Chapter 18
Growth and Form

Chapter Outline

Chapter Summary

All living things encounter the problem of scale: how to adapt and survive as their size changes from the beginning of life to maturity. There are physical limits to size, and the force of gravity has a profound effect on the size and shape that objects and living creatures can assume.

We can gain insight into the effects of size change by examining simple geometric examples. Geometrically, objects are considered similar if they have the same shape. The *scaling factor* tells us the change in *dimension* if we enlarge (or shrink) an object. Changing dimension induces a corresponding change in *area* or *volume*. For example, doubling the sides of a square increases area by a factor of four. Similarly, doubling the edges of a cube increases volume by a factor of eight (and surface area by a factor of four). Generally speaking, volume changes proportionally with the cube of the scaling factor; and area, proportionally with its square.

A real object (living or not) is composed of matter. This matter has a *mass* determined by the volume of the object and the *density* of its matter. The force of gravity acting on this mass is what gives the body *weight*. Finally, the object must be able to support this weight, which exerts pressure on that portion of the object supporting the weight. Now pressure increases proportionally with the scaling factor. Hence, when the scaling factor is made large enough, the object will crumble under its own weight. To solve this problem of scale, it may be necessary to change the material from which the object is made, change the object's shape, or both. In some cases (buildings, airplanes), we may be able to make the changes necessary for building larger and larger things. In other cases (mountains, trees, people), the demands are clearly beyond our capability.

The ability of an animal to survive falls, hold its breath under water, jump, and fly also depends on the size of the animal, and increases (or decreases) proportionally with the scaling factor. The need for animals to keep themselves warm (or to cool themselves off) produces an interesting interplay between their volume and surface area. The volume of the animal determines the amount of matter to be warmed or cooled, but the rate at which the warming or cooling takes place depends on its surface area. A few basic geometric and physical concepts have allowed us to show that none of the living beings and objects in our world could exist on vastly different scales, larger or smaller, without fundamental changes in shape or composition.

Skill Objectives

1. Determine the scaling factor when given the original dimensions of an object and its scaled dimensions.

2. Given the original dimensions of an object and its scaling factor, determine its scaled dimensions.

3. Calculate the change in area of a scaled object when its original area and the scaling factor are given.

4. Calculate the change in volume of a scaled object when its original volume and the scaling factor are given.

5. Determine whether two given geometric objects are similar.

6. Locate on a number line the new location of a scaled point.

7. When given the two-dimensional coordinates of a geometric object, its center, and the scaling factor, calculate its new coordinates after the scaling has taken place.

8. Calculate from given formulas the perimeter and area of a two-dimensional object.

9. Calculate from given formulas the surface area and volume of a three-dimensional object.

10. Describe the concept of area-volume tension.

11. Explain why objects in nature are restricted by a potential maximum size.

Teaching Tips

1. To explain why weight is proportional to volume, you might consider using the metric system example: 1 gram of water is defined to be the mass of 1 cubic centimeter of water.

2. The concept of geometric similarity may be introduced on an intuitive level by appealing to the student's visual sense. Using the overhead projector and prepared diagrams, many examples of similar and non-similar pairs of objects could be discussed. In particular, showing that any two squares are similar, but pointing out that two rectangles may or may not be similar, should help the student discriminate more clearly.

3. The concept of proportionality is important in many applications of mathematics. This chapter provides an opportunity to review the process of solving a proportion through cross-multiplication. Additional practice on the topic can be gained from finding the lengths of sides of a similar object when given the corresponding dimensions of the original object and the length of one side of its similar counterpart.

4. The Suggested Readings section of this chapter contains some interesting materials that, because of their visual nature, most students will be able to understand. You may wish to assign specific readings from these as extra-credit projects.

5. This chapter provides an unusually good opportunity for class discussion. You may find that students who rarely speak up in class will be motivated by the topic to participate.

6. Students who are amateur photographers can be encouraged to photograph examples of similar objects that exist in nature. If these are produced as slides, they can then be shown to the class.

Research Paper

Have students investigate the history of measurement. Although this is a broad topic, specific topics such as

- History of the cubit
- History of the Imperial system
- Origin of the inch, foot, and yard
- History of the metric system
- Units of type measurement
- Units of angular measurement

can be readily assigned. A website such as http://en.wikipedia.org/wiki/History_of_measurement can be very helpful to start research or assign topics. A more current topic of interest may be the "Metric Martyrs."

Spreadsheet Project

To do this project, go to http://www.whfreeman.com/fapp7e.

This spreadsheet project explores the impact of scale on the design of buildings.

Collaborative Learning

1. Photographic enlargement is an example of scaling that is familiar to almost everyone. Photocopy a photograph, preferably one that has a shape in it whose area can be easily calculated, such as a rectangle. Then enlarge the copy by a certain scaling factor, and ask the students to calculate the ratios of various linear measurements in the two photos, and then the ratios of the areas in the two photos. With some hints and prodding, the students will soon have a concrete example of how area scales as the square of the scaling factor.

Note: A scaling factor of 2 or 3 will help students discover this principle more quickly. Many copy machines allow you to set the scaling factor, although only in a limited range, so you may have to enlarge the original several times to obtain an integral scaling factor.

2. **a)** Ask the students to bring in graphics or other diagrams that visually "lie," similar to the ones at the end of the chapter (after the problem set), as well as others that they believe tell the truth. Working in pairs, have them analyze each other's examples.

b) Have the students construct their own diagrams or graphics that are examples of "How to Lie with Statistics." Again, have them work in pairs, criticizing each other's works.

Solutions

Skills Check:

1. c	2. c	3. a	4. c	5. b	6. c	7. c	8. b	9. c	10. b
11. a	12. a	13. c	14. c	15. a	16. a	17. b	18. c	19. a	20. a/b

Exercises:

1. (a) A contact print is the same size as the negative, so the scaling factor is 1.

(b) The scaling factor is the ratio of the length of the enlargement to the original, hence 3; the area of the enlargement will be $3^2 = 9$ times as much.

(c) Since we have $4 = 4\times1$ and $6 = 4\times1\frac{1}{2}$, the scaling factor is 4.

(d) Yes. For the negative, the width is $\frac{1.5}{1} = 1.5$ times the height; for the print, the ratio (with the length and height converted to thirty-seconds of an inch) is $\frac{147}{98} = 1.5$.

(e) The 5×7, 8×10, 11×14, and 16×20 are not geometrically similar to the negative, because their lengths are not exactly 1.5 times their heights; in printing one of those, the negative must be cropped.

(f) Based just on paper used, the 8×12 would cost $2^2 = 4$ times as much as the 4×6.

(g) The 5×7, with 35 in.^2 $\$1.79/35\ \text{in.}^2 \approx \$0.0511/\text{in.}^2$; the 8×10, costs about $\$0.0499/\text{in.}^2$. So you could expect the 11×14, with an area of 154 in.^2, to cost about 154×\$0.05 = \$7.70 (probably \$7.79 or \$7.99), based just on cost of the paper.

(h) The 20×30 is $600\,\text{in.}^2$. Since $\left(600\ \text{in.}^2\right)\left(\$0.0499/\text{in.}^2\right) = \$29.94$, the price should probably be \$29.99, based just on cost of the paper.

2. (a) $\frac{16\ \text{in.}}{10\ \text{in.}} = 1.6.$

(b) $(1.6)^2 = 2.56$ times as large.

(c) $\frac{\frac{1}{4}\pi(10\,\text{in.})^2}{\$6.30} \approx 12.5\,\text{in.}^2/\$$; $15.4\,\text{in.}^2/\$$; $18.4\,\text{in.}^2/\$$; $21.3\,\text{in.}^2/\$$; we assume that the thickness of the topping isn't scaled up and hence is the same for all the pizzas.

(d) No. the best deal, the extra large pizza, gives 12.0 in.2 per dollar.

(e) The new prices have been scaled linearly from the old ones.

3. (a) The linear scaling factor is $\frac{4\ \text{cm}}{160\ \text{cm}} = \frac{1}{40} = 0.025.$

(b) The volume of the real person goes up as the cube of the scaling factor and so is $40^3 = 64,000$ times as large as the volume of the Lego.

(c) 40×10 cm = 400 cm = 4.00m = 400 cm × $\frac{1\ \text{in}}{2.54\ \text{cm}} \approx 157.5\ \text{in.} \approx 13.1\ \text{ft.}$

4. (a) $\frac{1}{12}$.

(b) The dollhouse would weigh $\left(\frac{1}{12}\right)^3 = \frac{1}{1728} \approx 0.00058$ times as much.

5. The linear scaling factor for men compared to women (on average) is 1.08; if the brain scales as the cube of height, then men's brains (on average) would be $1.08^3 \approx 1.26$ times as large as women's, or 26% larger.

6. Geometrically similar: $\left(\frac{12}{10}\right)^3 = 1.73$ times as heavy. Geometrically similar except for thickness of metal: $\left(\frac{12}{10}\right)^2 = 1.44$ times as heavy.

7. (a) The new altar would have a volume 8 times as large − not "8 times greater than" or "8 times larger than", and definitely not twice as large, as the old altar.

(b) Since the volume scales as the cube of the side, the side scales as the cube root of the volume: $\sqrt[3]{2} \approx 1.26$.

8. (a) The resulting coin would have 8 times the weight of a quarter.

(b) $\frac{1}{16}$ in,

(c) Diameter $= \sqrt[3]{4} \times \frac{15}{16}$ in. ≈ 1.49 in., thickness $= \sqrt[3]{4} \times \frac{1}{16}$ in. ≈ 0.10 in. These are almost exactly the dimensions of the older Eisenhower dollar coin.

9. The writer meant that the volume was 2.5 times as much before packaging. Since 2.5 bags have been compressed to 1 bag, the new volume is $\frac{1}{2.5} = 0.4$ "times as much as" before. We could also correctly say that the peat moss has been compressed "to 40% of its original volume" or "by 60%," or that the compressed volume is "60% less than" the original volume.

10. (a) Nothing can be reduced by more than 100%.

(b) If it had improved 100%, there would be *no* lost luggage (and no room for further improvement!).

(c) It is a 50% reduction (and a drop of 5 percentage points).

11. You don't have to be an expert in management to see that answers A, B, and E are totally irrelevant (despite the claims at Microedu.com) and that the given statement is about costs being less than in country Q, thus eliminating D (with any percentage). If the original statement read instead "a car costs 120% more in Country Y than in Country Q," then C is correct. We can cast this into algebra, using q for the price in country Q, y for the price in Country Y, T for the tariff, and t for the transportation cost. The statements say that $q + t + T < y = q + 1.2q = 2.2q$, from which we find $T < t + T < 1.2q$, which is what C says.

12. Expenditures cannot go down by more than 100%. Oklahoma spends $100\% \times \frac{\$6.06}{\$13.17} \approx 46\%$ as much as, or $100\% \times \frac{\$13.17 - \$6.06}{\$13.17} \approx 54\%$ than, the national average. The regional expenditures of \$8.12 per capita is $100\% \times \frac{\$8.12 - \$6.06}{\$6.06} \approx 34\%$ more than in Oklahoma.

13. Answers will vary.

14. (a) The worth (according to the city) is $100\% \times \frac{\$259,000 - \$107,500}{\$259,000} \approx 58\%$ less.

(b) The charge can't be more than 100% less.

(c) The 7% in British Columbia is $100\% \times \frac{14.7\% - 7\%}{14.7\%} \approx 52\%$ less than Alberta's.

15. $\$0.80 = \$0.80 \times \frac{€1}{\$1.30} = €\frac{0.80}{\$1.30} \approx €0.62$. (This is only a little more than the cost of a domestic letter in Germany, € 0.55, which, however, is delivered overnight.)

16. $\text{US\$}0.60 \times \frac{\text{Cdn\$}1.23}{\text{US\$}1} \approx \text{Cdn\$}0.74$.

17. The car gets 100 km per 7.3 L, which is $\frac{100 \text{ km}}{7.3 \text{L}} = \frac{100 \times 0.621 \text{ mi}}{7.3 \times 0.2642 \text{gal}} \approx 32.2$ or 32 mpg.

18. $$60 \text{ mpg} = 60 \ \frac{\text{mi}}{\text{gal}} \approx 60 \ \frac{\text{mi}}{4 \text{qt}} \times \frac{1.61 \text{ km}}{1 \text{ mi}} \times \frac{1.057 \text{ qt}}{1 \text{ L}}$$

$$\approx \frac{60 \times 1.61 \times 1.057}{4} \frac{\text{km}}{\text{L}} \approx 25.5 \text{ km/L} = \frac{1 \text{ km}}{\frac{1}{25.5} \text{L}} \approx \frac{100 \text{ km}}{\frac{100}{25.5} \text{L}} \approx \frac{100 \text{ km}}{3.9 \text{ L}}.$$

So it uses 3.9 L per 100 km. The car is (barely) what the Germans call a "three-liter car".

19. (a) $\left(\frac{1}{87}\right)^3 \times 88 \text{ tons} \approx 0.00013364 \text{ tons}$.

(b) We assume that all parts of the scale model are made of the same materials as the real locomotive.

(c) 0.00013364 tons = 0.00013364 × 2000 lb ≈ 0.267 lb.

(d) $0.267 \text{ lb} = 0.267 \times 0.45359237 \text{ kg} \approx 0.121$ kg.

(e) $0.121 \text{ kg} = 0.121 \text{ kg} \times \frac{1 \text{ metric tonne}}{1000 \text{ kg}} = 0.000121$ metric tonnes.

20. A hectare equals 0.003861 mi^2 or 2.471 acres.

21. $\dfrac{€1.169}{1\text{L}}\times\dfrac{\$1.30}{€1}\times\dfrac{1\text{ L}}{1000\text{ cm}^3}\times\dfrac{(2.54\text{ cm})^3}{(1\text{ in})^3}\times\dfrac{231\text{ in}^3}{1\text{ gal}}\approx\$5.75/\text{gal.}$ or

$\dfrac{€1.169}{1\text{L}}\times\dfrac{\$1.30}{€1}\times\dfrac{1\text{ L}}{0.2642\text{ gal}}\approx\$5.75/\text{gal.}$

22. $45,000,000\text{ yen}\times\dfrac{\$1}{125\text{ yen}}=\$360,000$

23. 607 ft = 607 × 0.3048 m = 185 m.

24. Answers will vary with assumptions made. The top of the building is 1671 ft above ground level; that is an average of $\dfrac{1671\text{ ft}}{101\text{ floors}}\approx 16.5\text{ ft/floor}$. Assume that the floor of the elevator starts at ground level, and we neglect any additional height due to the roof or peak. The floor of the elevator needs to make it to the floor level of the 101st floor, which is 16.5 ft below the top of the building, at height $1671\text{ ft}-16.5\text{ ft}=1654.5\text{ ft}$. At 55 ft/s, the trip will take $\dfrac{1654.5\text{ ft}}{55\frac{\text{ft}}{\text{s}}}\approx 30.1\text{ s}$, not counting additional time for acceleration and deceleration.

25. (a) $55\text{ ft/s}=\dfrac{55\times 1\text{ ft}}{1\text{ s}}=\dfrac{55\times\frac{1}{5280}\text{ mi}}{\frac{1}{3600}\text{ h}}=\dfrac{55\times 3600\text{ mi}}{5280\text{ h}}=37.5\text{ mph.}$

(b) $1\text{ ft/s}=\dfrac{\frac{1}{5280}\text{ mi}}{\frac{1}{3600}\text{ h}}=\dfrac{3600}{5280}\text{ mph}\approx 0.68182\text{ mph}$, so $41\text{ ft/s}\approx 41\times 0.68182\text{ mph}\approx 28.0\text{ mph.}$

(c) $\dfrac{1\text{ mi}}{0.5\text{ min}}=\dfrac{1\text{ mi}}{0.5\text{ min}}\times\dfrac{60\text{ min}}{1\text{ h}}=120\text{ mph.}$

26. The home currency is the Middie and the target currency is the dollar.
We have (new value of 1 Middie − old value of 1 Middie) = \$1 − \$2 = −\$1.

(a) We divide the −\$1 by the new trading value, \$1, and multiply by 100, arriving at −100%, or a loss of 100% (this answer should make you question Option A, since the Middie did not become completely worthless!).

(b) We divide the −\$1 by the old trading value, \$2, and multiply by 100, arriving at −50%, a loss of 50%.

27. (a) $\dfrac{1.00\text{ Middie}-0.50\text{ Middie}}{1.00\text{ Middie}}\times 100\%=50\%.$

(b) $\dfrac{1.00\text{ Middie}-0.50\text{ Middie}}{0.50\text{ Middie}}\times 100\%=100\%.$

28. The home currency is the dollar and the target currency is the euro.
We have (new value of \$1 − old value of \$1) = (€0.7435 − €1.160)= €−0.4165.

(a) We divide the €−0.4165 by the new trading value, €0.7435, and multiply by 100, arriving at −56%, or a loss of 56%.

(b) We divide the €−0.4165 by the old trading value, EUR 1.160, and multiply by 100, arriving at −36%, a loss of 36%.

29. (a) $\dfrac{\$1.345 - \$0.862}{\$1.345} \times 100\% = 35.9\%.$

(b) $\dfrac{\$1.345 - \$0.862}{\$0.862} \times 100\% = 56.0\%.$

30. (a) Answers will vary. An analogous situation: Suppose that you make \$2000 per month and your friend makes \$4000 per month; you are making 50% less than she is, but she is making 100% more than you are. The relationship between a loss L and a corresponding gain G, when both are measured as signed numbers, is not additive but multiplicative. For Option A, we have $(1-L)(1-G)=1.$ (Check this for the Middie as home currency, where L = −100% = −1 from Exercise 26a and $G = 50\% = 0.5$ from Exercise 27a.) For Option B, the relationship is $(1+L)(1+G)=1.$ (Check this for the Middie as home currency, where $L = -50\% = -0.5$ from Exercise 26b and $G = 100\% = 1$ from Exercise 27b.) The connections between the two options (as can be shown easily using the notation in the solution for Exercise 31) are that $L_A = -G_B$ and $G_A = -A_B$, where L_A (respectively L_B) is the loss of currency 1 against currency 2 under Option A (respectively B) and G_A (respectively G_B) is the gain of currency 2 against currency 1 under option A (respectively B). (Check this for the results of Exercise 27.) For either Option, for a small change (under 5%, say), the size of the gain by the one currency is approximately the same as the size of the loss by the other. For example, under Option B, $1+G = \dfrac{1}{1+L} \approx 1-L,$ $G \approx -L$, or $|G| \approx |L|$.

(b) In light of the unnnatural result of Exercise 26a for Option A, we prefer Option B.

31. (a) For Option A, yes; for Option B, no. Let the previous value of the currency be C and the new value be D. Then Option A gives $\dfrac{D-C}{D} \times 100\% = \left(1 - \dfrac{C}{D}\right) \times 100\% < -100\%$ if $C > 2D$. Option B gives $\dfrac{D-C}{C} \times 100\% = \left(\dfrac{D}{C} - 1\right) \times 100\% \geq -100\%$ for all nonzero C, D.

(b) If the new trading value is higher than the old one, the percentage in Option B is higher than that in A: With $D > C$, Option A = $\dfrac{D-C}{D} \times 100\% < \dfrac{D-C}{C} \times 100\% =$ Option B. If the new trading value is lower than the old one, then both options give negative numbers but the absolute value of the percentage in Option B is higher than that in A: With $D < C$, Option A $\dfrac{D-C}{D} \times 100\% < \dfrac{D-C}{C} \times 100\% =$ Option B. In both cases, the absolute value of the percentage is higher for Option B.

(c) Either way, use Option B.

32. (a) $\frac{500 \text{ lb}}{144 \text{ in.}} \approx 3.47 \text{ lb/in.}^2$

(b) $3.47 \text{ lb/in.}^2 \times \frac{1 \text{ atm}}{14.7 \text{ lb/in.}^2} \approx 0.24 \text{ atm}$

33. (a) The layer of soil has volume $100 \text{ ft}^2 \times 0.5 \text{ ft} = 50 \text{ ft}^3$. The density is the weight divided by the volume, so $45{,}000 \text{ lb}/50 \text{ ft}^3 = 900 \text{ lb/ft}^3$.

(b) The density of steel is 500 lb/ft^3, so the claim is that the soil is $900/500 \approx 1.8 \approx 2$ times as dense as steel.

(c) Since 230 lb of compost is supposed to add about 5%, the original should be about 230 lb divided by 0.05, or 4,600 lb. The revised quotation should say that the mineral soil weighs about 4,500 to 4,600 lb.

34. (a) $\left(\frac{1}{2}\right)^3 \times 400 \text{ lb} = 50 \text{ lb}.$

(b) We assume that the gorilla undergoes geometric growth.

(c) $\frac{400 \text{ lb}}{1 \text{ ft}^2} = \frac{400 \text{ lb}}{1 \text{ ft}^2} \times \frac{1 \text{ ft}^2}{144 \text{ in.}^2} \approx 2.8 \text{ lb/in.}^2$

35. (a) Weight scales as the cube of the linear scaling factor, so KK would have to weigh $400 \text{ lb} \times 10^3 = 400{,}000 \text{ lb}$.

(b) The surface area of feet scales as the square of the linear scaling factor, so the area of KK's feet is $1 \text{ ft} \times 10^2 = 100 \text{ ft}^2$, and the pressure is $400{,}000 \text{ lb}/100 \text{ ft}^2 = 4{,}000 \text{ lb/ft}^2 = 4{,}000 \text{ lb}/144 \text{ in.}^2 \approx 28 \text{ lb/in}^2$.

36. (a) Solutions will vary. One strategy is to convert the measurements to feet, multiply to find the volume in cubic feet, convert cubic feet to cubic meters, convert cubic meters to liters, and then find the weight in kg and convert that to lb. The volume is $80 \text{ in.} \times 60 \text{ in.} \times 12 \text{ in.} \approx$
$6.6667 \text{ ft} \times 5 \text{ ft} \times 1 \text{ ft} \approx 33.333 \text{ ft}^3 = 33.333 \text{ ft}^3 \times \frac{1 \text{ m}^3}{35.31 \text{ ft}^3} \approx 0.944 \text{ m}^3 =$
$0.944 \text{ m}^3 \times \frac{1000 \text{ L}}{1 \text{ m}^3} = 944 \text{ L}$, which weighs 944 kg = $944 \text{ kg} \times \frac{2.205 \text{ lb}}{1 \text{ kg}} \approx 2080 \text{ lb}.$

(b) $\frac{2080 \text{ lb}}{4 \times (2 \text{ in.} \times 2 \text{ in.})} = \frac{2080 \text{ lb}}{16 \text{ in.}^2} = 130 \text{ lb/in}^2.$

(c) The person exerts $\frac{130 \text{ lb}}{0.25 \text{ ft}^2} = 520 \text{ lb/ft}^2 = 520 \text{ lb/ft}^2 \times \frac{1 \text{ ft}^2}{144 \text{ in.}^2} \approx 3.6 \text{ lb/in}^2.$

37. We have $r = 3$ ft and $h = 3.5$ ft, so the tub has volume $\pi r^2 h \approx (3.14)(3^2)(3.5) \text{ft}^3 \approx 99.0 \text{ ft}^3$. Per the text, 1 ft^3 of water weighs about 62 lb, so the water in the spa weighs $99.0 \text{ ft}^3 \times 62 \text{ lb/ft}^3 \approx 6100 \text{ lb} \approx 2800 \text{ kg}$.

38. The circumference at the base is $2\pi r = 40$ ft, so $r = \dfrac{40}{2\pi}$ ft ≈ 6.37 ft. So the volume is $\dfrac{1}{3}\pi r^2 h \approx \dfrac{3.14\times(6.37\text{ ft})^2\times 360\text{ ft}}{3} \approx 15,289\text{ ft}^3$. Since wood weighs about 31 lb/ ft^3, the weight is—$15{,}289\text{ ft}^3 \times 31\text{ lb/ ft}^3 = 473{,}959$ lb – over 230 tons.

39. The lights are strung around the outside of the tree branches, so in effect they cover the outside "area" of the tree (thought of as a cone). Hence, the number of strings needed grows in proportion to the square of the height: a 30-ft tree will need $5^2 = 25$ times as many strings as a 6-ft tree. However, you could also argue that a 30-ft tree is meant to be viewed from farther away, so that stringing the lights farther apart on the 30-ft tree would produce the same effect as with the shorter tree.

40. (a) Assuming pace is proportional to height (and vice versa), Hercules was 30% taller, hence $6\frac{1}{2}$ ft tall.

(b) With the shorter Hercules and the shorter cubit:

$$4\text{ cubits} = 4\text{ cubits} \times \frac{17\text{ in.}}{1\text{ cubit}} = 68\text{ in.} = 5\text{ ft } 8\text{ in.}$$

With the taller Hercules and the longer cubit:

$$4\text{ cubits} + 1\text{ "foot"} = 4\text{ cubits} \times \frac{22\text{ in.}}{1\text{ cubit}} + 12\text{ in.} = 88\text{ in.}+12\text{ in.} = 100\text{ in.} = 8\text{ ft } 4\text{ in.},$$

assuming that a Greek "foot" was equal in length to a modern ft.

41. The lower estimate of 17 in. for a cubit leads to a height of 6×17 in.+ 9 in. = 111 in. = 9 ft 3 in. = 111 in. × 2.54 cm/in. ≈ 282 cm. Similarly, the upper estimate of 22 in. for a cubit leads to a height of 11 ft 9 in ≈ 358 cm. In modern times, there have been men over 9 ft tall, but not over 11 ft tall.

42. $\text{BMI} = \dfrac{\text{weight (kg)}}{(\text{height (m)})^2} = \dfrac{65\text{ kg}}{(1.60\text{ m})^2} = \dfrac{65\text{ kg}}{2.56\text{ m}^2} \approx 25.4\ (\text{kg/m}^2)$; she is just barely overweight.

43. The weight W must satisfy BMI = $\dfrac{W}{h^2} = \dfrac{W}{1.90^2} < 25$, so $W < (1.90)^2 \times 25 = 90.25$ kg.

44. $\dfrac{1\text{ kg}}{(1\text{ m})^2} = \dfrac{2.205\text{ lb}}{(39.37\text{ in.})^2} \approx \dfrac{2.205\text{ lb}}{1550\text{ in.}^2} \approx \dfrac{1\text{ lb}}{703\text{ in.}^2}$. You must multiply the value calculated from pounds and inches by 703.

45. Answers will vary.

46. A Lilliputian would weigh $\left(\dfrac{1}{12}\right)^3 \approx 0.00058$ times as much as an adult human. A Lilliputian corresponding to a 140-lb adult human would weigh 0.08 lb, or about an ounce and a quarter. Human infants may be only a foot long at birth, barely twice Lilliputian size. Other mammals – either as infants (e.g., pandas) or as adults (e.g., mice) – are smaller than Lilliputians. So Lilliputians are not ruled out by area-volume considerations.

47. (a) If the species grew geometrically: Weight would scale as the cube of wingspan, so an individual with half the wingspan of an adult would have one-eighth the weight. However, the dimensions of this dinosaur probably did *not* grow geometrically; the wingspan probably grew not in proportion to the length of the dinosaur but more rapidly, so as to support the weight. Then an individual with half the wingspan would have one-fourth the wing area, so could support in flight only one-fourth the weight, or 25 lb.

(b) If the species grew geometrically: Weight would scale as the cube of wingspan, so an individual weighing half as much as an adult would have a wingspan $\sqrt[3]{\frac{1}{2}} \approx 0.79$ times as great. If instead the wingspan grew to support the weight: An individual weighing half as much as an adult would need half the wing area; with both length and width of the wing growing in the same proportion, wing area (and hence weight) would scale as the square of wingspan. Hence wingspan would scale as the square root of weight. The half-weight dinosaur would need a wingspan $\sqrt{\frac{1}{2}}$ times as large, or $50 \text{ ft} \times \sqrt{0.5} \approx 35 \text{ ft}$.

48. $\sqrt{12} \times 20 \text{ mph} \approx 69 \text{ mph}$.

49. Answers will also vary with assumptions about the height of Icarus. A 5-ft tall Icarus would have been about 15 times as long as a sparrow and hence had to fly $\sqrt{15} \times 20$ mph $\approx$ 77 mph. We assume that Icarus was a scaled-up sparrow, so that his wing loading was proportional to his length; with disproportionately large wings, the wing loading − and hence the minimum speed − would have been lower.

50. It has disproportionately large wings compared to geometric scaling up of a bird, hence lower wing loading and lower minimum flying speed. Also, in part it glides rather than flies.

51. (a) (i) The giant ants are 8 m = 800 cm long, compared to the 1-cm length of a common ant. So the linear scaling factor is 800.

(ii) Since area scales as the square of the linear scaling factor, the surface area of the giant ant is $800^2 = 640{,}000$ times as large.

(iii) Since volume scales as the cube of the linear scaling factor, the volume of the giant ant is $800^3 = 512{,}000{,}000$ times as great.

(b) The giant ant has 800 times as much volume per unit of surface area, so its skin could supply one eight-hundredth of what it would need.

(c) There couldn't be any such giant ants.

52. $20\left(\frac{60}{30}\right)^{1/4} = 20 \cdot 2^{1/4} \approx 23.8$ m.

53. Because $h \propto t^{1/4}$, we have $t \propto h^4$. For a tree to grow to 40 m tall, compared to a tree growing to 20 m tall, it will take $\left(\frac{40}{20}\right)^2 = 2^4 = 16$ times as long. So if it takes 30 years to grow to 20 m, it would take 16×30 = 480 years to grow to 40 m.

54. Since $h \propto t^{1/4}$, we have $t \propto h^4$, or $t \propto kh^4$ for a constant *k*. So 2 times as high will take $2^4 = 16$ times as long. In detail: Since *h* = 100 m when *t* = 1000 yr, we have $k = \frac{t}{h^4} = \frac{1000 \text{ yr}}{(100 \text{ m})^4} = 10^{-5} \text{ yr/m}^4$.

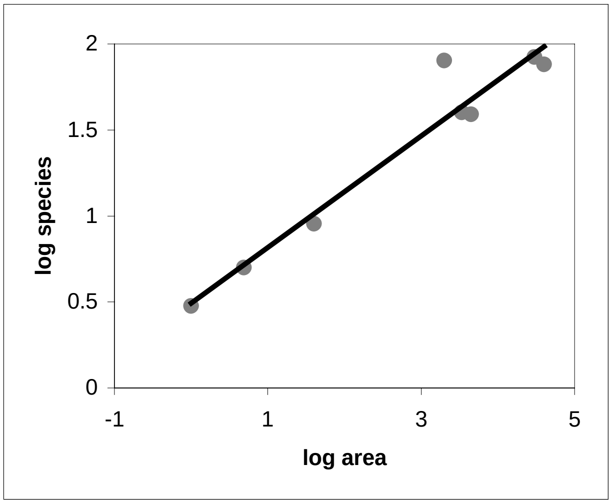
Thus, for *h* = 200 m, we have $t = 10^{-5} \text{ yr/m}^4 \times (200 \text{ m})^4 = 16,000 \text{ yr}$.

55. $A \propto d^2$ and $A \propto M^{3/4} \propto (d^2h)^{3/4} = d^{3/2}h^{3/4}$, so $d^2 \propto d^{3/2}h^{3/4}$, hence $d^{1/2} \propto h^{3/4}$, and $d \propto h^{3/2}$.

56. A major component of water loss is transpiration and evaporation through the skin. A taller person has a greater ratio of volume to surface area than a shorter person and hence loses less water per pound in a given time interval. On the other hand, the shorter person loses more heat per pound in a given time interval. (Note that deserts can be cold at night.)

57. A small warm-blooded animal has a large surface-area-to-volume ratio. Pound for pound, it loses heat more rapidly than a larger animal, hence must produce more heat per pound, resulting in a higher body temperature.

58. (a) On log-log paper:

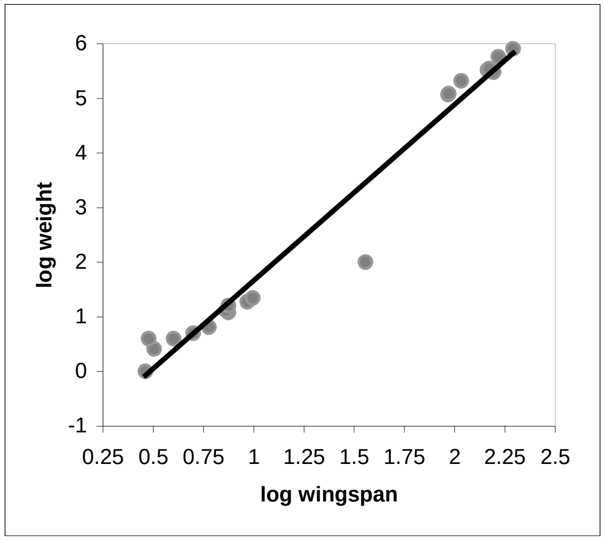


(b) The relationship is allometric, because on the log-log graph the points lie close to a straight line. Let *A* represent area and *S* represent species. The graph shows the line of best fit, whose equation is $\log_{10} S = 0.3423 \log_{10} A$, or $S = 10^{0.4931} A^{0.3423} \approx 3.11A^{0.3423}$.

(c) For an area of 400 sq mi, we have $\log_{10} A = \log_{10} 400 \approx 2.60$; so from the graph or from the line of best fit, $\log_{10} S \approx 1.3$, from which we get (using the 10^x key) approximately 20 species.

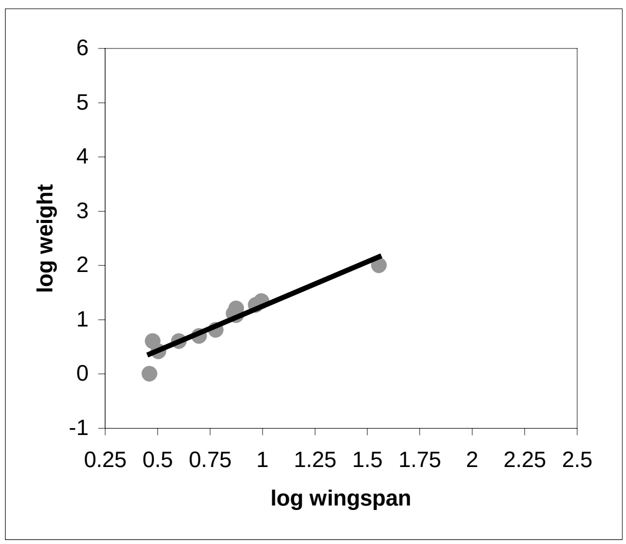
(d) It approximately doubles, since $10^{0.3423} \approx 2.2$.

59. (a) On log-log paper:



(b) Both relationships are allometric, since the results are good fits to straight lines whose slopes are not 1. For birds, the slope of the least-squares fit to the log-log graph is about 1.6; for planes, it is about 2.4. Because these slopes are not equal to 1, the relationships are not proportional. The data for birds and planes are graphed separately along with their corresponding lines on the same set of axes.

For Birds:



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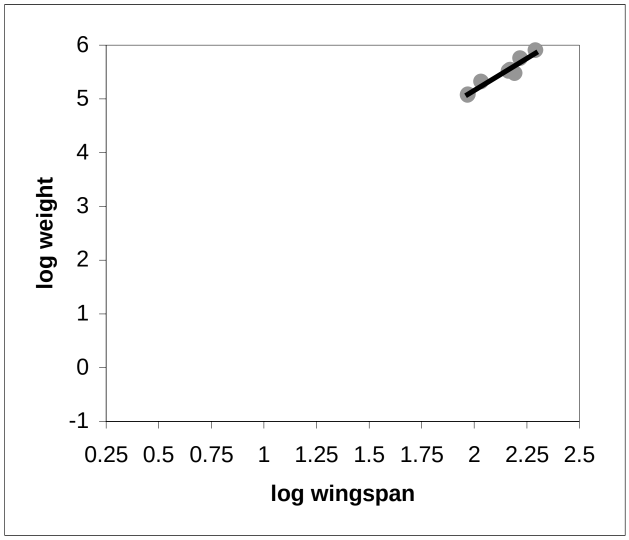
59. (b) continued
For Planes:



(c) The slope for birds (1.6) is less steep than for planes (2.4). The slope of the least-squares fit for both combined (a bad idea!) is about 3.4.

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