Braiding and Mixing

Periodic Boundary Conditions and Periodic Orbits

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Experiment of Boyland *et al.*

Four Basic Operations

\[ \sigma_1 \]

\[ \sigma_2 \]

\[ \sigma_1^{-1} \]

\[ \sigma_2^{-1} \]

\( \sigma_1 \) and \( \sigma_2 \) are referred to as the *generators* of the 3-braid group.
Two Stirring Protocols

\( \sigma_1 \sigma_2 \) protocol

\( \sigma_1^{-1} \sigma_2 \) protocol

Braiding

\( \sigma_1 \sigma_2 \) protocol  
\( \sigma_1^{-1} \sigma_2 \) protocol

Three-rod Mixer in a Bounded Domain
Three-rod Mixer in a Bounded Domain

[movie: bounded.mpg]
Computing the Line-stretching from a Braid

- How much are lines stretched by a given braid? What is the exponential rate? (could be zero)
- This rate is referred to as the braid’s topological entropy.
- The T.E. is obtained from a transition matrix.
- The really high-powered algorithms are variations on an idea called “train-tracks”.
- We use both train-tracks and more prosaic methods.
An interesting problem: what about singly-periodic boundary conditions?

Conformal map from cylinder to punctured plane:

\[ w = \exp(2\pi i z) \]

The origin in the \( w \)-plane acts as an extra rod!

So it should be possible to make a nontrivial braid with just two rods.
Two-rod Mixer on a Cylinder
Two-rod Mixer on a Cylinder

[movie: singly.mpg]
There is no corresponding conformal map for the torus.

So how do we compute entropies? Many chaotic systems live on doubly-periodic domains...
One-rod Mixer on a Torus: No Entropy
One-rod Mixer on a Torus: No Entropy

[movie: doubly.mpg]
Two-rod Mixer on a Torus: \( \tau_1 \sigma_1 \rho_1^{-1} \sigma_1 \)

[movie: torus_braid.mov]
Transition Matrix for Torus

in each column put what that colour maps to
Growth Rates for Lines
One Rod Mixer: The Kenwood Chef

[Image of a vintage Kenwood mixer]

[Image of an advertisement for the Kenwood Chef]

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Poincaré Section
Stretching of Lines
Motion of Islands

Make a braid from the motion of the rod and the periodic islands.

Most (74%) of the topological entropy is accounted for by this braid.
Motion of Islands and Unstable Periodic Orbits

Now we also include unstable periods orbits as well as the stable ones (islands).

Almost all (99%) of the topological entropy is accounted for by this braid.
Conclusion

All Chaos is Topological!