding method could be continuous. However, daily compounding also results in a yield of 6.18%, rounded to two decimal places. So does weekly compounding (rarely used), but not monthly, whose yield is 6.17%. This exercise will help reinforce the fact that continuous compounding is not as powerful as one might expect.

Solutions

Skills Check:

1. c	2. a/c	3. b	4. c	5. b	6. c	7. c	8. a	9. b	10. c
11. b	12. c	13. b	14. a	15. a	16. b	17. b	18. c	19. b	20. b

Exercises:

1. (a) By the pattern shown, there is an increase by a factor if 2^n every $3n$ days. By doing some calculations, we can see that $2^{232} \approx 6.9\times10^{69}$ and $2^{233} \approx 1.4\times10^{70}$. So n is between 232 and 233. Thus, $3n$ is between 696 and 699. Calculating $2^{697/3}$ and $2^{698/3}$, we see that the better choice is 698 days.

(b) We have $1000 = 10^3$. Since $\left(10^3\right)^{23} = 10^{69}$ and $\left(10^3\right)^{24} = 10^{72}$, an appropriate answer would be after 24 months.

2. There are $2^{64} - 1 \approx 1.845\times10^{19}$ kernels.

Since a kernel is about $\left(\frac{1}{4}\text{in.}\right)\left(\frac{1}{16}\text{in.}\right)\left(\frac{1}{16}\text{in.}\right) = \frac{1}{1024}\text{in.}^3$, the kernels occupy about $\left(1.845\times10^{19}\right)\left(\frac{1}{1024}\text{in.}^3\right) \approx 1.801\times10^{16}\ \text{in}^3$.

This occupies $1.801\times10^{16}\ \text{in.}^3 \times \frac{1\ \text{mi}^3}{\left(12\,\frac{\text{in}}{\text{ft}}\right)^3 \times \left(5280\,\frac{\text{ft}}{\text{mi}}\right)^3} \approx 70.8\ \text{mi}^3$.

(Answers in other units are possible).

3. (a) Since $A = P(1+rt)$, we have $A = \$1000(1+0.03\times1) = \$1000(1.03) = \$1030.00$. The annual yield is 3% since we have simple interest.

(b) Since $A = P(1+i)^n$, $n = 1$, and $i = \frac{0.03}{1} = 0.03$, we have the following.

$$A = \$1000(1+0.03) = \$1000(1.03) = \$1030.00$$

The annual yield is 3% since we are dealing with simple interest because the money is compounded once during the period of one year.

(c) Since $A = P(1+i)^n$, $n = 4$, and $i = \frac{0.03}{4}$, we have the following.

$$\$1000\times\left(1+\tfrac{0.03}{4}\right)^4 = \$1000(1.0075)^4 = \$1030.34$$

Since $APY = \left(1+\frac{r}{n}\right)^n - 1$, we have $APY = \left(1+\frac{0.03}{4}\right)^4 - 1 \approx 3.034\%$.

(d) $\$1000\times\left(1+\frac{0.03}{365}\right)^{365} = \$1000(1.000082192)^{365} = \$1030.45$, with the same result for a 360-day or 366-day year.

Since $APY = \left(1+\frac{r}{n}\right)^n - 1$, we have $APY = \left(1+\frac{0.03}{365}\right)^{365} - 1 \approx 3.045\%$, with the same result for a 360-day or 366-day year.

4. (a) Since $A = P(1+rt)$, we have $A = \$1000(1+0.06\times1) = \$1000(1.06) = \$1060.00$. The annual yield is 6% since we have simple interest.

(b) Since $A = P(1+i)^n$, $n = 1$, and $i = \frac{0.06}{1} = 0.06$, we have the following.

$$A = \$1000(1+0.06) = \$1000(1.06) = \$1060.00$$

The annual yield is 6% since we are dealing with simple interest because the money is compounded once during the period of one year.

(c) Since $A = P(1+i)^n$, $n = 4$, and $i = \frac{0.06}{4}$, we have the following.

$$\$1000\times\left(1+\tfrac{0.06}{4}\right)^4 = \$1000(1.025)^4 = \$1061.36$$

Since $APY = \left(1+\dfrac{r}{n}\right)^n - 1$, we have $APY = \left(1+\dfrac{0.06}{4}\right)^4 - 1 \approx 6.136\%$.

(d) $\$1000\times\left(1+\frac{0.06}{365}\right)^{365} = \$1000(1.000164384)^{365} = \$1061.83$, with the same result for a 360-day or 366-day year.

Since $APY = \left(1+\dfrac{r}{n}\right)^n - 1$, we have $APY = \left(1+\dfrac{0.06}{365}\right)^{365} - 1 \approx 6.183\%$, with the same result for a 360-day or 366-day year.

5. Using 365-day years: The daily interest rate $i = \frac{0.03}{365}$ is in effect for $n = 8\times365 = 2920$ days. We have in the compound interest formula $A = \$10,000 = P(1+i)^n$, so we get $P = \frac{\$10,000}{1.2712366} =$ \$7866.36. (Fine point: In fact, the 8 years must contain two Feb. 29 days. Calculating interest for $n = 6\times365 = 2190$ days at $i = \frac{0.03}{365}$ and for $n = 2\times366 = 732$ days at $i = \frac{0.03}{366}$ gives a result that differs by less than one one-hundredth of a cent.)

6. Using 365-day years: The daily interest rate $i = \frac{0.04}{365}$ is in effect for $n = 5\times365 = 1825$ days. We have in the compound interest formula $A = \$10,000 = P(1+i)^n$, so we get $P = \frac{\$10,000}{1.221389374} =$ \$8187.40. (Fine point: In fact, the 5 years must contain one or two Feb. 29 days, but yields a negligible difference)

7. The interest is \$26.14 on a principal of \$7744.70, or $\frac{\$26.14}{\$7744.70}\times100\% = 0.3375211435\%$ over 34 days. The daily interest rate is $\left(1.003375211435^{1/34} - 1\right)\times100\% = 0.0099109\%$. The annual rate is then $\left(1.000099109^{365} - 1\right)\times100\% = 3.68\%$.

8. The interest is \$22.16 on a principal of \$7722.54, or $\frac{\$22.16}{\$7722.54}\times100\% = 0.2869522204\%$ over 27 days. The daily interest rate is $\left(1.002869522204^{1/27} - 1\right)\times100\% = 0.0106132038\%$. The annual rate is then $\left(1.000106132038^{365} - 1\right)\times100\% = 3.95\%$.

9. (a) 3%: Predicted doubling time is $\frac{72}{100 \times 0.03} = \frac{72}{3} = 24.$

Since $A = P(1+i)^n$, $n = 24,$ and $i = \frac{0.03}{1} = 0.03,$ we have the following.

$$A = \$1000(1+0.03)^{24} = \$1000(1.03)^{24} = \$2032.79$$

4%: Predicted doubling time is $\frac{72}{100 \times 0.04} = \frac{72}{4} = 18.$

Since $A = P(1+i)^n$, $n = 18,$ and $i = \frac{0.04}{1} = 0.04,$ we have the following.

$$A = \$1000(1+0.04)^{18} = \$1000(1.04)^{18} = \$2025.82$$

6%: Predicted doubling time is $\frac{72}{100 \times 0.06} = \frac{72}{6} = 12.$

Since $A = P(1+i)^n$, $n = 12,$ and $i = \frac{0.06}{1} = 0.06,$ we have the following.

$$A = \$1000(1+0.06)^{12} = \$1000(1.06)^{12} = \$2012.20$$

(b) 8%: Predicted doubling time is $\frac{72}{100 \times 0.08} = \frac{72}{8} = 9.$

Since $A = P(1+i)^n$, $n = 9,$ and $i = \frac{0.08}{1} = 0.08,$ we have the following.

$$A = \$1000(1+0.08)^{9} = \$1000(1.08)^{9} = \$1999.00$$

9%: Predicted doubling time is $\frac{72}{100 \times 0.09} = \frac{72}{9} = 8.$

Since $A = P(1+i)^n$, $n = 8,$ and $i = \frac{0.09}{1} = 0.09,$ we have the following.

$$A = \$1000(1+0.09)^{8} = \$1000(1.09)^{8} = \$1992.56$$

(c) 12%: Predicted doubling time is $\frac{72}{100 \times 0.12} = \frac{72}{12} = 6.$

Since $A = P(1+i)^n$, $n = 6,$ and $i = \frac{0.12}{1} = 0.12,$ we have the following.

$$A = \$1000(1+0.12)^{6} = \$1000(1.12)^{6} = \$1973.82$$

24%: Predicted doubling time is $\frac{72}{100 \times 0.24} = \frac{72}{24} = 3.$

Since $A = P(1+i)^n$, $n = 3,$ and $i = \frac{0.24}{1} = 0.24,$ we have the following.

$$A = \$1000(1+0.24)^{3} = \$1000(1.24)^{3} = \$1906.62$$

36%: Predicted doubling time is $\frac{72}{100 \times 0.36} = \frac{72}{36} = 2.$

Since $A = P(1+i)^n$, $n = 2,$ and $i = \frac{0.36}{1} = 0.36,$ we have the following.

$$A = \$1000(1+0.36)^{2} = \$1000(1.36)^{2} = \$1849.60$$

(d) For small and intermediate interest rates, the rule of 72 gives good approximations to the doubling time.

10. 23.1, 11.55, 7.7 yrs, all close to the predictions. The number 72 has the convenience of being evenly divisible by many small numbers.

It's a "rule of 110." Divide 110 by 100r; for r = 0.05, you get 22 yrs.

11. (a) 2, 2.59, 2.705, 2.7169, 2.718280469

(b) 3, 6.19, 7.245, 7.3743, 7.389041321

(c) $e = 2.718281828\ldots$; $e^2 = 7.389056098\ldots$. Your calculator may give slightly different answers, because of its limited precision.

12. (a) 0, 0.35, 0.366, 0.3677, 0.367879257

(b) −1, 0.11, 0.133, 0.1351, 0.135335013

(c) $e^{-1} = \frac{1}{e} = 0.367879441\ldots$; $e^{-2} = \frac{1}{e^2} = 0.135335283\ldots$. Your calculator may give slightly different answers, because of its limited precision.

13. (a) Since $A = Pe^{rt}$ we have $A = \$1000e^{(0.03)(1)} = \$1000e^{0.03} = \$1030.45$. Thus, the interest is $\$1030.45 - \$1000.00 = \$30.45$.

(b) Since $A = P\left(e^{r/360}\right)^{360}$ we have $A = \$1000\left(e^{0.03/360}\right)^{360} = \$1030.45$. Thus, the interest is $\$1030.45 - \$1000.00 = \$30.45$.

(c) Since $A = P\left(e^{r/365}\right)^{365}$ we have $A = \$1000\left(e^{0.03/365}\right)^{365} = \$1030.45$. Thus, the interest is $\$1030.45 - \$1000.00 = \$30.45$.

In all cases, \$30.45, not taking into account any rounding to the nearest cent of the daily posted interest.

14. (a) $i = \$4632.10 - \$4532.10 = \$100.00$

Since effective rate would be $\dfrac{100}{4532.10} = 2.2065\%$.

(b) The bank is using the formula $A = P\left(e^{r/365}\right)^{365}$. Since $\$4532.10\left(e^{0.021825/365}\right)^{365} \approx \$4532.10$, 2.1825% will be the approximate nominal rate.

(c) In this case the difference is negligible, not taking into account any rounding in posting interest.

15. (a) $\left(e^{0.04} - 1\right) \times 100\% = 4.08108\%$.

(b) The approximation for effective rate is $r + \frac{1}{2}r^2 = 0.05 + \frac{1}{2} \times (0.05)^2 = 0.05 + 0.00125 = 0.05125$ or 5.125%, very slightly less than the true effective rate.

16. (a) Continuous compounding yields $\$1000e^{(0.04)(10)}$ = \$1491.82.

True Difference: $D = \$1000\left[e^{(0.04)(10)} - \left(1 + \dfrac{0.04}{4}\right)^{(4)(10)}\right] \approx \$2.96$

Approximate Difference: $D \approx \$1000 \times \dfrac{(0.04)^2 \times 10 \times e^{(0.04)(10)}}{2 \times 4 + \dfrac{4 \times 0.04}{3} + \dfrac{(0.04)^2 \times 10}{2}} \approx \$2.96$

Continued on next page

16. continued

(b) Continuous compounding yields $\$1000e^{(0.18)(10)}$ = \$6049.65.

$$\text{True Difference: } D=\$1000\left[e^{(0.18)(10)}-\left(1+\frac{0.18}{365}\right)^{(365)(10)}\right]\approx\$2.68$$

$$\text{Approximate Difference: } D\approx\$1000\times\frac{(0.18)^2\times10\times e^{(0.18)(10)}}{2\times365+\frac{4\times0.18}{3}+\frac{(0.18)^2\times10}{2}}\approx\$2.68$$

17. Use the savings formula $A=d\left[\frac{(1+i)^n-1}{i}\right]$, with A = \$2000, $i=\frac{0.05}{12}$, and n = 24.

$$\$2000=d\left[\frac{\left(1+\frac{0.05}{12}\right)^{24}-1}{\frac{0.05}{12}}\right]\approx25.18592053d$$

$$d=\frac{\$2000}{25.18592053}=\$79.41$$

18. Use the savings formula $A=d\left[\frac{(1+i)^n-1}{i}\right]$, with A = \$2000, $i=\frac{0.07}{12}$, and n = 24.

$$\$2000=d\left[\frac{\left(1+\frac{0.07}{12}\right)^{24}-1}{\frac{0.07}{12}}\right]\approx25.68103157d$$

$$d=\frac{\$2000}{25.68103157}=\$77.88$$

19. Use the savings formula $A=d\left[\frac{(1+i)^n-1}{i}\right]$, with d = \$400, $i=\frac{0.055}{12}$, and n = 144.

$$A=d\left[\frac{(1+i)^n-1}{i}\right]=\$400\left[\frac{\left(1+\frac{0.055}{12}\right)^{144}-1}{\frac{0.055}{12}}\right]=\$81{,}327.45$$

20. Use the savings formula, $A=d\left[\frac{(1+i)^n-1}{i}\right]$ with n = 35×12 = 420.

(a) With d = \$100 and $i=\frac{0.05}{12}$, we have the following.

$$A=d\left[\frac{(1+i)^{504}-1}{i}\right]=\$100\left[\frac{\left(1+\frac{0.05}{12}\right)^{504}-1}{\frac{0.05}{12}}\right]=\$171{,}134.87$$

(b) With d = \$100 and $i=\frac{0.075}{12}$, we have the following.

$$A=d\left[\frac{(1+i)^{504}-1}{i}\right]=\$100\left[\frac{\left(1+\frac{0.075}{12}\right)^{504}-1}{\frac{0.075}{12}}\right]=\$353{,}734.73$$

(c) With d = \$100 and $i=\frac{0.10}{12}$, we have the following.

$$A=d\left[\frac{(1+i)^{504}-1}{i}\right]=\$100\left[\frac{\left(1+\frac{0.10}{12}\right)^{504}-1}{\frac{0.10}{12}}\right]=\$774{,}429.65$$

21. Use the savings formula $A = d\left[\frac{(1+i)^n - 1}{i}\right]$, with A = \$1,000,000, $i = \frac{0.05}{12}$, and $n = 35\times12 = 420$.

$$\$1{,}000{,}000 = d\left[\frac{\left(1+\frac{0.05}{12}\right)^{420} - 1}{\frac{0.05}{12}}\right] \approx 1136.092425d$$

$$d = \frac{\$1{,}000{,}000}{1136.092425} = \$880.21$$

22. (a) Use the savings formula $A = d\left[\frac{(1+i)^n - 1}{i}\right]$, with d = \$100, $i = \frac{0.06}{12}$, and $n = 30\times12 = 360$.

$$A = d\left[\frac{(1+i)^n - 1}{i}\right] = \$100\left[\frac{\left(1+\frac{0.06}{12}\right)^{360} - 1}{\frac{0.06}{12}}\right] = \$100{,}451.50$$

(b) Use the savings formula $A = d\left[\frac{(1+i)^n - 1}{i}\right]$, with A = \$250,000, $i = \frac{0.075}{12}$, and $n = 20\times12 = 240$.

$$\$250{,}000 = d\left[\frac{\left(1+\frac{0.075}{12}\right)^{240} - 1}{\frac{0.075}{12}}\right] \approx 553.7307525d$$

$$d = \frac{\$250{,}000}{553.7307525} = \$451.48$$

23. (a) $\frac{\$100}{1-0.32} = \$147.06$

(b) Use the savings formula $A = d\left[\frac{(1+i)^n - 1}{i}\right]$, with d = \$147.06, $i = \frac{0.075}{12}$, and $n = 40\times12 = 480$ to calculate $A = \$147.06\left[\frac{\left(1+\frac{0.075}{12}\right)^{480} - 1}{\frac{0.075}{12}}\right] = \$444{,}683.29$.

(c) $0.68\times\$444,683.29 = \$302{,}384.64$

24. (a) \$302,382.22. But when you take it out, you owe 32% tax on the part that is interest: \$302,382.22 − 480×\$100 = \$254,382.22. Your net is the \$48,000 contributed on which taxes were already paid plus 0.68 × \$254,382.22 = \$172,979.91, for a total of \$220,979.91.

(b) The entire \$302,382.22 is yours, with taxes already paid, at age 65. The answer in Exercise 23b is \$9.68 more because of rounding of \$151.515152 up to \$151.52.

25. (a) Write the series as $\frac{1}{2}\left(1+\frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}\right) = \frac{1}{2}\times\frac{\left(\frac{1}{2}\right)^5 - 1}{\frac{1}{2}-1} = \frac{31}{32}$.

(b) $\frac{2^n - 1}{2^n}$

(c) 1

26. (a) $\dfrac{\left(-\frac{1}{3}\right)^5-1}{-\frac{1}{3}-1}=\dfrac{-\frac{1}{243}-1}{-\frac{1}{3}-1}=\dfrac{-\frac{244}{243}}{-\frac{4}{3}}=\dfrac{61}{81}$

(b) $\dfrac{3\left[1-\left(-\frac{1}{3}\right)^{n+1}\right]}{4}$, via the formula for sum of a geometric series.

(c) $\dfrac{3}{4}$

27. (a) $A=P(1.15)(1.07)(0.80)=0.98440P$, so $r=(0.98440)^{1/3}-1=-0.00523=-0.523\%$.

(b) It is the effective rate.

28. (a) – (c) Answers will vary.

(d) The ordinary IRA and the Roth IRA are equivalent provided the tax rate remains the same (however, they have different provisions regarding withdrawals and other considerations); the ordinary after-tax investment is worst.

(e) The Roth IRA is better in the first case (you pay the tax now at the lower rate), the ordinary IRA is better in the second case (you pay the lower tax rate later).

29. (a) $\$(1.04)^3=\$1.12$

(b) \$1/1.12 = \$0.89

30. \$10,000 × $(1-0.12)^3$ = \$10,000 × 0.68 = \$6800

31. $\$10,000\times(1-0.12)^6\times\left(\dfrac{1}{1+0.03}\right)^6\approx\$3900$

32. (a) Since $\dfrac{\text{cost in 2006}}{\$4.98}=\dfrac{\text{CPI for 2006}}{\text{CPI for 1965}}=\dfrac{200.5}{31.5}\approx 6.363079365$, we have the following.

$$\text{cost in 2006}=\$4.98(6.363079365)=\$31.70$$

Additional answers will vary.

(b) Since $\dfrac{\text{cost in 2006}}{\$40}=\dfrac{\text{CPI for 2006}}{\text{CPI for 1940}}=\dfrac{200.5}{14.0}\approx 14.32142857$, we have the following.

$$\text{cost in 2006}=\$40(14.32142857)=\$572.86$$

33. (a) Since $\dfrac{\text{cost in 2006}}{\$10.75}=\dfrac{\text{CPI for 2006}}{\text{CPI for 1962}}=\dfrac{200.5}{30.9}\approx 6.4898673139$, we have the following.

$$\text{cost in 2006}=\$10.75(6.4898673139)=\$69.75\approx\$70$$

$$\$10.75\times(6.4898673)=\$69.75\approx\$70$$

Additional answers will vary.

(b) Since $\dfrac{\text{cost in 2006}}{\$0.25}=\dfrac{\text{CPI for 2006}}{\text{CPI for 1970}}=\dfrac{200.5}{38.8}\approx 5.167525773$, we have the following.

$$\text{cost in 2006}=\$0.25(5.167525773)=\$1.29$$

Since $\dfrac{\text{cost in 2006}}{\$0.25}=\dfrac{\text{CPI for 2006}}{\text{CPI for 1974}}=\dfrac{200.5}{49.3}\approx 4.06693712$, we have the following.

$$\text{cost in 2006}=\$0.70(4.06693712)=\$2.85$$

34. (a) $\dfrac{90.9-82.4}{82.4}=10.3\%$

(b) $184\times(1.03)^3\approx 201.1$

35. Let the purchasing power of the original salary be P. Then the purchasing power of the new salary is $P\times 1.10\times\dfrac{1}{1+0.20}\approx 0.917P$, an 8.3% loss.

36. $\$2000\times\left[1+\left(\dfrac{1}{1.03}\right)^1+\ldots+\left(\dfrac{1}{1.03}\right)^{39}\right]=\$2000\times\dfrac{\left(\frac{1}{1.03}\right)^{40}-1}{\frac{1}{1.03}-1}\approx\$47{,}616.43$

37. Nowhere close

(a) As she ends her 35th year of service, her salary will be \$166,973.02, which we multiply by $\frac{1}{1.03^{35}}$ to get the equivalent in today's dollars: \$59,339.44. (We do not take into account that annual salaries are normally rounded to the nearest dollar or hundred dollars.) The result is easily obtained by use of a spreadsheet, proceeding through her salary year by year and then adjusting at the end for inflation. Here is the corresponding formula, using Fisher's effect with r = 0.04 and a = 0.03.

$$\left\{\left[\$42{,}000\left(1+\frac{0.01}{1.03}\right)^7+\$1500\left(\frac{1}{1.03}\right)^7\right]\left(1+\frac{0.01}{1.03}\right)^7+\$1500\left(\frac{1}{1.03}\right)^{14}\right\}$$
$$\times\left\{\left(1+\frac{0.01}{1.03}\right)^{20}\left(\frac{1}{1.03}\right)\right\}$$

The last factor is for inflation during her 35th year of service.

(b) \$57,394.20.

38. (a) Answers will vary with salary protocol that the student devises. Keeping the two promotion raises at \$1500 and varying only the initial salary requires an initial salary of \$64,737.85. Keeping the starting salary at \$42,000 but adjusting the raises for inflation to be \$1500 in 2005 dollars requires annual raises of 5.25%. (Answers can be derived by programming adaptations of the formula from the solution to Exercise 37 into a spreadsheet, calculator, or computer algebra system, and using either successive approximation or a Solve routine.)

(b) Answers will vary.

39. $\$2{,}000{,}000\times\left(1+x+\ldots+x^{19}\right)$, with $x=\frac{1}{1+a}=\frac{1}{1.03}$, giving \$29.8 million. If you can expect to earn interest rate r on funds once you receive them, through the last payment, then the present value of your stream of income of annual lottery payments P plus interest (with inflation rate a) is as follows.

$$P\left[\left(\frac{1+r}{1+a}\right)^{19}\cdot 1+\left(\frac{1+r}{1+a}\right)^{18}\left(\frac{1}{1+a}\right)^{1}+\left(\frac{1+r}{1+a}\right)^{17}\left(\frac{1}{1+a}\right)^{2}+\ldots\right.$$
$$\left.\ldots+\left(\frac{1+r}{1+a}\right)^{1}\left(\frac{1}{1+a}\right)^{18}+1\cdot\left(\frac{1}{1+a}\right)^{19}\right]=P\frac{1}{(1+a)^{19}}\left[\frac{(1+r)^{20}-1}{r}\right]$$

For P = \$2 million, r = 4%, and a = 3%, we get \$33.4 million. If you can earn 4% forever but inflation stays at 3%, the present value is infinite!

40. Not taking into account interest earned on funds received, the present value is still \$29.8 million. Using the formula from the solution to Exercise 39, if you can earn 6% through the last payment, the present value of the income stream through then is \$42.0 million.

41. (a) Use the savings formula with A = \$100,000, n = 35 × 4 = 140 quarters, and $i = \frac{0.072}{4}$ per quarter. You find d = \$161.39.

(b). $\dfrac{\$100{,}000}{(1.04)^{35}} = \$25{,}341.55$

(c) $\dfrac{\$100{,}000}{(1.04)^{65}} = \$7813.27$

42. (a) Since she wants income each year of \$50,000 in 2005 dollars, the present value is $45 \times$ \$50,000 = \$2.25 million.

(b) She needs \$2.25 million in 2005 dollars, which at 4% inflation per year will be (\$2.25 million)$\times$ 1.0435 ≈ \$8.88 million in 2040. We find the quarterly contribution d to this sinking fund by applying the savings formula, $A = d\left[\dfrac{(1+i)^n - 1}{i}\right]$, with fund A = $\$8{,}878{,}700.24$, quarterly interest $i = \frac{0.072}{4} = 0.018$, and n = 140 quarters.

$$\$8{,}878{,}700.24 = d\left[\frac{(1+0.018)^{140} - 1}{0.018}\right] \approx 619.6195407d$$

Perhaps your roommate should reassess her plans!

43. (a) $\frac{1.0453}{1.031} - 1$ = 0.01387 = 1.39%.

(b) $(1 - 0.30) \times$ 1.39% = 0.97% (however, some states and cities do not tax interest earned on U.S. government securities).

44. $\dfrac{r-i}{1+i}(1-t)$

45. The price before should have been about $\dfrac{D(1.03)}{0.15-0.03} = 8.583D$, the price after should have been $\dfrac{D(1.03)}{0.1475-0.03} = 8.766D$, so the percentage change expected was $\dfrac{8.766D - 8.583D}{8.583D} = 2.13\%$, which applied to the Dow Jones should have produced a rise of 188 pts. The answer does not depend on the value of D.

46. (a) $S\left[\dfrac{-\Delta r}{r + \Delta r - g}\right]$

(b) $0.0213S$

(c) r = 3.25%

47. Programming the savings formula into the spreadsheet and varying the value of i until you find $A \geq \$5000$, using the Solver command in Excel, or otherwise: i = 1.60% per month, or an annual rate of 12 × 1.60% = 19.2%.

48. (a) \$12,000

(b) The resulting equation is $\frac{100\left[(1+i)^{120}-1\right]}{i} = 37,747$. Replacing i by $1+x$ and rearranging gives $x^{120} - 377.47x + 376.47 = 0$. The solution is x = 1.016714122, for an annual nominal rate of $12(0.016714122) = 20.06\%$.

The effective annual yield (APY) is $(1.016714122)^{12} - 1$ = 22.01%.

49. 4.97%. It is the effective rate.

50. 0.00634% per month, or 7.61% annual rate.

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