

Math 234	Exam I	Spring 2017
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- You may **not** use a calculator, notes, a notecard, or books.
- Show your work and write clearly.
- Make sure that your answers stand out.
- *Good Luck!*

Question	Points	Score
1	24	
2	24	
3	16	
4	20	
5	16	
Total:	100	

1. Consider the points $A = (-2, 1, 1)$, $B = (1, -2, 1)$, and $C = (-3, 3, 3)$.

(a) [8 points] Find the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

Answer:

(b) [8 points] Find the area of the triangle formed by A , B , and C .

Answer:

(c) [8 points] Find an equation for the plane containing A , B , and C .

Answer:

2. Consider the vector function $\vec{x}(t) = \begin{pmatrix} \cos t \\ \sin t \\ t^2/2 \end{pmatrix}$.

- (a) [8 points] Set up *but do not evaluate* an integral for the length of the curve parameterized by $\vec{x}(t)$ from $t = 0$ to $t = \pi$.

- (b) [8 points] Find the unit tangent vector $\vec{T}(t)$ to the curve parameterized by $\vec{x}(t)$.

- (c) [8 points] Find the curvature vector $\vec{\kappa}(t)$ when $t = \pi$.

3. (a) [8 points] For which values of the constant C is $f(x, y) = x^2 + 6xy + Cy^2$ positive definite?

Answer:

- (b) [8 points] Consider the quadratic form $g(x, y) = 2x^2 - 8xy - 10y^2$.
- Classify it as positive definite, negative definite, semidefinite, or indefinite.
 - Draw the zero set and indicate where g is positive and where it is negative.

4. Consider the function $f(x, y) = \frac{x^2 + 1}{x + y}$.

(a) [8 points] Find an equation for the tangent plane to $f(x, y)$ at the point $(2, 3, 1)$.

Answer:

(b) [6 points] Find a vector normal to the graph of $z = f(x, y)$ at $(2, 3, 1)$.

Answer:

(c) [6 points] Find a vector normal to the level set $f^{-1}(1)$ at the point $(2, 3)$.

Answer:

5. Suppose that $f(x, y)$ is a function such that $f_x(x, y) = xy^2 + x$ and $f_y(x, y) = x^2y$.
- (a) [8 points] Consider the parameterization of the unit circle given by $x(t) = \cos t$ and $y(t) = \sin t$. Find $\frac{df}{dt}$ as a function of t .

Answer:

- (b) [8 points] Let $g(u, v) = f(u^2, uv)$. Find $\frac{\partial g}{\partial u}(u, v)$.

Answer: