Math 234	Final Exam	Spring 2017

Name: ______________________ TA: ☐ Jing Hao ☐ Vlad A. Matei

Section: ______________________ ☐ Moises Herradon ☐ Tony Yuan

☐ Brandon J. Legried ☐ Yuhua Zhu

- Show your work and write clearly.
- Make sure that your answers stand out.
- *Have a great Summer!*

Question	Points	Score
1	12	
2	16	
3	18	
4	18	
5	12	
6	12	
7	12	
Total:	100	

1. Consider the points $A = (1, 2, 3)$, $B = (1, -2, 2)$, and $C = (2, 1, 4)$.
   (a) [6 points] Find the area of the triangle formed by A, B, and C.

Answer:

   (b) [6 points] Find t such that $D = (3, -2, t)$ is a point on the plane formed by A, B, and C.

Answer:

2. (a) [6 points] Consider the quadratic form $f(x, y) = x^2 + Cxy + 2y^2$, where C is a constant. For which values of C is $f(x, y)$ indefinite?

Answer:

(b) [10 points] Consider the function $f(x, y) = x^3 + 6xy + 3y^2 - 9x$. Find all of the critical points of f. For each critical point, classify it as a local maximum, local minimum, or saddle point of f.

Answer:

3. (a) [8 points] Consider the line segment joining the points $(1,2,3)$ and $(1,-2,2)$. Let $\vec{\boldsymbol{v}} = \begin{pmatrix} yz \\ x^2 \\ y+z \end{pmatrix}$. Find $\int_{\mathcal{C}} \vec{\boldsymbol{v}} \cdot d\vec{\boldsymbol{x}}$.

Answer:

(b) [5 points] Let $\vec{\boldsymbol{F}} = \begin{pmatrix} 2xe^y \\ x^2e^y + 1 \end{pmatrix}$. Find a function $z = f(x, y)$ such that $\vec{\boldsymbol{\nabla}} f = \vec{\boldsymbol{F}}$.

Answer:

(c) [5 points] Take $\vec{\boldsymbol{F}}$ as defined in part (b). Let $\mathcal{C}$ be a curve with initial point $(0, 5)$ and final point $(2, 3)$. Find $\int_{\mathcal{C}} \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{x}}$.

Answer:

4. (a) [6 points] Let $\mathcal{R}$ be the region in the first quadrant of the plane bounded above by $y = 4 - x^2$. Let $\mathcal{C}$ be the boundary of $\mathcal{R}$ with counterclockwise orientation and let $\vec{\boldsymbol{v}} = \begin{pmatrix} xy \\ 2y \end{pmatrix}$. Use Green's theorem to find $\int_{\mathcal{C}} \vec{\boldsymbol{v}} \cdot d\vec{\boldsymbol{x}}$.

Answer:

(b) [6 points] Let $\mathcal{C}$ be as in part (a) and let $\vec{\boldsymbol{v}} = \begin{pmatrix} x^2 \\ xy \end{pmatrix}$. Find the outward flux of $\vec{\boldsymbol{v}}$ across $\mathcal{C}$, i.e., find $\int_{\mathcal{C}} \vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{N}} \, ds$, where $\vec{\boldsymbol{N}}$ is the outward pointing unit normal to $\mathcal{C}$.

Answer:

(c) [6 points] Let $\mathcal{R} = \{(x, y, z) \colon 0 \le x \le 1, 0 \le y \le 2, 0 \le z \le 3\}$. Let $\mathcal{S}$ be the boundary of $\mathcal{R}$ and let $\vec{\boldsymbol{N}}$ be the outward pointing unit normal to $\mathcal{S}$. Finally, let $\vec{\boldsymbol{v}} = \begin{pmatrix} 2x + \sin(z^2) \\ \cos(z^2) \\ yz \end{pmatrix}$. Find $\iint_{\mathcal{S}} \vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{N}} \, dA$, i.e., the outward flux of $\vec{\boldsymbol{v}}$ across $\mathcal{S}$.

Answer:

5. [12 points] Let $\vec{\boldsymbol{F}} = \begin{pmatrix} y \\ x \\ z \end{pmatrix}$ and let $\mathcal{S}$ be the surface in three dimensional space given by $z = x^2$ for $-1 \leq x \leq 1$ and $0 \leq y \leq 2$. Find $\iint_{\mathcal{S}} \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{N}}\, dA$, where $\vec{\boldsymbol{N}}$ is the unit normal to $\mathcal{S}$ that points upward (i.e., with positive z-coordinate).

Answer:

6. [12 points] Find the surface area of the part of the paraboloid $z = x^2 + y^2$ for which $x^2 + y^2 \leq 1$.

Answer:

7. [12 points] Find the maximum and minimum values of $f(x, y) = x^2 + 2y^2 - x$ on the unit circle.

Answer: