Apportionment

JWR

July 27, 2006

Contents

1 Introduction

The constitution says that apportionment of representatives to a state should be proportional to its population. If taken literally, this would mean giving fractional representatives, so some method of converting fractions to whole numbers must be used. Various methods of doing this have been used over the years. Often the result seems unfair to many people and bitter disputes have resulted. Here is a list of surprising things that happened as a result of using one method or another.

1.1. The Alabama Paradox. An increase in the total number of seats to be apportioned can cause a state to lose a seat. The Alabama Paradox first surfaced after the 1870 census. With 270 members in the House of Representatives, Rhode Island got 2 representatives but when the House size was increased to 280, Rhode Island lost a seat. After the 1880 census, C. W. Seaton (chief clerk of U. S. Census Office) computed apportionments for all House sizes between 275 and 350 members. He then wrote a letter to Congress pointing out that if the House of Representatives had 299 seats, Alabama would get 8 seats but if the House of Representatives had 300 seats, Alabama would only get 7 seats. The method used at the time was Hamilton's method explained below in paragraph **??**.

1.2. The Population Paradox. An increase in a state's population can cause it to lose a seat. The Population Paradox was discovered around 1900, when it was shown that a state could lose seats in the House of Representatives as a result of an increase in its population. (Virginia was growing much faster than Maine, but Virginia lost a seat in the House while Maine gained a seat.) The method used at the time was Hamilton's method explained below in paragraph **??**.

1.3. The New States Paradox. Adding a new state with its fair share of seats can affect the number of seats due other states. The New States Paradox was discovered in 1907 when Oklahoma became a state. Before Oklahoma became a state, the House of Representatives had 386 seats. Comparing Oklahoma's population to other states, it was clear that Oklahoma should have 5 seats so the House size was increased by five to 391 seats. The intent was to leave the number of seats unchanged for the other states. However, when the apportionment was mathematically recalculated, Maine gained a seat (4 instead of 3) and New York lost a seat (from 38 to 37). The method used at the time was Hamilton's method explained below in paragraph **??**.

1.4. The Quota Paradox. The percentage of a state's population times the house size is called the *quota* for that state; the *lower quota* is the quota rounded down and the *upper quota* is the quota rounded up. It can happen that a state receives a number of seats which is smaller than its lower quota or larger than its upper quota. For example, in the apportionment based on the 1820 census New York had a population of 1,368,775, the total U.S. population was 8,969,878, and the size of the house was 213. New York's quota was thus $q = \frac{1{,}368{,}775}{8{,}969{,}878} \times 213 = 32.503$. However the apportionment method used at the time (Jefferson's method explained below in paragraph **??**) awarded New York 34 seats.

2 Notation and Terminology

2.1. Assume that there are n states numbered $i = 1, 2, \ldots, n$. We call a vector

$$\mathbf{p} = (p_1, p_2, \ldots, p_n) \in \mathbb{R}^n_+$$

of positive numbers a **population vector**; p_i represents the population of the ith state. We suppose that h denotes the total number of seats to be allocated and call it the **house size**. An **apportionment** of house size h among n states is a vector

$$\mathbf{a} = (a_1, a_2, \ldots, a_n) \in \mathbb{N}^n$$

of nonnegative integers such that

$$h = a_1 + a_2 + \cdots + a_n.$$

The number a_i is the number of seats assigned to the ith state by the apportionment. Let

$$\mathbb{N}^n_h := \{\mathbf{a} \in \mathbb{N}^n : \mathbf{a} = (a_1, a_2, \ldots, a_n),\ \ a_1 + a_2 + \cdots + a_n = h\}$$

denote the set of all apportionments of house size h among n states. Roughly speaking an apportionment method is a function assigns an apportionment to each population vector. However, both to allow for ties and to ease the exposition, we allow multivalued functions. Thus an **apportionment function** for n states and house size h is a function M which assigns to each population vector $\mathbf{p}$ a nonempty set $M(\mathbf{p}) \subset \mathbb{N}_h^n$ of apportionments of house size h among n states. An **apportionment system** is a function which assigns apportionment function for n states and house size h to each positive integer n and each nonnegative integer h.

2.2. The quantity

$$\bar{p}_i = \frac{p_i}{p}$$

is the ith states percentage of the total population. (In order to allow for rescaling, we do not impose the condition that the population p_i of the ith state is an integer.) The vector

$$\bar{\mathbf{p}} = (\bar{p}_1, \bar{p}_2, \dots, \bar{p}_n)$$

is called the **normalized population vector**. For each nonnegative integer h the quantity

$$q_i = \bar{p}_i \times h$$

is called the **quota** of the ith state for house size h: it represents the number of seats that would get if fractional numbers were allowed. The numbers $\lfloor q_i \rfloor$ and $\lceil q_i \rceil$ are called the lower and upper quotes respectively. Thus $\lfloor q_i \rfloor$ and $\lceil q_i \rceil$ are nonnegative integers and

$$\lfloor q_i \rfloor \leq q_i \leq \lceil q_i \rceil.$$

3 Apportionment Methods

3.1. Hamilton's Method. For each i calculate the quota $q_i = p_i/h$ of the ith state. The sum $h' = \lfloor q_1 \rfloor + \lfloor q_2 \rfloor + \cdots + \lfloor q_n \rfloor$ will satisfy $h - n \leq h' < h$. (The equality $h = h'$ occurs only in the unlikely event that all the quotas q_i are whole numbers.) Award the ith state either its lower quota $\lfloor q_i \rfloor$ or its upper quota $\lceil q_i \rceil$. The states which receive their upper quota are the $h - h'$ states with the larger fractional parts $q_i - \lfloor q_i \rfloor$.

3.2. Divisor Methods. These methods all involve a notion of "rounding" as explained below. The idea is that the (exact) quota for each state can be determined by dividing the population of each state by the number of persons represented (ideally) by each representative, i.e.

$$q_i = \frac{p_i}{p} \times h = \frac{p_i}{\lambda_0}, \text{ where } \lambda_0 = \frac{p}{h}.$$

For a divisor λ which is near λ_0 the numbers p_i/λ will be close to quotas $q_i = p_i/\lambda_0$ but rounding p_i/λ might give a different whole number than rounding

p_i/λ_0. The various divisor methods differ in how they define "rounding". The various notions of rounding involve the choice, for each nonnegative integer a of a number $\mu(a)$ between a and $a+1$. Then the result of rounding a number q is $\lfloor q \rfloor$ if $\lfloor q \rfloor \leq q < \mu(\lfloor q \rfloor)$ and $\lceil q \rceil$ if $\mu(\lfloor q \rfloor) \leq q < \lceil q \rceil$. Here are the most important divisor methods.

Jefferson's Method. This method rounds down, i.e. the result of rounding q is $\lfloor q \rfloor$, the largest integer less than or equal to q. For example,

$$\lfloor 2.2 \rfloor = \lfloor 2.9 \rfloor = 2 = \lfloor 2 \rfloor.$$

For this method the function μ is $\mu_{\mathrm{J}}(a) = a + 1$.

Adams' Method. This method rounds up, i.e. the result of rounding q is $\lceil q \rceil$, the smallest integer greater than or equal to q. For example,

$$\lceil 2.2 \rceil = \lceil 2.9 \rceil = 3 = \lceil 3 \rceil.$$

For this method the function μ is $\mu_{\mathrm{A}}(a) = a$.

Webster's Method. This method rounds in the usual way, i.e. the result of rounding q is $\langle q \rangle$ is defined by

$$\langle q \rangle = \begin{cases} \lfloor q \rfloor & \text{if } \lfloor q \rfloor \leq q < A \\ \lceil q \rceil & \text{if } A \leq q < \lceil q \rceil \end{cases}$$

where $A = (\lfloor q \rfloor + \lceil q \rceil)/2$ is the arithmetic mean (average) of $\lfloor q \rfloor$ and $\lceil q \rceil$. For example, the average A of 2 and 3 is 2.5 so,

$$\langle 2.4 \rangle = 2 = \langle 2 \rangle, \qquad \langle 2.5 \rangle = \langle 2.7 \rangle = 3.$$

The notation $\langle q \rangle$ is not standard. For this method the function μ is $\mu_{\mathrm{W}}(a) = a + \frac{1}{2}$.

Huntington-Hill Method. This method rounds by comparing q with the geometric mean G of $\lfloor q \rfloor$ and $\lceil q \rceil$, i.e. the result of rounding q is $\langle\langle q \rangle\rangle$ defined by

$$\langle\langle q \rangle\rangle = \begin{cases} \lfloor q \rfloor & \text{if } \lfloor q \rfloor \leq q < G \\ \lceil q \rceil & \text{if } G \leq q < \lceil q \rceil \end{cases}$$

where $G = \sqrt{\lfloor q \rfloor \times \lceil q \rceil}$. For example, the geometric mean G of 2 and 3 is $\sqrt{2 \times 3} = \sqrt{6} = 2.449$ so

$$\langle\langle 2.43 \rangle\rangle = 2 = \langle\langle 2 \rangle\rangle, \qquad \langle\langle 2.7 \rangle\rangle = 3 = \langle\langle \sqrt{6} \rangle\rangle.$$

The notation $\langle\langle q \rangle\rangle$ is not standard. For this method the function μ is $\mu_{\mathrm{H}}(a) = \sqrt{a(a+1)}$.

Dean's Method.[1] This method rounds by comparing q with the harmonic mean H of $\lfloor q \rfloor$ and $\lceil q \rceil$, i.e. the result of rounding q is $[[q]]$ defined by

$$[[q]] = \begin{cases} \lfloor q \rfloor & \text{if } \lfloor q \rfloor \le q < H \\ \lceil q \rceil & \text{if } H \le q < \lceil q \rceil \end{cases}$$

where $H = \dfrac{2 \times \lfloor q \rfloor \times \lceil q \rceil}{\lfloor q \rfloor + \lceil q \rceil}$. For example, the harmonic mean H of 2 and 3 is $12/5 = 2.4$ so

$$[[2.23]] = 2 = [[2]], \qquad [[2.7]] = 3 = [[2.4]].$$

This method is not mentioned in the text [6] and the notation $[[q]]$ is not standard. For this method the function μ is $\mu_{\mathrm{D}}(a) = 2a(a+1)/(2a+1)$.

The table in Figure 2 shows the outputs given by five different apportionment methods based on the U.S. census data for 1990. Here we have $n = 50$, $h = 435$, and $p = 249{,}022{,}783$. Wisconsin is the 49th state in alphabetical order so $p_{49} = 4{,}906{,}735$. Four of the apportionment methods award Wisconsin 9 representatives, but Jefferson's Method awards Wisconsin only 8.

4 Fairness

Let a_i and p_i be the number of seats for state i and its population, respectively. The fraction a_i/p_i represents the number of representatives per person in state i. Ideally the persons of state i have the same number of representatives per person as those of state j, i.e.

$$\frac{a_i}{p_i} = \frac{a_j}{p_j}$$

but of course this generally won't happen because the numbers a_i must be whole numbers. If

$$\frac{a_i}{p_i} > \frac{a_j}{p_j} \qquad (*)$$

the people in ith state have more representatives per person than those of the jth state and are thus "better represented". A function $T(p_i, a_i, p_j, a_j)$ such that

$$T(p_i, a_i, p_j, a_j) > 0 \iff \frac{a_i}{p_i} > \frac{a_j}{p_j}$$

is called a **fairness measure**. The idea is that the bigger $T(p_i, a_i, p_j, a_j)$ is the more unfair is the gap between the representation of ith state and the jth state. An allocation is called **stable** for a given fairness measure iff for each pair of states, switching a representative from the better represented state i to

[1] James Dean was a mathematician from the University of Vermont and Dartmouth in the nineteenth century.

Figure 1: Apportionment of the U.S. House of Representatives

		Hamilton		Jefferson ($\lambda = 546000$)		Adams ($\lambda = 605400$)		Webster ($\lambda = 574150$)		Huntington-Hill ($\lambda = 575000$)	
State	Pop.	p_i/λ_0	a_i	p_i/λ	a_i	p_i/λ	a_i	p_i/λ	a_i	p_i/λ	a_i
AL	4062608	7.097	7	7.441	7	6.711	7	7.076	7	7.065	7
AK	551947	0.964	1	1.011	1	0.912	1	0.961	1	0.960	1
AZ	3677985	6.425	6	6.736	6	6.075	7	6.406	6	6.396	6
AR	2362239	4.126	4	4.326	4	3.902	4	4.114	4	4.108	4
CA	29839250	52.124	52	54.651	54	49.288	50	1.971	52	51.894	52
CO	3307912	5.778	6	6.058	6	5.464	6	5.761	6	5.753	6
CT	3295669	5.757	6	6.036	6	5.444	6	5.740	6	5.732	6
DE	668696	1.168	1	1.225	1	1.105	2	1.165	1	1.163	1
FL	13003362	22.715	23	23.816	23	21.479	22	22.648	23	22.615	23
GA	6508419	11.369	11	11.920	11	10.751	11	11.336	11	11.319	11
HI	1115274	1.948	2	2.043	2	1.842	2	1.942	2	1.940	2
ID	1011986	1.768	2	1.853	1	1.672	2	1.763	2	1.760	2
IL	11466682	20.030	20	21.001	21	18.941	19	19.972	20	19.942	20
IN	5564228	9.720	10	10.191	10	9.191	10	9.691	10	9.677	10
IA	2787424	4.869	5	5.105	5	4.604	5	4.855	5	4.848	5
KS	2485600	4.342	4	4.552	4	4.106	5	4.329	4	4.323	4
KY	3698969	6.461	6	6.775	6	6.110	7	6.443	6	6.433	6
LA	4238216	7.403	7	7.762	7	7.001	8	7.382	7	7.371	7
ME	1233223	2.154	2	2.259	2	2.037	3	2.148	2	2.145	2
MD	4798622	8.382	8	8.789	8	7.926	8	8.358	8	8.345	8
MA	6029051	10.532	11	11.042	11	9.959	10	10.501	11	10.485	10
MI	9328784	16.296	16	17.086	17	15.409	16	16.248	16	16.224	16
MN	4387029	7.663	8	8.035	8	7.246	8	7.641	8	7.630	8
MS	2586443	4.518	4	4.737	4	4.272	5	4.505	5	4.498	5
MO	5137804	8.975	9	9.410	9	8.487	9	8.949	9	8.935	9
MT	803655	1.404	1	1.472	1	1.327	2	1.400	1	1.398	1
NE	1584617	2.768	3	2.902	2	2.617	3	2.760	3	2.756	3
NV	1206152	2.107	2	2.209	2	1.992	2	2.101	2	2.098	2
NH	1113915	1.946	2	2.040	2	1.840	2	1.940	2	1.937	2
NJ	7748634	13.536	14	14.192	14	12.799	13	13.496	13	13.476	13
NM	1521779	2.658	3	2.787	2	2.514	3	2.650	3	2.647	3
NY	18044505	31.521	31	33.049	33	29.806	30	31.428	31	31.382	31
NC	6657630	11.630	12	12.193	12	10.997	11	11.596	12	11.578	12
ND	641364	1.120	1	1.175	1	1.059	2	1.117	1	1.115	1
OH	10887325	19.018	19	19.940	19	17.984	18	18.963	19	18.934	19
OK	3157604	5.516	5	5.783	5	5.216	6	5.500	5	5.491	6
OR	2853733	4.985	5	5.227	5	4.714	5	4.970	5	4.963	5
PA	11924710	20.830	21	21.840	21	19.697	20	20.769	21	20.739	21
RI	1005984	1.757	2	1.842	1	1.662	2	1.752	2	1.750	2
SC	3505707	6.124	6	6.421	6	5.791	6	6.106	6	6.097	6
SD	699999	1.223	1	1.282	1	1.156	2	1.219	1	1.217	1
TN	4896641	8.554	9	8.968	8	8.088	9	8.529	9	8.516	9
TX	17059805	29.801	30	31.245	31	28.179	29	29.713	30	29.669	30
UT	1727784	3.018	3	3.164	3	2.854	3	3.009	3	3.005	3
VT	564964	0.987	1	1.035	1	0.933	1	0.984	1	0.983	1
VA	6216568	10.859	11	11.386	11	10.269	11	10.827	11	10.811	11
WA	4887941	8.538	9	8.952	8	8.074	9	8.513	9	8.501	9
WV	1801625	3.147	3	3.300	3	2.976	3	3.138	3	3.133	3
WI	4906745	8.571	9	8.987	8	8.105	9	8.546	9	8.533	9
WY	455975	0.797	1	0.835	1	0.753	1	0.794	1	0.793	1
Total	249022783	435	435	435	435	435	435	435	435	435	435

the other state j makes state j better represented and the measure larger, i.e. iff

$$T(p_j, a_j + 1, p_i, a_i - 1) \geq T(p_i, a_i, p_j, a_j)$$

for all pairs of states.

Theorem 4.1 (Huntington). *Assume the apportionment* $(a_1, a_2, \ldots, a_n)$ *results from one of the aforementioned divisor methods. Then*

- *For* $T(p_i, a_i, p_j, a_j) = a_j - a_i(p_j/p_i)$*, Adams' Method is stable.*
- *For* $T(p_i, a_i, p_j, a_j) = p_i/a_i - p_j/a_j$*, Dean's Method is stable.*
- *For* $T(p_i, a_i, p_j, a_j) = (p_j a_i/p_i a_j) - 1$*, Huntington-Hill's Method is stable.*
- *For* $T(p_i, a_i, p_j, a_j) = a_j/p_j - a_i/p_i$*, Webster's Method is stable.*
- *For* $T(p_i, a_i, p_j, a_j) = a_i(p_j/p_i) - a_i$*, Jefferson's Method is stable.*

(In all definitions the ith state is better represented, i.e. $p_i/a_i \geq p_j/a_j$*.)*

Remark 4.2. It is easy to see the appeal of Webster's Method; it is the method which attempts to make the number of representatives per person the same in all states by minimizing the differences $p_i/a_i - p_j/a_j$. Similarly Dean's Method attempts to minimize the differences $a_j/p_j - a_i/p_i$ in the average district size for the various states. The Huntington-Hill Method attempts to minimize the relative differences of these quantities as we now explain.

The **relative difference** of two numbers is the result of subtracting the larger from the smaller and then dividing by the smaller. Now the inequality $(*)$ can be written in four ways:

$$\frac{a_i}{p_i} > \frac{a_j}{p_j} \iff \frac{p_j}{a_j} > \frac{p_i}{a_i} \iff a_i > a_j\frac{p_i}{p_j} \iff a_i\frac{p_j}{p_i} > a_j.$$

We can get four of the five fairness measures in Theorem 4.1 from these four ways by subtracting the right side from the left. The differences of the two sides in each of these ways. The relative difference between the two sides is the same for all four ways, i.e.

$$\frac{\frac{a_i}{p_i} - \frac{a_j}{p_j}}{\frac{a_j}{p_j}} = \frac{\frac{p_j}{a_j} - \frac{p_i}{a_i}}{\frac{p_i}{a_i}} = \frac{a_i - a_j\frac{p_i}{p_j}}{\frac{p_i}{p_j}} = \frac{a_i\frac{p_j}{p_i} - a_j}{a_j} = \frac{p_j a_i}{p_i a_j} - 1.$$

The last expression is the fairness measure for the Huntington-Hill Method. Perhaps this is why Huntington thought his fairness measure was the best, but which measure is the best is a value judgment, not a mathematical question.

Remark 4.3. In [4] Balinski and Young devised an ingenious method which they call the **Quota Method**, which avoids both the Quota Paradox and the Alabama paradox.However, their Quota Method does not avoid the Population Paradox. In their book [5] they show that

- Divisor methods avoid the Alabama Paradox, the Population Paradox, and the New State Paradox, but
- No divisor method can avoid the Quota Paradox. In other words, for any divisor method there exists $(p_1, p_2, \ldots, p_n)$ and h such that the method assigns $(a_1, a_2, \ldots, a_n)$ where either $a_i < \lfloor q_i \rfloor$ for some i or $\lceil q_i \rceil < q_i$ for some i. (Note however, that the Jefferson Method always gives $\lfloor q_i \rfloor \leq a_i$ and the Adams Method always gives $a_i \leq \lceil q_i \rceil$.)

4.4. Figure 2 shows the apportionment of the U.S. House of Representatives using various apportionment methods and the 1990 census data. Each of the apportionment methods has been modified, where necessary, so that every state gets at least one seat as provided for under the Constitution.

A U.S. Constitution

The way the U.S. House of Representatives is to be constituted is specified in Article 1, Section 2 of the U.S. Constitution.

> *The House of Representatives shall be composed of members chosen every second year by the people of the several states... No person shall be a representative who shall not have attained to the age of twenty-five years, and been seven years a citizen of the United States, and who shall not, when elected, be an inhabitant of that state in which he shall be chosen.*
>
> *Representatives and direct taxes shall be apportioned among the several states which may be included within this Union, according to their respective numbers... The actual enumeration shall be made within three years after the first meeting of the Congress of the United States, and within every subsequent term of ten years in such manner as they shall by law direct. The number of representatives shall not exceed one for every thirty thousand, but each state shall have at least one representative; and until such enumeration shall be made, the state of New Hampshire shall be entitled to choose three, Massachusetts eight, Rhode Island and Providence Plantations one, Connecticut five, New-York six, New- Jersey four, Pennsylvania eight, Delaware one, Maryland six, Virginia ten, North-Carolina five, South-Carolina five, and Georgia three.*

B Apportionment Timeline

1787 Constitution drafted by the Constitutional Convention

1790 First Census

1791 After much debate, Congress approved a bill for a 120 member House and Alexander Hamilton's method to apportion seats among the states. Hamilton's method won out over Thomas Jefferson's method. Hamilton's method was supported by the Federalists while Jefferson's method was supported by the Republicans. President Washington vetoes the above bill (first veto in U.S. history!). The House, unable to override the veto, passed a new bill for a 105 member House and Jefferson's method to apportion seats among the states. (This method was used until 1840.)

1822 Rep. William Lowndes (SC) proposed an apportionment method now known as the Lowndes's method. It never passed.

1832 John Quincy Adams (former President and, at this time, a representative from Massachusetts) proposes the Adams's method for apportionment. It fails. Senator Daniel Webster (MA) proposes Webster's method. It fails. Congress passes a bill that retains Jefferson's method but changes the size of the House to 240.

1842 Webster's method is adopted and the size of the House is reduced to 223.

1852 Rep. Samuel Vinton (OH) proposed a bill adopting Hamilton's method with a House size of 233. Congress passes this bill with a change to a House size of 234, a size for which Hamilton's and Webster's methods give the same apportionment.

1872 A very confusing year! First the House size was chosen to be 283 so that Hamilton's and Webster's methods would again agree. After much political infighting, 9 more seats were added and the final apportionment did not agree with either method.

1876 Rutherford B. Hayes became president based on the botched apportionment of 1872. The electoral college vote was 185 for Hayes and 184 for Tilden. Tilden would have won if the correct apportionment as required by law had been used.

1880 The Alabama Paradox surfaced as a major flaw of Hamilton's method.

1882 Concerns continued over the flaws in Hamilton's method. Congress passed a bill that kept Hamilton's method but changed the House size to 325 so that Hamilton's method gave the same apportionment as Webster's.

1901 The Census Bureau gave Congress tables showing apportionments based on Hamilton's method for all House sizes between 350 and 400. For all House sizes in this range (except for 357) Colorado would get 3 seats. For 357, Colorado would get 2 seats. Rep. Albert Hopkins (IL), chairman of the House Committee on Apportionment, submitted a bill using a House size of 357–causing an uproar. Congress defeated Hopkins' bill and instead adopted Webster's method with a House size of 386.

1907 Oklahoma joined the union and the New States Paradox was discovered as a result.

1911 Webster's method was readopted with a House size of 433. A provision was made to give Arizona and New Mexico each 1 seat if they were admitted to the union.

1921 No reapportionment was done after the 1920 census.

1929 The Speaker of the House, Nicholas Longworth, suggested that the National Academy of Sciences make an objective study of the problem. A committee of mathematicians consisting of G.A. Bliss (1876-1951), E.W. Brown (1866-1938), L. P. Eisenhart (1876-1965) and Raymond Pearl (1879-1940) was formed to investigate the situation. Their report indicated support for Huntington's method. (in 1949 an even more prestigious group of mathematicians provided a report to the President of the National Academy of Sciences on apportionment methods. The report was signed by Harold Marston Morse (1892-1977), John Von Neumann (1903-1957), and Luther Eisenhart. Their report again supported the Huntington-Hill method.)

1931 Webster's method was adopted with a House size of 435.

1941 Senator Vandenburg of Michigan suggested that Arkansas prefered the Huntington-Hill Method over the Webster Method in order to gain a single seat. The only difference between these methods for the 1940 census was that the former gave Arkansas 7 and Michigan 17 while the latter gave Arkansas 6 and Michigan 18. The Huntington-Hill method was adopted with a House size of 435.

1990 The U.S. Census Bureau, for only the second time since 1900, allocated Defense Department overseas employees for apportionment purposes. This resulted in Massachusetts losing a seat to Washington. Massachusetts filed suit.

1992 Overruling a U. S. district court decision, the U. S. Supreme Court ruled against Massachusetts on technical grounds involving "the separation of powers and the unique constitutional position of the President." (The President is charged with calculating and transmitting the apportionment to Congress.)

1992 Montana challenged the constitutionality of the Huntington-Hill method (Montana v. U.S. Dept. of Commerce). The Supreme Court upheld the method. Montana was upset because it lost a seat to Washington based on the results of the 1990 census.

C Notes

I downloaded the time line, Figure 2, and the material in pargraphs 1.1-1.3 from [1]. The material in paragraph 1.4 comes from [6].

References

[1] Larry Bowen: MATH 103: Introduction to Contemporary Mathematics, `http://www.ctl.ua.edu/math103`

[2] `http://www.ams.org/featurecolumn/archive/apportion1.html`

[3] `http://www.ams.org/featurecolumn/archive/apportionII1.html`

[4] Michel Balinski and H. Peyton Young: The quota method of apportionment, Amer. Math. Monthly 82 (1975) 701-730.

[5] Michel Balinski and H. Peyton Young: *Fair Representation*, Yale University Press, New Haven, 1982, (2nd. ed.), Brookings Institution, 2001.

[6] Comap: *For all Practical Purposes*, 7th Edition, W.H. Freeman, 2005.