EXERCISES 9.2

9.2.1. Consider
\[
\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} + Q(x, t)
\]
\[u(x, 0) = g(x).\]

In all cases obtain formulas similar to (9.2.20) by introducing a Green's function.

(a) Use Green's formula instead of term-by-term spatial differentiation if
\[u(0, t) = 0\quad \text{and} \quad u(L, t) = 0.\]

(b) Modify part (a) if
\[u(0, t) = A(t)\quad \text{and} \quad u(L, t) = B(t).\]

Do not reduce to a problem with homogeneous boundary conditions.

(c) Solve using any method if
\[\frac{\partial u}{\partial x}(0, t) = 0\quad \text{and} \quad \frac{\partial u}{\partial x}(L, t) = 0.\]

* (d) Use Green's formula instead of term-by-term differentiation if
\[\frac{\partial u}{\partial x}(0, t) = A(t)\quad \text{and} \quad \frac{\partial u}{\partial x}(L, t) = B(t).\]

9.2.2. Solve by the method of eigenfunction expansion
\[c \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left( K_0 \frac{\partial u}{\partial x} \right) + Q(x, t)\]
subject to \[u(0, t) = 0, \ u(L, t) = 0, \ \text{and} \ u(x, 0) = g(x),\] if \(c\rho\) and \(K_0\) are functions of \(x\). Assume that the eigenfunctions are known. Obtain a formula similar to (9.2.20) by introducing a Green's function.

* 9.2.3. Solve by the method of eigenfunction expansion
\[\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + Q(x, t)\]
\[u(0, t) = 0 \quad u(x, 0) = f(x)\]
\[u(L, t) = 0 \quad \frac{\partial u}{\partial t}(x, 0) = g(x).\]
Define functions (in the simplest possible way) such that a relationship similar to (9.2.20) exists. It must be somewhat different due to the two initial conditions. (Hint: See Exercise 8.5.1.)

9.2.4. Modify Exercise 9.2.3 (using Green's formula if necessary) if instead

(a) \[\frac{\partial u}{\partial x}(0, t) = 0 \quad \text{and} \quad \frac{\partial u}{\partial x}(L, t) = 0\]

(b) \[u(0, t) = A(t) \quad \text{and} \quad u(L, t) = 0\]

(c) \[\frac{\partial u}{\partial x}(0, t) = 0 \quad \text{and} \quad \frac{\partial u}{\partial x}(L, t) = B(t)\]