Exercises for André's FRG tutorials, Madison, May 2009

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Please e-mail solutions to André by June 15.

The student with the best solutions will receive a free copy of Andre's book "Computability and randomness".

The upper bound on the length of solution in typed lines is given.

1. Cost functions

All cost functions will be monotonic. For a cost function $c(x, s)$ we let $\widehat{c}(x) = \sup_s c(x, s)$. We may assume this quantity is finite for each x.

1. (a) *(4 lines)* Characterize the functions of the form $\widehat{c}$ for a cost function c.

    (b) *(14 lines)* Define cost functions c, d that are incomparable in the lower semilattice of cost functions (modulo cost function equivalence). That is, $c \not\to d$ and $d \not\to c$.

2. *(5 lines)* Show that there is a cost function c such that for c.e. A, if $A \models c$ then A is strongly jump traceable. Can c be benign?

2. Randomness and Computability

3. *(5 lines)* A set Z is called 2-random if Z is ML-random relative to $\emptyset'$. Show that a 2-random set can be Low_2.

4. *(12 lines)* Show the following. (i) Some weakly random set is K-trivial. (ii) No Schnorr random set is K-trivial.

3. Effective descriptive set theory

In the following you can use language such as "enumerate x into a S at stage α", where S is a Π_1^1 set under construction and α a computable ordinal.

5. *(15 lines)* Indicate the cost function construction of a $\underline{K}$-trivial Π_1^1 set that is not hyperarithmetical.

6. *(25 lines)* Sketch an argument that there are Π_1^1 sets $P, Q \subseteq \mathbb{N}$ that are incomparable under $\leq_{fin-h}$.

7. *(5 lines)* Give an example of a singleton Π_1^1 class $\mathring{A} \subseteq 2^\omega$ that is no Δ_1^1.