Chapter 16
Identification Numbers

Chapter Outline

Chapter Summary

Coding data facilitates the accurate and efficient transfer of information. Grocery stores use the *Universal Product Code* (*UPC*), and books use the *International Standard Book Number* (*ISBN*) method. In this chapter several different methods used in our daily lives of encoding information will be discussed.

To at least partially ensure accuracy, a coding scheme should have some means of *detecting errors*. Many codes use *check digits* for this purpose. Many places such as blood banks and libraries use *Codabar*. What types of errors can be detected and the rates of success for detecting them depend on the design of the code. Efficiency will be enhanced if it is easy to encode data and if the code represents data compactly. Good codes thus perform a delicate balancing act: sophisticated enough to detect high percentages of (perhaps) several kinds of errors, yet simple enough to provide for quick and compact coding of data. Not surprisingly, number theory, the mathematics of the integers, has proved useful in designing good codes. Our current "Information Age" would seem to ensure that codes will be a topic of interest for some time to come.

Identification numbers serve to unambiguously identify individuals (people, books, products, or whatever). Most schemes used to produce these numbers use check digit codes. The need to ensure accuracy is perhaps obvious if the identification number happens to be that of an individual's bank account. A situation that may not come so quickly to mind, except to the person in inventory control, is the parts identification numbers that help fill orders and keep track of available stock. Indeed, there are many numbers that enter our lives as identification numbers of one kind or another. Because of the availability of different examples of identification numbers, this chapter should be very accessible and interesting to its readers.

Skill Objectives

1. Understand the purpose of a check digit and be able to determine one for various schemes.

2. Given an identification number and the scheme used to determine it; be able to decide if the number is a valid number for that scheme.

3. Be able to convert a given ZIP code to its corresponding bar code, and vice versa.

4. Be able to convert a given UPC number to its corresponding bar code.

Teaching Tips

1. Have students analyze the UPC bar codes on products they have purchased recently.

2. Have students think of various numbers that are a regular part of their lives; then have them decide which of these numbers are identification numbers and which are not.

3. Joseph Gallian gave an informative and entertaining talk on identification numbers during the 75th anniversary celebration of Pi Mu Epsilon, the National Mathematics Honor Society. This tape may be borrowed from Robert Woodside, Department of Mathematics, East Carolina University, Greenville, NC. Professor Woodside is the National Secretary for Pi Mu Epsilon.

4. Some coding schemes fail to detect certain kinds of errors: either some substitutions of one digit for another, or some types of transpositions. Why this is the case can be illustrated very easily by a few examples. This is especially true in schemes that use remainders modulo some fixed integer. Gallian's College Math Journal article listed in the suggested readings is a good source for this.

Research Paper

Have students research the history of the United States Postal Service and the introduction of the ZIP code. Also delivery, in general, is rich in history. Around 500 B.C. the Greek philosopher Herodotus of Halicarnassus penned the phrase, "Neither snow nor rain nor heat or gloom of night stays these couriers from the swift completion of their appointed rounds." He was describing the Persian "postal" system. His quote is inscribed on the New York Post Office building.

Collaborative Learning

Bar Codes

To motivate learning about ZIP codes, have students bring in an article of mail for them to compare. Since most students should live in the same general area, they should see some common patterns in the first several groupings of bars. You could also ask the students to bring in envelopes after the lesson and ask them to decode each other's envelopes.

Solutions

Skills Check:

1. a	2. c	3. b	4. b	5. b	6. c	7. b	8. a	9. a	10. a
11. b	12. b	13. c	14. b	15. b	16. c	17. c	18. c	19. c	20. a

Exercises:

1. Since $3+9+5+3+8+1+6+4+0=48=9\times5+3$, the check digit is 3.

2. Since $7+2+3+4+5+4+1+7+8+0=41=9\times4+5$, the check digit is 5.

3. Since $873345672=7\times124763667+3$, the check digit is 3.

4. Since $2+7+7+5+0+4+2+1+1+6=35=9\times3+8$, the error is not detected.

5. Since $30860422052=7\times4408631721+5$, the check digit is 5.

6. Since $540047=7\times77149+4$, the check digit is 4.

7. Since $3\cdot3+8+3\cdot1+3+3\cdot7+0+3\cdot0+9+3\cdot2+1+3\cdot3=69$, the check digit is 1.

8. Since $3\cdot0+5+3\cdot0+7+3\cdot4+3+3\cdot1+1+3\cdot5+0+3\cdot2=52$, the check digit is 8.

9. Since $10\cdot0+9\cdot6+8\cdot6+7\cdot9+6\cdot1+5\cdot9+4\cdot4+3\cdot9+2\cdot3=265=11\times24+1$, the check digit is X.

10. Since $10\cdot0+9\cdot6+8\cdot6+7\cdot9+6\cdot3+5\cdot3+4\cdot9+3\cdot0+2\cdot7=248=11\times22+6$, the check digit is 5.

11. Since $7\cdot0+3\cdot9+9\cdot1+7\cdot9+3\cdot0+9\cdot2+7\cdot0+3\cdot4=129$, the check digit is 9.

12. Since $7\cdot0+3\cdot9+9\cdot1+7\cdot0+3\cdot0+9\cdot0+7\cdot0+3\cdot1=39$, the check digit is 9.

13. Since $4+6+1+2+1+2+0+2+3=21$, the check digit is 6.

14. Since $(3+4+0+3+0+3+2+7)\times2=44$ and one of the summands exceeds 4, we have $44+1+(5+1+2+2+0+3+2+0)=60$. So, the number is valid.

15. Since $7\cdot3+8+7\cdot1+3+7\cdot7+0+7\cdot0+9+7\cdot2+1+7\cdot3=133$, the check digit is 7. This check-digit scheme will detect all single-digit errors.

16. The check digit is 7. The errors are not detected because the errors increase the sum by 10, and so the new weighted sum is still divisible by 10. In the odd numbered positions, if a digit a is replaced by the digit b where $a-b=\pm5$ the error is not detected.

17. In the odd-numbered positions, if a digit a is replaced by the digit b where $a-b$ is even, the error is not detected.

18. The weight 9 will detect all errors in its position. The weights 4, 6, and 8 will not detect all errors.

19. We begin with $(3+0+2+6+0+9+4+1)\times 2 = 50$. Adding 2, we obtain 52 and have the following.

$$52+0+1+5+0+1+6+3=68$$

So, the check digit is 2.

20. First convert JM1GD222J1581570 to 1417422211581570. Then we have the following.

$$8\cdot 1+7\cdot 4+6\cdot 1+5\cdot 7+4\cdot 4+3\cdot 2+2\cdot 2+10\cdot 2+$$
$$9\cdot 1+8\cdot 1+7\cdot 5+6\cdot 8+5\cdot 1+4\cdot 5+3\cdot 7+2\cdot 0=269=11\times 24+5$$

Thus, the check digit is 5.

21. (a) Since $1\cdot 0+1\cdot 1+3\cdot 2+3\cdot 1+1\cdot 6+3\cdot 9+1\cdot 0=43$, the check digit is 7.

(b) Since $1\cdot 0+1\cdot 2+3\cdot 7+3\cdot 4+1\cdot 5+3\cdot 5+1\cdot 1=56$, the check digit is 4.

(c) Since $1\cdot 0+1\cdot 7+3\cdot 6+3\cdot 0+1\cdot 0+3\cdot 2+1\cdot 2=33$, the check digit is 7.

(d) Since $1\cdot 0+1\cdot 4+3\cdot 9+3\cdot 6+1\cdot 5+3\cdot 8+1\cdot 0=78$, the check digit is 2.

22. (a) Since $1\cdot 0+1\cdot 7+3\cdot 5+1\cdot 4+3\cdot 7+3\cdot 0=47$, the check digit is 3

(b) Since $1\cdot 0+1\cdot 7+3\cdot 7+1\cdot 4+3\cdot 7+3\cdot 1=56$, the check digit is 4

(c) Since $1\cdot 0+1\cdot 7+3\cdot 2+1\cdot 4+3\cdot 4+3\cdot 4=41$, the check digit is 9

23. First observe that the given number 0669039254 results in a weighted sum that has a remainder of 5 after division by 11. So all we need to do is check for successive pairs of digits of this number that results in a contribution to the weighted sum of 5 less or 6 more, since either of these will make the weighted sum divisible by 11. Checking each pair of consecutive digits, we see that 39 contributes $5\cdot 3+4\cdot 9=51$ whereas 93 contributes $5\cdot 9+4\cdot 3=57$. So, the correct number is 0669093254.

24. In the bank scheme 751 contributes $7\cdot 7+3\cdot 5+9\cdot 1=73$ to the sum, whereas 157 contributes $7\cdot 1+3\cdot 5+9\cdot 7=85$ to the sum. Since the last digit of 73 and the last digit of 85 do not match, the check digit for the correct number will not match the last digit of the sum obtained with 157. Thus, the error is detected. In the UPC scheme both 751 and 157 contribute the same amount to the relevant sum so that the sum for the incorrect number would still end in a zero. Thus, the error is not detected.

25. Notice that when we add the weighted sum used for the actual check digit:

$$7a_1+3a_2+9a_3+7a_4+3a_5+9a_6+7a_7+3a_8$$

and the weighted sum

$$3a_1+7a_2+a_3+3a_4+7a_5+a_6+3a_7+7a_8,$$

we obtain

$$10a_1+10a_2+10a_3+10a_4+10a_5+10a_6+10a_7+10a_8,$$

which always ends with 0. So, the actual check digit and the check digit calculated with the weighted sum $3a_1+7a_2+a_3+3a_4+7a_5+a_6+3a_7+7a_8$ are both 0 or their sum is 10.

26. Since the check digit is determined by the sum of the noncheck digits, transposing any two noncheck digits does not change the sum. This means that the remainder after division by 9 will remain the same. Transposing the check digit does change the sum if the transposed digits are distinct.

27. Replacing Z by 9 or vice versa is not detected.

28. Since the sum of the digits is not changed by rearranging the terms, the sum of the rearranged digits is divisible by 9.

29. Since the value of the weighted sum determines whether or not a number is valid, the position of the check digit is not relevant.

30. Replacing a 0 by a 9 or vice versa changes the sum by 9 or -9. In either case, the remainder after division by 9 is unchanged.

31. Since the remainder after dividing by 9 is between 0 and 8, 9 cannot be a check digit.

32. If the remainder after dividing the sum by 9 is *k*, then the check digit is $9-k$ unless $k=0$. When $k=0$, the check digit is 0. So, 9 can never be a check digit.

33. If the remainder of the sum of the noncheck digits after dividing by 7 is *k*, then the check digit is $7-k$ if $k \neq 0$ and 0 if $k=0$. So, 7,8, and 9 can never be a check digit.

34. All adjacent transposition are detectable except the transposition of the last two digits.

35. Yes. The ISBN scheme detects all transposition errors.

36. In the UPS scheme the substitution *a* for *b* is undetectable if and only if $a-b=\pm 7$.

37. For the transposition to go undetected, it must be the case that the difference of the correct number and the incorrect number is evenly divisible by 11. That is,

$$(10a_1+9a_2+8a_3+\cdots+a_{10})-(10a_3+9a_2+8a_1+\cdots+a_{10})$$

is divisible by 11. This reduces to $2a_1-2a_3=2(a_1-a_3)$ is divisible by 11. But $2(a_1-a_3)$ is divisible by 11 only when a_1-a_3 is divisible by 11 and this only happens when $a_1-a_3=0$. In this case, there is no error. The same argument works for the fourth and sixth digits.

38. The check digit is the same in both cases. To see this, note that because both

$$11a_1+11a_2+\cdots+11a_9+11a_{10}$$

and

$$10a_1+9a_2+\cdots+2a_9+a_{10}$$

are divisible by 11, so is their difference $a_1+2a_2+\cdots+9a_9+10a_{10}$.

39. The combination 72 contributes $7\cdot 1+2\cdot 3=13$ or $7\cdot 3+2\cdot 1=23$ (depending on the location of the combination) towards the total sum, while the combination 27 contributes $2\cdot 1+7\cdot 3=23$ or $2\cdot 3+7\cdot 1=13$. So, the total sum resulting from the number with the transposition is still divisible by 10. Therefore, the error is not detected. When the combination 26 contributes $2\cdot 1+6\cdot 3=20$ towards the total sum, the combination 62 contributes $6\cdot 1+2\cdot 3=12$ toward the total sum; so the new sum will not be divisible by 10. Similarly, when the combination 26 contributes $2\cdot 3+6\cdot 1=12$ to the total, the combination 62 contributes $6\cdot 3+2\cdot 1=20$ to the total. So, the total for the number resulting from the transposition will not be divisible by 10 and the error is detected. In general, an error that occurs by transposing *ab* to *ba* is undetected if and only if $a-b=\pm 5$

40. The 53 contributes $7\cdot 5+3\cdot 3=44$ towards the weighted sum, whereas 35 contributes $7\cdot 3+3\cdot 5=36$ towards the weighted sum. Thus the transposition changes the last digit and consequently the error is detected. In 237 the 2 and 7 contribute $7\cdot 2+9\cdot 7=77$ towards the weighted sum, whereas in 732 the 7 and 2 contribute $7\cdot 7+9\cdot 2=67$. Thus, the last digit of the weighted sum remains unchanged after the transposition. This means that the error is undetected.

41. The error $\cdots abc \cdots \to \cdots cba \cdots$ is undetectable if and only if $a - c = \pm 5$. To see this in the case that the weights for *abc* are 7, 3, 9, notice that *a* and *c* contribute $7a + 9c$ toward the weighted sum, whereas in the case of *cba*, the c and a contribute $7c + 9a$. Thus, the error is undetectable if and only if $7a + 9c$ and $7c + 9a$ contribute equal amounts to the last digit of the weighted sum. This means that they differ by a multiple of 10. That is, $-2a + 2c = 2(c - a)$ is a multiple of 10. This occurs when $c - a = 0$ or $c - a = \pm 5$. When $c - a = 0$, there is no error.

42. (a) Since $9 \cdot 1 + 8 \cdot 4 + 7 \cdot 9 + 6 \cdot 1 + 5 \cdot 0 + 4 \cdot 5 + 3 \cdot 7 + 2 \cdot 3 = 157$, the check digit is 3.

(b) Since $9 \cdot 1 + 8 \cdot 4 + 7 \cdot 9 + 6 \cdot 1 + 5 \cdot 0 + 4 \cdot 5 + 3 \cdot 2 + 2 \cdot 6 + 7 = 155$ is not divisible by 10, the number is not valid. The correct number cannot be determined because the mistake could have occurred in several possible positions or there could have been more than one error. If you knew the error occurred in the seventh position, the error could be corrected since 7 is the only digit that results in a weighted sum divisible by 10.

(c) Since the number 199105767 gives the weighted sum

$$9 \cdot 1 + 8 \cdot 9 + 7 \cdot 9 + 6 \cdot 1 + 5 \cdot 0 + 4 \cdot 5 + 3 \cdot 7 + 2 \cdot 6 + 7 = 210,$$

which is divisible by 10, the error is undetected.

(d) Say the transposition is $\cdots ab \cdots \to \cdots ba \cdots$ with a weighted with $i+1$ and b weighted with i. The transposition is undetected only if $a(i+1) + bi$ and $b(i+1) + ai$ end in the same digit. This simplifies to their difference, $a - b$, ending in 0. Since a and b are distinct and between 0 and 9, this can never happen.

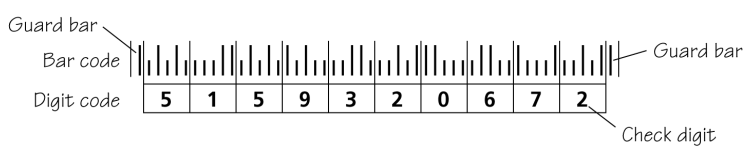
43. Since any error in the position with weight 10 does not change the last digit of the weighted sum, no error in that position is detected. In the position with weight 5, replacing an even digit by any other even digit is not detected. In positions with weights 12, 8, 6, 4, or 2, replacing a by b is undetectable if $a - b = \pm 5$. In positions 11, 9, 7, 3, or 1, all errors are detected.

44. They predate computers.

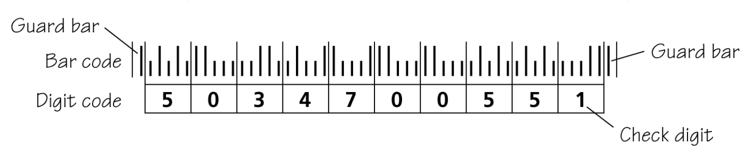
45. Since both numbers are valid the difference of the weighted sums is divisible by 10. That is, $(7w + 3 + 2w + 1 + 5w + 6 + 7w + 4) - (7w + 3 + 2w + 1 + 5w + 6 + 6w + 1)$ is divisible by 10. The difference simplifies to $w + 3$. So, $w = 7$.

46. Suppose that *x* is a weighted sum of all the digits of the U.S. ISBN excluding the check digit and *c* is the check digit for the German ISBN. Since both $30 + x + c$, the weighted sum of the German ISBN, and $x + 1$, the weighted sum of the U.S. ISBN, are divisible by 11 so is their difference $29 + c$. Thus, the check digit is 4.

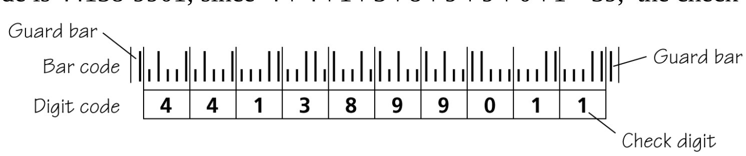
47. (a) The code is 51593-2067; since $5 + 1 + 5 + 9 + 3 + 2 + 0 + 6 + 7 = 38$, the check digit is 2.



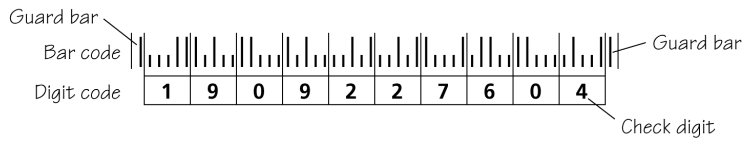
(b) The code is 50347-0055; since $5 + 0 + 3 + 4 + 7 + 0 + 0 + 5 + 5 = 29$, the check digit is 1.



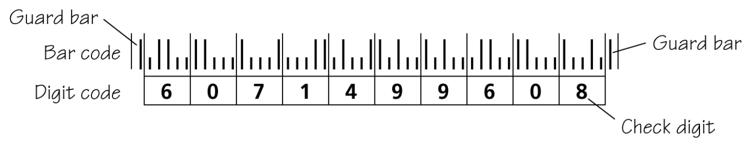
(c) The code is 44138-9901; since $4 + 4 + 1 + 3 + 8 + 9 + 9 + 0 + 1 = 39$, the check digit is 1.



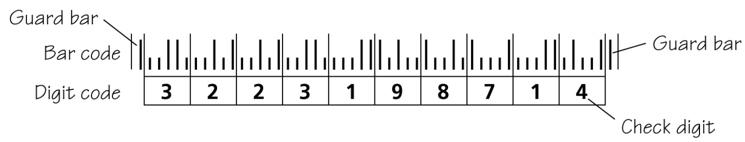
48. (a) The code is 19092-2760; since $1+9+0+9+2+2+7+6+0=36$, the check digit is 4.



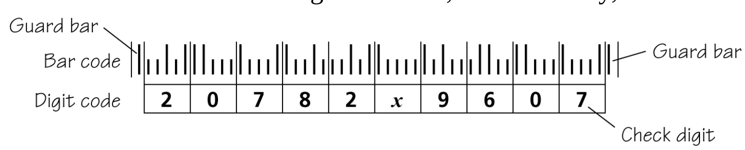
(b) The code is 60714-9960; since $6+0+7+1+4+9+9+6+0=42$, the check digit is 8.



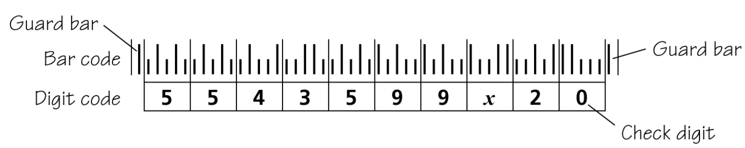
(c) The code is 32231-9871; since $3+2+2+3+1+9+8+7+1=36$, the check digit is 4.



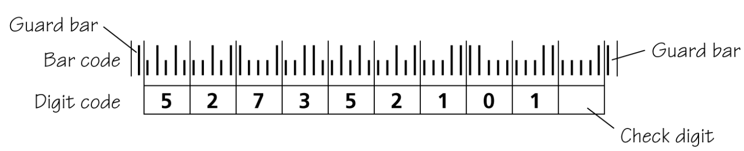
49. (a) Since the sixth block of five bars (ignoring the first bar) has one long bar and four short bars, that block is incorrect. Call the digit corresponding to that block *x*. Then the code is 20782*x*960. Since the sum of the digits is $x+41$, $x=9$. Finally, we write 20782-9960.



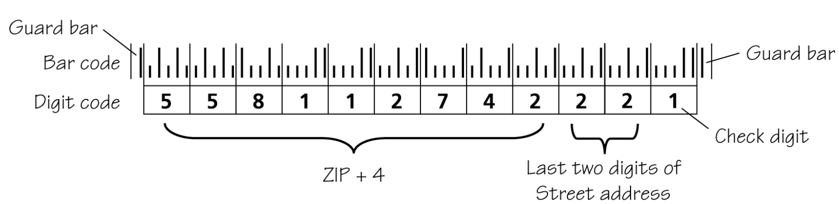
(b) Since the eighth block of five bars (ignoring the first bar) has three long bars and two short bars, that block is incorrect. Call the digit corresponding to that block *x*. Then the code is 5543599*x*2. Since the sum of the digits is $x+42$, $x=8$. Finally, we write 55435-9982.



(c) Since the tenth block of five bars (ignoring the first bar) has one long bar and four short bars, that block is incorrect. Since this is the check digit, the nine-digit ZIP code is 52735-2101.



50. The ZIP+4 is 55811-2742. The last two digits of the street address are 22, and the check digit is 1.



51. If a double error in a block results in a new block that does not contain exactly two long bars, we know this block has been misread. If a double error in a block of five results in a new block with exactly two long bars, the block now gives a different digit from the original one. If no other digit is in error, the check digit catches the error, since the sum of the 10 digits will not end in 0. So, in every case an error has been detected. Errors of the first type can be corrected just as in the case of a single error. When a double error results in a legitimate code number, there is no way to determine which digit is incorrect.

52. 000111000100110 where 0 represents a short bar and 1 a long bar.

53. The strings are *aaabb*, *aabab*, *aabba*, *abaab*, *ababa*, *abbaa*, *baaab*, *baaba*, *babaa*, and *bbaaa* (in alphabetical order). If you replace each short bar in the bar code table (page 603) by an a and replace each long bar in the bar code table by a b, the resulting strings are listed in alphabetical order.

54. The UPC number for books beginning with a 9 are followed by two blocks of six digits; whereas, on grocery items, the lead digit is followed by two blocks of five digits. Moreover, for books, the check digit is printed within the guard pattern; whereas, for grocery items it is not. The UPC number begins with 978 and is followed by the ISBN number with the ISBN check digit replaced by a UPC check digit. The check digit is chosen so that the number $a_1a_2\cdots a_{13}$, including the check digit, has the property that the following ends with a 0.

$$a_1+3a_2+a_3+3a_4+a_5+3a_6+a_7+3a_8+a_9+3a_{10}+a_{11}+3a_{12}+a_{13}$$

55. Since there is an even number of 1's in 1000100, the scanner is reading from right to left.

56. The manufacturer's number and the product number are 6 digits long instead of 5. This bar code is the European UPC code.

57. Wyoming, Nevada, and Alaska.

58. States have lower numbers than those states to their west. States have lower numbers than those directly south of them.

59. The size of the population.

60. Nevada is the most likely to need a new allotment.

61. The Canadian scheme detects any transposition error involving adjacent characters. Also, there are $26^3\times10^3=17,576,000$ possible Canadian codes but only $10^5=100,000$ U.S. five-digit ZIP codes. Hence the Canadian scheme can target a location more precisely.

62. S-530 for each name.
Smith $\to$ Smit $\to$ 2503 $\to$ 2503 $\to$ 503 $\to$ 53 $\to$ S-530
Schmid $\to$ Scmid $\to$ 22503 $\to$ 2503 $\to$ 503 $\to$ 53 $\to$ S-530
Smyth $\to$ Smyt $\to$ 2503 $\to$ 2503 $\to$ 503 $\to$ 53 $\to$ S-530
Schmidt $\to$ Scmidt $\to$ 225033 $\to$ 2503 $\to$ 503 $\to$ 53 $\to$ S-530

63. Skow $\to$ Sko $\to$ 220 $\to$ 20 $\to$ 0 $\to$ all numbers gone $\to$ S-000
Sachs $\to$ Sacs $\to$ 2022 $\to$ 202 $\to$ 02 $\to$ 2 $\to$ S-200
Lennon $\to$ Lennon $\to$ 405505 $\to$ 0505 $\to$ 55 $\to$ L-550
Lloyd $\to$ Lloyd $\to$ 44003 $\to$ 403 $\to$ 03 $\to$ 3 $\to$ L-300
Ehrheart $\to$ Ereart $\to$ 060063 $\to$ 06063 $\to$ 6063 $\to$ 663 $\to$ E-663
Ollenburger $\to$ Ollenburger $\to$ 04405106206 $\to$ 0405106206 $\to$ 405106206 $\to$ 451626 $\to$ O-451

64. 42 are the first two digits. The remaining three come from $40(7-1)+18+500=758$. Thus, the number is 42758

65. A person born in 1999 is too young for a driver's license.

66. Since the birthyear is 1942, 42 are the first two digits. For the months of January through May, we have $31(5)=155$. Add to this the 18 days of June we have $155+18=173$. Since the individual is male, the five digits are 42173.

67. For a woman born in November or December the formula $40(m-1)+b+600$ gives a number requiring four digits.

68. The first two digits indicate the birthyear of 1977. Since $61=31+30$, the birthday would have to be February 30.

69. The 58 indicates that the year of birth is 1958. Since 818 is larger than $12\cdot 31=372$, we subtract 600 from 818 to obtain 218. Then $218=7\cdot 31+1$ tells us that the person was born on the first day of the eighth month. So, the birth date is August 1, 1958.

70. 42218: Since 42 are the first two digits, the birthyear is 1942. Also, $218=40(6-1)+18$. Thus the birthday is June 18.

53953: Since 53 are the first two digits, the birthyear is 1953. Since 953 exceeds $40(12-1)+31=471$, the person is female and we subtract 500. Since $953-500=453$ and $453=40(12-1)+13$, the birthday is December 13.

71. Since $248=63(3)+58+1$, the number 248 corresponds to a female born on March 29; since $601=63(9)+34$ the number corresponds to a male born on September 17.

72. Since many people don't like to make their age public, this method is used to make it less likely that people would notice that the license number encodes year of birth.

73. Likely circumstances could be twins; sons named after their fathers (such as John L. Smith, Jr.); common names such as John Smith and Mary Johnson; and states that do not include year of birth in the code.

74. In many cases the code will be the same.

75. Because of the short names and large population there would be a significant percentage of people whose names would be coded the same.

Word Search Solution

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S I H A E E M O A E N Q S I G W H A K G M L E S A
M N T P S L E D O E P E N N D L X U C G S K E E A
E T F O A C T D R L J M A H H Q Z Q E S M G B N I
A C I P L A K O C N G X H O Q S X E E E N A M E I
E H H D N I D W H S I T Q Y L G N I D O C N E C E
M E T S Y S G N I D O C X E D N U O S N R A F A E
I C F R O X N S S S A E H O E N C I S I A A W E N
N K J A S E D O C G N I T C E T E D R O R R E O F
R D H E H B A E H R A R H M C Q T N W J E J V R C
L I P S T W S E R K R N E U F H B W T A M I E R N
O G E J S E E R E D S E D T E L Y K I G C F E M S
E I C N E I N P O D J O T I N U N A O H J L C O F
S T R X L C O S E D R A F R N C O N S A S K M F Y
C N A P W E T L I P C O D E E I O O O K M N I S F
E B S J J H E P L O A H N R O G U D O O E L U N H
B Z R L G T R A B S H T E D O C Y R A N I B S T P
E D N I T N S Y A T Q A J X E M O K T B X Y N O N
A U E K A R I R R N Z T E D J R E O G T A Y D N E
I W E E E X S D C E S E F F R L C J R F F R I V E
F A G V O E L R O T P P C T S H C F D G A M R F E
P I I E A M O G D C K E T R C E O A T A E C E A T
S N S K L S E H E O E I B R D O M O A R N O G H Q
U I E P M H V M H D T D E S W F A B G C B G G L G
E R R J L I U E T E M N P E I G N T A Y N M A J S
I N N T H L A G O A A N A Z M S E R O W E O T C Q
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