Stirring by squirmers

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A ‘gas’ of swimmers

[movie 1] 100 cylinders, box size = 1000
Displacement by a moving body

Suggests mechanism for stirring by swimming organisms. (Katija & Dabiri, 2009; Thiffeault & Childress, 2010)
A sequence of kicks

Inspired by Einstein’s theory of diffusion (Einstein, 1905): a test particle initially at $x(0) = 0$ undergoes $N$ encounters with an axially-symmetric swimming body:

$$x(t) = \sum_{k=1}^{N} \Delta_L(a_k, b_k) \hat{r}_k$$

$\Delta_L(a, b)$ is the displacement, $a_k$, $b_k$ are impact parameters, and $\hat{r}_k$ is a direction vector.

($a > 0$, but $b$ can have either sign.)
Effective diffusivity

Putting this together,

\[ \langle |x|^2 \rangle = \frac{2 U n t}{L} \int \Delta_L^2(a, b) \, da \, db = 4 \kappa t, \quad 2D \]

\[ \langle |x|^2 \rangle = \frac{2\pi U n t}{L} \int \Delta_L^2(a, b) a \, da \, db = 6 \kappa t, \quad 3D \]

which defines the effective diffusivity \( \kappa \).

Valid for low number density is low \( (nL^d \ll 1) \).

(Lin, Thiffeault & Childress, JFM, in press)
Considerable literature on transport due to microorganisms: Wu & Libchaber (2000); Hernandez-Ortiz et al. (2006); Saintillian & Shelley (2007); Underhill et al. (2008); Ishikawa (2009); Leptos et al. (2009)

Lighthill (1952), Blake (1971), and more recently Ishikawa et al. (2006) have considered squirmers:

- Sphere in Stokes flow;
- Steady velocity specified at surface, to mimic cilia;
- Steady swimming condition imposed (no net force on fluid).
3D axisymmetric streamfunction for a typical squirmer, in cylindrical coordinates $(\rho, z)$:

$$\psi = -\frac{1}{2} \rho^2 + \frac{1}{2r^3} \rho^2 + \frac{3\beta}{4r^3} \rho^2 z \left( \frac{1}{r^2} - 1 \right)$$

where $r = \sqrt{\rho^2 + z^2}$, $U = 1$, radius of squirmer = 1.

$\beta$ is the amplitude of the stresslet (distinguishes pushers/pullers).

We will use $\beta = 5$ for most of the remainder.
Squirmer displacements $a^2 \Delta_L^2(a, b)$
Squirmers: Transport
Squirmers: Trajectories

The two peaks in the displacement plot come from ‘incomplete’ trajectories:

For long path length, the effective diffusivity is independent of the swimming path length, and yet the dominant contribution arises from the finiteness of the path (uncorrelated turning directions).
• Variance exhibits similar short-time anomalous scaling as in Wu & Libchaber (2000);
• PDF matches experiments of Leptos et al. (2009). In our case, exponential tails are due to sticking at the stagnation points on the squirmer’s body.
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References