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# Appendix

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## TABLES

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**Table I Trigonometric Functions**

| Degrees | Radians | Sin  | Tan  | Cot    | Cos   |         |         |
|---------|---------|------|------|--------|-------|---------|---------|
| 0       | .000    | .000 | .000 |        | 1.000 | 1.571   | 90      |
| 1       | .017    | .017 | .017 | 57.29  | 1.000 | 1.553   | 89      |
| 2       | .035    | .035 | .035 | 28.64  | .999  | 1.536   | 88      |
| 3       | .052    | .052 | .052 | 19.081 | .999  | 1.518   | 87      |
| 4       | .070    | .070 | .070 | 14.301 | .998  | 1.501   | 86      |
| 5       | .087    | .087 | .087 | 11.430 | .996  | 1.484   | 85      |
| 6       | .105    | .105 | .105 | 9.514  | .995  | 1.466   | 84      |
| 7       | .122    | .122 | .123 | 8.144  | .993  | 1.449   | 83      |
| 8       | .140    | .139 | .141 | 7.115  | .990  | 1.431   | 82      |
| 9       | .157    | .156 | .158 | 6.314  | .988  | 1.414   | 81      |
| 10      | .175    | .174 | .176 | 5.671  | .985  | 1.396   | 80      |
| 11      | .192    | .191 | .194 | 5.145  | .982  | 1.379   | 79      |
| 12      | .209    | .208 | .213 | 4.705  | .978  | 1.361   | 78      |
| 13      | .227    | .225 | .231 | 4.331  | .974  | 1.344   | 77      |
| 14      | .244    | .242 | .249 | 4.011  | .970  | 1.326   | 76      |
| 15      | .262    | .259 | .268 | 3.732  | .966  | 1.309   | 75      |
| 16      | .279    | .276 | .287 | 3.487  | .961  | 1.292   | 74      |
| 17      | .297    | .292 | .306 | 3.271  | .956  | 1.274   | 73      |
| 18      | .314    | .309 | .325 | 3.078  | .951  | 1.257   | 72      |
| 19      | .332    | .326 | .344 | 2.904  | .946  | 1.239   | 71      |
| 20      | .349    | .342 | .364 | 2.747  | .940  | 1.222   | 70      |
| 21      | .367    | .358 | .384 | 2.605  | .934  | 1.204   | 69      |
| 22      | .384    | .375 | .404 | 2.475  | .927  | 1.187   | 68      |
| 23      | .401    | .391 | .424 | 2.356  | .921  | 1.169   | 67      |
| 24      | .419    | .407 | .445 | 2.246  | .914  | 1.152   | 66      |
|         |         | Cos  | Cot  | Tan    | Sin   | Radians | Degrees |

(Table I is continued on the next page.)

**Table I    Trigonometric Functions (continued)**

| Degrees | Radians | Sin  | Tan   | Cot   | Cos  |         |         |
|---------|---------|------|-------|-------|------|---------|---------|
| 25      | .436    | .423 | .466  | 2.144 | .906 | 1.134   | 65      |
| 26      | .454    | .438 | .488  | 2.050 | .899 | 1.117   | 64      |
| 27      | .471    | .454 | .510  | 1.963 | .891 | 1.100   | 63      |
| 28      | .489    | .469 | .532  | 1.881 | .883 | 1.082   | 62      |
| 29      | .506    | .485 | .554  | 1.804 | .875 | 1.065   | 61      |
| 30      | .524    | .500 | .577  | 1.732 | .866 | 1.047   | 60      |
| 31      | .541    | .515 | .601  | 1.664 | .857 | 1.030   | 59      |
| 32      | .559    | .530 | .625  | 1.600 | .848 | 1.012   | 58      |
| 33      | .576    | .545 | .649  | 1.540 | .839 | .995    | 57      |
| 34      | .593    | .559 | .675  | 1.483 | .829 | .977    | 56      |
| 35      | .611    | .574 | .700  | 1.428 | .819 | .960    | 55      |
| 36      | .628    | .588 | .727  | 1.376 | .809 | .942    | 54      |
| 37      | .646    | .602 | .754  | 1.327 | .799 | .925    | 53      |
| 38      | .663    | .616 | .781  | 1.280 | .788 | .908    | 52      |
| 39      | .681    | .629 | .810  | 1.235 | .777 | .890    | 51      |
| 40      | .698    | .643 | .839  | 1.192 | .766 | .873    | 50      |
| 41      | .716    | .656 | .869  | 1.150 | .755 | .855    | 49      |
| 42      | .733    | .669 | .900  | 1.111 | .743 | .838    | 48      |
| 43      | .750    | .682 | .933  | 1.072 | .731 | .820    | 47      |
| 44      | .768    | .695 | .966  | 1.036 | .719 | .803    | 46      |
| 45      | .785    | .707 | 1.000 | 1.000 | .707 | .785    | 45      |
|         |         | Cos  | Cot   | Tan   | Sin  | Radians | Degrees |

**Table II    Greek Alphabet**

|                   |         |                     |         |
|-------------------|---------|---------------------|---------|
| A $\alpha$        | Alpha   | N $\nu$             | Nu      |
| B $\beta$         | Beta    | $\Xi \xi$           | Xi, Si  |
| $\Gamma \gamma$   | Gamma   | O $\omicron$        | Omicron |
| $\Delta \delta$   | Delta   | $\Pi \pi$           | Pi      |
| E $\epsilon$      | Epsilon | P $\rho$            | Rho     |
| Z $\zeta$         | Zeta    | $\Sigma \sigma$     | Sigma   |
| H $\eta$          | Eta     | T $\tau$            | Tau     |
| $\Theta \theta$   | Theta   | $\Upsilon \upsilon$ | Upsilon |
| I $\iota$         | Iota    | $\Phi \phi$         | Phi     |
| K $\kappa$        | Kappa   | X $\chi$            | Chi     |
| $\Lambda \lambda$ | Lambda  | $\Psi \psi$         | Psi     |
| M $\mu$           | Mu      | $\Omega \omega$     | Omega   |

Table III    Exponential Functions

| $x$ | $e^x$ | $e^{-x}$ | $x$ | $e^x$ | $e^{-x}$ |
|-----|-------|----------|-----|-------|----------|
| 0.0 | 1.00  | 1.00     | 3.0 | 20.1  | .050     |
| 0.1 | 1.11  | .905     | 3.1 | 22.2  | .045     |
| 0.2 | 1.22  | .819     | 3.2 | 24.5  | .041     |
| 0.3 | 1.35  | .741     | 3.3 | 27.1  | .037     |
| 0.4 | 1.49  | .670     | 3.4 | 30.0  | .033     |
| 0.5 | 1.65  | .607     | 3.5 | 33.1  | .030     |
| 0.6 | 1.82  | .549     | 3.6 | 36.6  | .027     |
| 0.7 | 2.01  | .497     | 3.7 | 40.4  | .025     |
| 0.8 | 2.23  | .449     | 3.8 | 44.7  | .022     |
| 0.9 | 2.46  | .407     | 3.9 | 49.4  | .020     |
| 1.0 | 2.72  | .368     | 4.0 | 54.6  | .018     |
| 1.1 | 3.00  | .333     | 4.1 | 60.3  | .017     |
| 1.2 | 3.32  | .301     | 4.2 | 66.7  | .015     |
| 1.3 | 3.67  | .273     | 4.3 | 73.7  | .014     |
| 1.4 | 4.06  | .247     | 4.4 | 81.5  | .012     |
| 1.5 | 4.48  | .223     | 4.5 | 90.0  | .011     |
| 1.6 | 4.95  | .202     | 4.6 | 99.5  | .010     |
| 1.7 | 5.47  | .183     | 4.7 | 110   | .0091    |
| 1.8 | 6.05  | .165     | 4.8 | 122   | .0082    |
| 1.9 | 6.69  | .150     | 4.9 | 134   | .0074    |
| 2.0 | 7.39  | .135     | 5.0 | 148   | .0067    |
| 2.1 | 8.17  | .122     | 5.1 | 164   | .0061    |
| 2.2 | 9.02  | .111     | 5.2 | 181   | .0055    |
| 2.3 | 9.97  | .100     | 5.3 | 200   | .0050    |
| 2.4 | 11.0  | .091     | 5.4 | 221   | .0045    |
| 2.5 | 12.2  | .082     | 5.5 | 245   | .0041    |
| 2.6 | 13.5  | .074     | 5.6 | 270   | .0037    |
| 2.7 | 14.9  | .067     | 5.7 | 299   | .0033    |
| 2.8 | 16.4  | .061     | 5.8 | 330   | .0030    |
| 2.9 | 18.2  | .055     | 5.9 | 365   | .0027    |
|     |       |          | 6.0 | 403   | .0025    |

Table IV    Natural Logarithms

| $n$ | .0    | .1    | .2    | .3    | .4    | .5    | .6    | .7    | .8    | .9    |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0*  |       | 7.697 | 8.391 | 8.796 | 9.084 | 9.307 | 9.489 | 9.643 | 9.777 | 9.895 |
| 1   | 0.000 | 0.095 | 0.182 | 0.262 | 0.336 | 0.405 | 0.470 | 0.531 | 0.588 | 0.642 |
| 2   | 0.693 | 0.742 | 0.788 | 0.833 | 0.875 | 0.916 | 0.956 | 0.993 | 1.030 | 1.065 |
| 3   | 1.099 | 1.131 | 1.163 | 1.194 | 1.224 | 1.253 | 1.281 | 1.308 | 1.335 | 1.361 |
| 4   | 1.386 | 1.411 | 1.435 | 1.459 | 1.482 | 1.504 | 1.526 | 1.548 | 1.569 | 1.589 |
| 5   | 1.609 | 1.629 | 1.649 | 1.668 | 1.686 | 1.705 | 1.723 | 1.740 | 1.758 | 1.775 |
| 6   | 1.792 | 1.808 | 1.825 | 1.841 | 1.856 | 1.872 | 1.887 | 1.902 | 1.917 | 1.932 |
| 7   | 1.946 | 1.960 | 1.974 | 1.988 | 2.001 | 2.015 | 2.028 | 2.041 | 2.054 | 2.067 |
| 8   | 2.079 | 2.092 | 2.104 | 2.116 | 2.128 | 2.140 | 2.152 | 2.163 | 2.175 | 2.186 |
| 9   | 2.197 | 2.208 | 2.219 | 2.230 | 2.241 | 2.251 | 2.262 | 2.272 | 2.282 | 2.293 |
| 10  | 2.303 | 2.313 | 2.322 | 2.332 | 2.342 | 2.351 | 2.361 | 2.370 | 2.380 | 2.389 |

\* Subtract 10 if  $n < 1$ ; for example,  $\ln 0.3 \approx 8.796 - 10 = -1.204$ .

Table V Powers and Roots

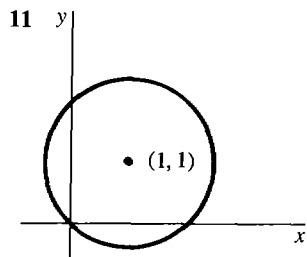
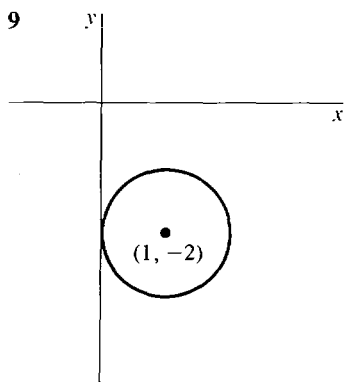
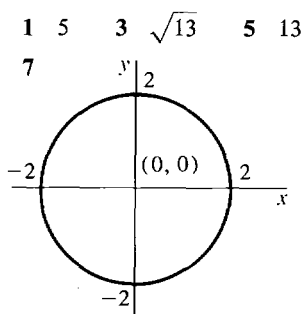
| $n$ | $n^2$ | $\sqrt{n}$ | $n^3$   | $\sqrt[3]{n}$ | $n$ | $n^2$  | $\sqrt{n}$ | $n^3$     | $\sqrt[3]{n}$ |
|-----|-------|------------|---------|---------------|-----|--------|------------|-----------|---------------|
| 1   | 1     | 1.000      | 1       | 1.000         | 51  | 2,601  | 7.141      | 132,651   | 3.708         |
| 2   | 4     | 1.414      | 8       | 1.260         | 52  | 2,704  | 7.211      | 140,608   | 3.733         |
| 3   | 9     | 1.732      | 27      | 1.442         | 53  | 2,809  | 7.280      | 148,877   | 3.756         |
| 4   | 16    | 2.000      | 64      | 1.587         | 54  | 2,916  | 7.348      | 157,464   | 3.780         |
| 5   | 25    | 2.236      | 125     | 1.710         | 55  | 3,025  | 7.416      | 166,375   | 3.803         |
| 6   | 36    | 2.449      | 216     | 1.817         | 56  | 3,136  | 7.483      | 175,616   | 3.826         |
| 7   | 49    | 2.646      | 343     | 1.913         | 57  | 3,249  | 7.550      | 185,193   | 3.849         |
| 8   | 64    | 2.828      | 512     | 2.000         | 58  | 3,364  | 7.616      | 195,112   | 3.871         |
| 9   | 81    | 3.000      | 729     | 2.080         | 59  | 3,481  | 7.681      | 205,379   | 3.893         |
| 10  | 100   | 3.162      | 1,000   | 2.154         | 60  | 3,600  | 7.746      | 216,000   | 3.915         |
| 11  | 121   | 3.317      | 1,331   | 2.224         | 61  | 3,721  | 7.810      | 226,981   | 3.936         |
| 12  | 144   | 3.464      | 1,728   | 2.289         | 62  | 3,844  | 7.874      | 238,328   | 3.958         |
| 13  | 169   | 3.606      | 2,197   | 2.351         | 63  | 3,969  | 7.937      | 250,047   | 3.979         |
| 14  | 196   | 3.742      | 2,744   | 2.410         | 64  | 4,096  | 8.000      | 262,144   | 4.000         |
| 15  | 225   | 3.873      | 3,375   | 2.466         | 65  | 4,225  | 8.062      | 274,625   | 4.021         |
| 16  | 256   | 4.000      | 4,096   | 2.520         | 66  | 4,356  | 8.124      | 287,496   | 4.041         |
| 17  | 289   | 4.123      | 4,913   | 2.571         | 67  | 4,489  | 8.185      | 300,763   | 4.062         |
| 18  | 324   | 4.243      | 5,832   | 2.621         | 68  | 4,624  | 8.246      | 314,432   | 4.082         |
| 19  | 361   | 4.359      | 6,859   | 2.668         | 69  | 4,761  | 8.307      | 328,509   | 4.102         |
| 20  | 400   | 4.472      | 8,000   | 2.714         | 70  | 4,900  | 8.367      | 343,000   | 4.121         |
| 21  | 441   | 4.583      | 9,261   | 2.759         | 71  | 5,041  | 8.426      | 357,911   | 4.141         |
| 22  | 484   | 4.690      | 10,648  | 2.802         | 72  | 5,184  | 8.485      | 373,248   | 4.160         |
| 23  | 529   | 4.796      | 12,167  | 2.844         | 73  | 5,329  | 8.544      | 389,017   | 4.179         |
| 24  | 576   | 4.899      | 13,824  | 2.884         | 74  | 5,476  | 8.602      | 405,224   | 4.198         |
| 25  | 625   | 5.000      | 15,625  | 2.924         | 75  | 5,625  | 8.660      | 421,875   | 4.217         |
| 26  | 676   | 5.099      | 17,576  | 2.962         | 76  | 5,776  | 8.718      | 438,976   | 4.236         |
| 27  | 729   | 5.196      | 19,683  | 3.000         | 77  | 5,929  | 8.775      | 456,533   | 4.254         |
| 28  | 784   | 5.292      | 21,952  | 3.037         | 78  | 6,084  | 8.832      | 474,552   | 4.273         |
| 29  | 841   | 5.385      | 24,389  | 3.072         | 79  | 6,241  | 8.888      | 493,039   | 4.291         |
| 30  | 900   | 5.477      | 27,000  | 3.107         | 80  | 6,400  | 8.944      | 512,000   | 4.309         |
| 31  | 961   | 5.568      | 29,791  | 3.141         | 81  | 6,561  | 9.000      | 531,441   | 4.327         |
| 32  | 1,024 | 5.657      | 32,768  | 3.175         | 82  | 6,724  | 9.055      | 551,368   | 4.344         |
| 33  | 1,089 | 5.745      | 35,937  | 3.208         | 83  | 6,889  | 9.110      | 571,787   | 4.362         |
| 34  | 1,156 | 5.831      | 39,304  | 3.240         | 84  | 7,056  | 9.165      | 592,704   | 4.380         |
| 35  | 1,225 | 5.916      | 42,875  | 3.271         | 85  | 7,225  | 9.220      | 614,125   | 4.397         |
| 36  | 1,296 | 6.000      | 46,656  | 3.302         | 86  | 7,396  | 9.274      | 636,056   | 4.414         |
| 37  | 1,369 | 6.083      | 50,653  | 3.332         | 87  | 7,569  | 9.327      | 658,503   | 4.431         |
| 38  | 1,444 | 6.164      | 54,872  | 3.362         | 88  | 7,744  | 9.381      | 681,472   | 4.448         |
| 39  | 1,521 | 6.245      | 59,319  | 3.391         | 89  | 7,921  | 9.434      | 704,969   | 4.465         |
| 40  | 1,600 | 6.325      | 64,000  | 3.420         | 90  | 8,100  | 9.487      | 729,000   | 4.481         |
| 41  | 1,681 | 6.403      | 68,921  | 3.448         | 91  | 8,281  | 9.539      | 753,571   | 4.498         |
| 42  | 1,764 | 6.481      | 74,088  | 3.476         | 92  | 8,464  | 9.592      | 778,688   | 4.514         |
| 43  | 1,849 | 6.557      | 79,507  | 3.503         | 93  | 8,649  | 9.644      | 804,357   | 4.531         |
| 44  | 1,936 | 6.633      | 85,184  | 3.530         | 94  | 8,836  | 9.695      | 830,584   | 4.547         |
| 45  | 2,025 | 6.708      | 91,125  | 3.557         | 95  | 9,025  | 9.747      | 857,375   | 4.563         |
| 46  | 2,116 | 6.782      | 97,336  | 3.583         | 96  | 9,216  | 9.798      | 884,736   | 4.579         |
| 47  | 2,209 | 6.856      | 103,823 | 3.609         | 97  | 9,409  | 9.849      | 912,673   | 4.595         |
| 48  | 2,304 | 6.928      | 110,592 | 3.634         | 98  | 9,604  | 9.899      | 941,192   | 4.610         |
| 49  | 2,401 | 7.000      | 117,649 | 3.659         | 99  | 9,801  | 9.950      | 970,299   | 4.626         |
| 50  | 2,500 | 7.071      | 125,000 | 3.684         | 100 | 10,000 | 10.000     | 1,000,000 | 4.642         |

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# ANSWERS TO SELECTED PROBLEMS

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## Section 1.1



13  $(x - 3)^2 + y^2 = 4$

15  $(x - 1)^2 + (y - \sqrt{3})^2 = 4, (x - 1)^2 + (y + \sqrt{3})^2 = 4$

17  $(x - 2)^2 + (y - 4)^2 = 25$

## Section 1.2

|   | -1   | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 1   |
|---|------|----------------|---|---------------|-----|
| 1 | -1/3 | -1/6           | 0 | 1/6           | 1/3 |
| 3 | -6   | 3/8            | 2 | 9/8           | 0   |
| 5 | 1    | $1/\sqrt{2}$   | 0 | *             | *   |
| 7 | 2    | 1              | 1 | 1             | 2   |

9 yes

$$11 \quad f(2) = 7, f(t) = 1 + t + t^2, f(t + \Delta t) = 1 + (t + \Delta t) + (t + \Delta t)^2, \\ f(1 + t + t^2) = 1 + (1 + t + t^2) + (1 + t + t^2)^2, f(g(t)) = 1 + g(t) + (g(t))^2$$

$$13 \quad f(t) = t\sqrt{t}, f(t + \Delta t) = (t + \Delta t)^{3/2}, f(t^2) = |t^3|, f(\sqrt{t}) = t^{3/4}, f(g(t)) = (g(t))^{3/2}$$

$$15 \quad 4\Delta x$$

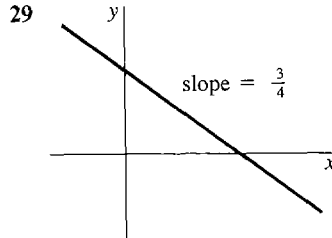
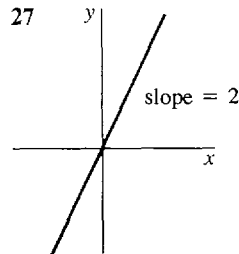
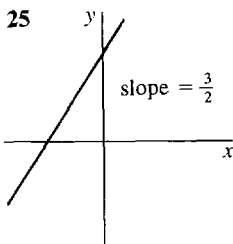
$$17 \quad \frac{1}{x^2 + 2x\Delta x + \Delta x^2} - \frac{1}{x^2} = -\frac{(2x + \Delta x)\Delta x}{x^2(x^2 + 2x\Delta x + \Delta x^2)}$$

$$19 \quad \sqrt{x + \Delta x} - \sqrt{x} = \frac{\Delta x}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$21 \quad \text{everywhere except } x = 1 \text{ and } x = -1 \quad 23 \quad x \geq 0 \quad 25 \quad (-1, 1)$$

## Section 1.3

$$1 \quad m = 1, y = x + 1 \quad 3 \quad \text{vertical, } x = -4 \quad 5 \quad m = -\frac{1}{3}, y = (-\frac{1}{3})x + 1 \\ 7 \quad m = 0, y = 3 \quad 9 \quad y = 2x - 3 \quad 11 \quad y = -(\frac{1}{2})x - \frac{7}{2} \quad 13 \quad y = 5x \quad 15 \quad y = 4 \\ 17 \quad v = 2, y = 2t \quad 19 \quad v = -\frac{1}{2}, y = -(\frac{1}{2})t + \frac{9}{2} \quad 21 \quad v = 0, y = 4 \quad 23 \quad y = 3t + 2$$



$$33 \quad (-c/a, 0), (0, -c/b)$$

## Section 1.4 (no problem set)

## Section 1.5

$$1 \quad \text{infinitesimal} \quad 3 \quad \text{infinite} \quad 5 \quad \text{infinite} \quad 7 \quad \text{infinite} \quad 9 \quad \text{infinitesimal} \\ 11 \quad \text{infinitesimal} \quad 13 \quad \text{infinite} \quad 15 \quad \text{infinite} \quad 17 \quad \text{finite} \quad 19 \quad \text{finite} \quad 21 \quad \text{infinite} \\ 23 \quad \text{infinitesimal} \quad 25 \quad \text{infinite} \quad 27 \quad \text{finite} \quad 29 \quad \text{finite} \quad 31 \quad \text{finite} \quad 33 \quad \text{infinite} \\ 35 \quad \text{infinitesimal} \quad 37 \quad \text{infinite} \quad 39 \quad \text{finite} \\ 41 \quad (a) \varepsilon > \varepsilon^2 \quad (b) 1/\varepsilon^3 < 1/\varepsilon^4 \quad (c) H < H^2 \quad (d) \varepsilon < \sqrt{\varepsilon} \quad (e) H > \sqrt{H} \quad (f) \sqrt{H} > \sqrt[3]{H} \\ 43 \quad (a) a = 0 \text{ and } b \neq 0 \quad (b) a \neq 0 \text{ and } b \neq 0, \text{ or } a = b = 0 \quad (c) a \neq 0 \text{ and } b = 0$$

## Section 1.6

$$1 \quad 2 \quad 3 \quad 2/5 \quad 5 \quad x^{12} \quad 7 \quad 0 \quad 9 \quad 2 \quad 11 \quad 3 \quad 13 \quad 0 \quad 15 \quad 1/2 \quad 17 \quad 1/2 \\ 19 \quad x + y \quad 21 \quad 3x^2 \quad 23 \quad 10 \quad 25 \quad -1/6 \quad 27 \quad 1 \quad 29 \quad 1/2 \quad 31 \quad -1/16 \\ 33 \quad \frac{1}{1 + \sqrt{2}} \text{ or } \sqrt{2} - 1$$

**Extra Problems for Chapter 1.**

- 1  $\sqrt{122}$     3  $-5/3$     5  $y = -2x + 13$     7  $(x-1)^2 + (y-3)^2 = 5$   
 9 finite    11 infinite    13 infinite    15 infinitesimal    17 48  
 19  $4/7$     21  $-15/4$     23  $\frac{1}{\sqrt{x+\Delta x}} - \frac{1}{\sqrt{x}}$     25  $A = 6V^{2/3}$

**Section 2.1**

- 1  $2x$     3  $-4x$     5 4    7  $12t^2$     9  $5/(2\sqrt{u})$     11  $\frac{3}{2}\sqrt{x}$     13  $-2t^{-3}$   
 15  $-3y^{-2} + 4$     17  $a$     19  $a/(2\sqrt{ax+b})$     21  $2(3-2x)^{-2}$     23  $1/4$     25  $4t^3$   
 27 \$600, \$0,  $-\$1000$     29  $-3x^2$  if  $x \leq 0$ ,  $3x^2$  if  $x > 0$ .

**Section 2.2**

- 1  $\Delta y = 2x \Delta x + \Delta x^2$ ,  $dy = 2x \Delta x$ ,  $\Delta y = dy + \Delta x^2$   
 3  $\Delta y = \frac{2\Delta x}{\sqrt{x+\Delta x} + \sqrt{x}}$ ,  $dy = \frac{\Delta x}{\sqrt{x}}$ ,  $\Delta y = dy + \left[ \frac{2}{\sqrt{x+\Delta x} + \sqrt{x}} - \frac{1}{\sqrt{x}} \right] \Delta x$   
 5  $\Delta y = -\frac{\Delta x}{x(x+\Delta x)}$ ,  $dy = -\frac{\Delta x}{x^2}$ ,  $\Delta y = dy + \left[ \frac{1}{x^2} - \frac{1}{x(x+\Delta x)} \right] \Delta x$   
 7  $\Delta y = \left( 1 + \frac{1}{x(x+\Delta x)} \right) \Delta x$ ,  $dy = \left( 1 + \frac{1}{x^2} \right) \Delta x$ ,  $\Delta y = dy + \left[ \frac{1}{x(x+\Delta x)} - \frac{1}{x^2} \right] \Delta x$   
 9  $\Delta y = 4x \Delta x + 2\Delta x^2$ ,  $\Delta z = 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3$ ,  $dy = 4x dx$ ,  $dz = 3x^2 dx$     11  $2 dx$   
 13  $dx/(2\sqrt{x+1})$     15  $a dx$     17  $(-2/x^2) dx$     19  $-\frac{1}{2}x^{-3/2} dx$   
 21  $d(y+z) = \left( \frac{1}{2\sqrt{x}} + 3 \right) dx$ ,  $d\left(\frac{y}{z}\right) = -\frac{1}{6}x^{-3/2}$     23  $y = 4x - 4$     25  $y = 0$   
 27  $y = 3x - 4$     29  $y = -32x - 48$     31  $y = (2x_0)x - x_0^2$

**Section 2.3**

- 1  $6x + 5$     3  $5(x+8)^4$     5  $-3(4-t)^2$     7  $6x(x^2+5)^2$     9  $-12x(6-2x^2)^2$   
 11  $24x^2(1-4x^3)^{-3}$     13  $5(1+x^{-2})$     15  $-8(4x-1)(2x^2-x+3)^{-3}$   
 17  $t^{-2}(1+t^{-1})^{-2}$  or  $(t+1)^{-2}$     19  $3(8t-2)(4t^2-2t+1)^{-2}$     21  $-3x^2 + 5x - 1$   
 23  $6(3t^2+1)(2t-4)^2 + 6t(2t-4)^3$  or  $6(2t-4)^2(5t^2-4t+1)$     25  $-2(x-1)^{-2}$   
 27  $4x(x^2+1)^{-2}$     29  $(s^2-6s+7)(s-3)^{-2}$     31  $-5(3x-4)^{-2}$     33 0  
 35  $6(x^2+1)(2x^2-1) + 12x(x^2+1)(2x+3) + 6x(2x^2-1)(2x+3)$   
 37  $-(4x-5)(2x+1)^{-2}(x-3)^{-2}$     39  $2(2x+1)^{-2}[(2x+1)^{-1}+3]^{-2}$   
 41  $4x(2x+1)^3(x^2+1) + 6(2x+1)^2(x^2+1)^2$  or  $2(2x+1)^2(x^2+1)(7x^2+2x+3)$   
 43  $\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$     45  $\frac{dy}{dx} = 4\frac{du}{dx} + 2v\frac{dv}{dx}$     47  $\frac{dy}{dx} = -\left[ u\frac{dv}{dx} + v\frac{du}{dx} \right] (uv)^{-2}$   
 49  $y = 6x - 2$     51  $b = -2$ ,  $c = 4$

**Section 2.4**

- 1  $(9y^2+2)^{-1}$     3  $-\frac{1}{4}y^{-1}$     5  $-\frac{1}{2}y^{-1}(y^2+2)^2$  or  $-1/(2x^2y)$     7  $\frac{4}{3}x^{1/3}$   
 9  $-\frac{1}{\sqrt{x}(\sqrt{x}-1)^2}$     11  $\frac{2}{3}x^{-2/3} + \frac{8}{3}x^{-1/3} + 6$     13  $-2(\frac{2}{3}x^{2/3}-1)(x^{5/3}-x)^{-3}$   
 15  $(y^{-2/3}+2)^{-1}$     17  $y = \frac{1}{k}(x-c)$ ,  $\frac{dy}{dx} = \frac{1}{k}$     19  $y = \sqrt{\frac{x-1}{2}}$ ,  $\frac{dy}{dx} = \frac{1}{2\sqrt{2}\sqrt{x-1}}$   
 21  $y = (x+3)^{1/4}$ ,  $\frac{dy}{dx} = \frac{1}{4}(x+3)^{-3/4}$

$$\begin{aligned}
 23 \quad y &= \left[ \frac{\sqrt{4x-3}-1}{2} \right]^{1/2}, \frac{dy}{dx} = (4y^3 + 2y)^{-1} \text{ or } \frac{dy}{dx} = \frac{1}{2} \left[ \frac{\sqrt{4x-3}-1}{2} \right]^{-1/2} (4x-3)^{-1/2} \\
 25 \quad y &= \left[ \frac{\sqrt{1+8x}-1}{4} \right]^2 = \frac{1+4x-\sqrt{1+8x}}{8}, \frac{dy}{dx} = \left( \frac{1}{2\sqrt{y}} + 2 \right)^{-1} \\
 &\text{or } \frac{dy}{dx} = \frac{1}{2} - \frac{1}{2\sqrt{1+8x}}
 \end{aligned}$$

## Section 2.5

$$\begin{aligned}
 1 \quad &-2 \sin \theta \cos \theta & 3 \quad &2 \cos x - 3 \sin x & 5 \quad &\sin z (\cos z)^{-2} & 7 \quad &n \sin^{n-1} \theta \cos \theta \\
 9 \quad &t \cos t + \sin t & 11 \quad &e^x + x e^x & 13 \quad &(2 \ln x)/x & 15 \quad &e^x/x + e^x \ln x \\
 17 \quad &-e^v \sqrt{v} + (1 - e^v)/2\sqrt{v} & 19 \quad &x^n/x + n x^{n-1} \ln x \\
 21 \quad &y - 1/2 = (\sqrt{3}/2)(x - \pi/6) & 23 \quad &y = x - x/e
 \end{aligned}$$

## Section 2.6

$$\begin{aligned}
 1 \quad &\frac{1}{2}(x+2)^{-1/2} & 3 \quad &-\frac{1}{2}(5-x)^{-1/2} & 5 \quad &-\frac{3}{2}(2+3x)^{-3/2} & 7 \quad &2(6x+1)^{-2/3} \\
 9 \quad &x(x^2+1)^{-1/2} & 11 \quad &3 \cos(3x) & 13 \quad &-2x^{-3} \cos(x^{-2}) & 15 \quad &4e^{4x} \\
 17 \quad &-\sin x e^{\cos x} & 19 \quad &-e^x \sin(e^x) & 21 \quad &-40(1-4x)^9 \\
 23 \quad &-2x \cos(1-x^2) + 2 \cos(2x-1) \\
 25 \quad &\frac{e^{\sqrt{\sin x}} \cos x}{2\sqrt{\sin x}} & 27 \quad &-\frac{1}{3}x(1+\sqrt{x^2-1})^{-4/3}(x^2-1)^{-1/2} \\
 29 \quad &-12[1+(x^3+1)^{-1}]^3 x^2 (x^3+1)^{-2} \\
 31 \quad &\frac{1}{3}[\sqrt{x^2-1} + \sqrt{x^2+1}]^{-2/3} [x(x^2-1)^{-1/2} + x(x^2+1)^{-1/2}] \\
 33 \quad &6x \cos(2x-1) + 3 \sin(2x-1) \\
 35 \quad &-\frac{\cos(3t)}{\sin(3t)} & 37 \quad &\frac{2 \cos(2t)}{\cos t} \\
 39 \quad &2t(t+1) & 41 \quad &\sqrt{t^2-4}/\sqrt{t^2+4} \\
 43 \quad &(t+1)^{2/3}(t+2)^{-2/3} & 45 \quad &(9/2)t
 \end{aligned}$$

## Section 2.7

$$\begin{aligned}
 1 \quad &2x^{-3} & 3 \quad &-10(x+1)^{-3} & 5 \quad &-(1/4)x^{-3/2} + (3/4)x^{-5/2} & 7 \quad &(3/4)t^{-1/2} \\
 9 \quad &-\sin x & 11 \quad &-AB^2 \sin(Bx) & 13 \quad &a^2 e^{ax} & 15 \quad &-x^{-2} \\
 17 \quad &-2(t^2+1)^{-2} + 8t^2(t^2+1)^{-3} & 19 \quad &-14(x+2)^{-3} \\
 21 \quad &-\frac{1}{4}x(x+1)^{-3/2} + (x+1)^{-1/2} \text{ or } (\frac{3}{4}x+1)(x+1)^{-3/2} \\
 23 \quad &\frac{3}{4}t^{1/2}(t+3)^{-5/2} - \frac{1}{2}t^{-1/2}(t+3)^{-3/2} - \frac{1}{4}t^{-3/2}(t+3)^{-1/2} \text{ or } -(3t^{-1/2} + \frac{9}{4}t^{-3/2})(t+3)^{-5/2} \\
 25 \quad &-6t^{-4} & 27 \quad &3 \frac{d^2u}{dx^2} & 29 \quad &2u \frac{d^2u}{dx^2} + 2 \left( \frac{du}{dx} \right)^2
 \end{aligned}$$

## Section 2.8

$$\begin{aligned}
 1 \quad &-(y/x) & 3 \quad &-(x/y)^2 & 5 \quad &\frac{-y}{2y+x} & 7 \quad &-(y/x)^3 & 9 \quad &-\frac{2x+3y}{3x+2y} & 11 \quad &\frac{5x^4}{2y-1} \\
 13 \quad &\frac{y}{2y-x} & 15 \quad &\frac{6xy-y^2-1}{2xy-3x^2} & 17 \quad &\frac{y \cos(xy)}{1-x \cos(xy)} & 19 \quad &-\frac{1}{2 \sin y \cos y} \\
 21 \quad &\frac{y}{1-2y} & 23 \quad &ye^x & 25 \quad &2(6y^2+4xy-3)^{-1}
 \end{aligned}$$



- 27 at (1, 2),  $y' = -4/5$ ; at (-1, 3),  $y' = -1/5$   
 29 at (2, 1),  $y' = 2$ ; at (2, -1),  $y' = -2$ ; at  $(\sqrt{3}, 0)$ , vertical  
 31  $-2/3$     33  $e$     35  $\frac{2x}{y}, \frac{2y^2 - 4x^2}{y^3}$

### Extra Problems for Chapter 2

- 1  $12x^2 - 2$     3 8    5  $\Delta y = \frac{1}{(x + \Delta x)^3} - \frac{1}{x^3} = -\frac{3x^2 + 3x\Delta x + \Delta x^2}{x^3(x + \Delta x)^3}\Delta x$ ,  
 $dy = -3x^{-4}\Delta x$     7  $(2x - 2x^{-3})dx$     9  $y = -x$     11  $-9x^2 - 5$   
 13  $9t^2 + 8t - 15$     15  $(-2v^4 + 10v^3 - 3v^2 - 16v + 20)(v^3 - 4)^{-2}$   
 17  $\frac{1}{2}x^{-1/2} + 6x^{1/2}$     19  $\frac{1}{3}x^{-2/3} + \frac{1}{4}x^{-5/4}$     21  $(2y + \frac{1}{2}y^{-1/2})^{-1}$  or  $\frac{2\sqrt{y}}{4y\sqrt{y} + 1}$   
 23  $\frac{-3}{2\sqrt{1-3x}}$     25  $-\frac{5}{2}(5x + 4)^{-3/2}$     27  $-(3 + \frac{1}{2}t^{-1/2})(t^{-2} + 2t)^{-1}$   
 29  $-4(4x - 1)^{-3/2}$     31  $v = \frac{3t + 6}{2\sqrt{t + 3}}, a = \frac{3t + 12}{4(t + 3)^{3/2}}$     33  $-\frac{3y^3 + 6x^2y}{9xy^2 + 2x^3}$   
 35  $f'(x) = \begin{cases} 2x & \text{if } |x| > 1 \\ \text{undefined} & \text{if } |x| = 1 \\ -2x & \text{if } |x| < 1 \end{cases}$

### Section 3.1

- 1  $p = 4\sqrt{A}$     3  $V = S^{3/2}/(6\sqrt{\pi})$     5  $z = \sqrt{x^4 - x^2 + 1}$     7  $V = (s - 2x)^2x$   
 9  $s = 3y/(10 - y)$ ,  $0 < y < 10$     11  $b/16$     13  $A = (1 + t)(w - 2t)$   
 15  $P(x) = x(100 - \sqrt{x}) - 10x$  or  $P(x) = x(90 - \sqrt{x})$

### Section 3.2

- 1  $100 \text{ cm}^2/\text{sec}$     3  $-2 \text{ in.}^2/\text{sec}$     5  $10\sqrt{61} \text{ mph}$     7  $10/3 \text{ in.}^2/\text{sec}$   
 9  $40(3g)^{1/2} \text{ ft/sec}$  where  $g = 32$ , or  $160\sqrt{6} \text{ ft/sec}$     11  $-120 \text{ mph}$   
 13  $\frac{800 \cdot 400}{\sqrt{(800)^2 + 6^2}} = \frac{160,000}{\sqrt{160,009}} \text{ mph}$     15  $-2 \text{ ft/sec}$   
 17  $1,000,000t - 300,000\sqrt{t} \text{ people/year}$     19  $\$60 \text{ per person per year}$   
 21  $(55/144)\pi \text{ in.}^2/\text{min}$     23  $-0.01$     25  $2 - (x/500)$     27  $-100 \text{ radians/hr}$   
 29 22

### Section 3.3

- 1 53    3 does not exist    5 does not exist    7  $-1/(4\sqrt{2})$     9  $1/3$   
 11 does not exist    13 does not exist    15 1    17  $(1 + \sqrt{2})^{1/2}$     19 1  
 21  $-1/2$     23  $2x$     25  $1/(2\sqrt{t})$     27  $3x^2$     29  $-2$     31 does not exist

### Section 3.4

- 1  $R$     3  $> -2$     5  $R$     7 all  $x$  except  $x = 0, x = -1$     9  $-2 < x < 2$   
 11 all  $x$  except  $x = (-1 \pm \sqrt{5})/2$  and  $x = -1$   
 13 all  $x$  except  $x = 2, x = 3$     15  $x < i$

## Section 3.5

- 1 0, min    3 0, min    5 0, neither    7  $-1/3$ , min    9 0, min    11 0, max  
 13 0, neither    15  $-1/(2^{1/3})$ , min    17  $3/2$ , min    19 0, max    21  $\pi$ , neither  
 23 0, max    25 0, min    27  $-1$ , min    29 1, min    31 3, min    33 0, max  
 35  $1/2$ , max    37  $1/\sqrt{17}$  at  $(4/17, 1/17)$     39  $1 - m$  at  $x = 1$   
 41 max of  $3/4$  at  $x = 2$ , min of  $-1/4$  at  $x = -2$   
 43 max of  $3/(4 \cdot 3^{1/4})$  at  $x = 1/(3^{1/4})$ , min of  $-3/(4 \cdot 3^{1/4})$  at  $x = -1/(3^{1/4})$

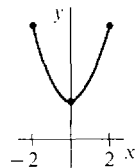
## Section 3.6

- 1  $x = 20/3, y = 40/3$     3  $x = 50, y = 1$     5  $x = 3/4, y = 1/4, x^3y = 27/256$   
 7 width =  $10\sqrt{3}$  in., length =  $40\sqrt{3}$  in.    9 base = height =  $1/\sqrt{2}$   
 11 side of square =  $10/3$  in., height of triangle =  $10/3$  in.    13 area =  $r^2$   
 15 base of radius  $2/3$ , height 1    17 base of radius  $4/(3\pi)$ , height  $4/3$     19 50  
 21  $x = 50,000, p = \$5$     23 256 sec    25  $1/2$     27 base 4, height 2  
 29  $1/\sqrt{17}$  at  $x = 4/17, y = 1/17$     31 height  $(3/2) \cdot \text{side of base}$   
 33  $r = (2\pi)^{-1/3}, h = (4/\pi)^{1/3}$     35  $x = (a + b)/2$   
 37  $A = 2/r, dA/dr$  is never zero,  $0 < r < \infty$     39  $10\sqrt[3]{2}$  hours    41 62,500

## Section 3.7

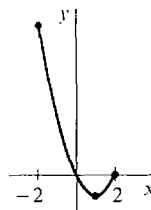
1

| $x$ | $f(x)$ | $f'(x)$ | $f''(x)$ |       |        |
|-----|--------|---------|----------|-------|--------|
| -2  | 6      | -4      | +        | decr. | $\cup$ |
| 0   | 2      | 0       | +        | min.  | $\cup$ |
| 2   | 6      | 4       | +        | incr. | $\cup$ |



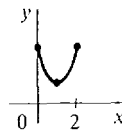
3

| $x$ | $f(x)$ | $f'(x)$ | $f''(x)$ |       |        |
|-----|--------|---------|----------|-------|--------|
| -2  | 8      | -6      | +        | decr. | $\cup$ |
| 1   | -1     | 0       | +        | min.  | $\cup$ |
| 2   | 0      | 2       | +        | incr. | $\cup$ |



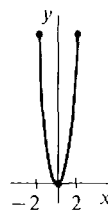
5

| $x$ | $f(x)$ | $f'(x)$ | $f''(x)$ |       |        |
|-----|--------|---------|----------|-------|--------|
| 0   | 3      | -4      | +        | decr. | $\cup$ |
| 1   | 1      | 0       | +        | min.  | $\cup$ |
| 2   | 3      | 4       | +        | incr. | $\cup$ |



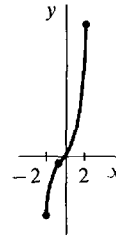
7

| $x$ | $f(x)$ | $f'(x)$ | $f''(x)$ |       |        |
|-----|--------|---------|----------|-------|--------|
| -2  | 16     | -32     | +        | decr. | $\cup$ |
| 0   | 0      | 0       | +        | min.  | $\cup$ |
| 2   | 16     | 32      | +        | incr. | $\cup$ |



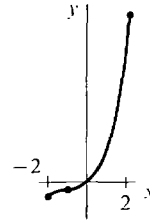
9

| $x$            | $f(x)$          | $f'(x)$       | $f''(x)$ |       |        |
|----------------|-----------------|---------------|----------|-------|--------|
| -2             | -6              | 9             | -        | incr. | $\cap$ |
| $-\frac{1}{3}$ | $-\frac{7}{27}$ | $\frac{2}{3}$ | 0        | incr. | infl.  |
| 2              | 14              | 17            | +        | incr. | $\cup$ |



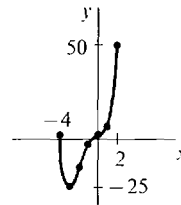
11

| $x$ | $f(x)$         | $f'(x)$ | $f''(x)$ |        |        |
|-----|----------------|---------|----------|--------|--------|
| -2  | $-\frac{2}{3}$ | 1       | -        | incr.  | $\cap$ |
| -1  | $-\frac{1}{3}$ | 0       | 0        | horiz. | infl.  |
| 2   | $8\frac{2}{3}$ | 9       | +        | incr.  | $\cup$ |



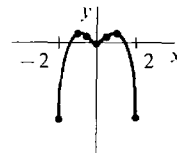
13

| $x$ | $f(x)$ | $f'(x)$ | $f''(x)$ |        |        |
|-----|--------|---------|----------|--------|--------|
| -4  | 2      | -64     | +        | decr.  | $\cap$ |
| -3  | -25    | 0       | +        | min.   | $\cup$ |
| -2  | -14    | 16      | 0        | incr.  | infl.  |
| -1  | -1     | 8       | -        | incr.  | $\cap$ |
| 0   | 2      | 0       | 0        | horiz. | infl.  |
| 1   | 7      | 16      | +        | incr.  | $\cup$ |
| 2   | 50     | 96      | +        | incr.  | $\cup$ |



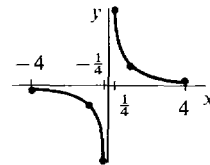
15

| $x$           | $f(x)$         | $f'(x)$                | $f''(x)$ |       |        |
|---------------|----------------|------------------------|----------|-------|--------|
| -2            | -4             | 12                     | -        | incr. | $\cap$ |
| -1            | $\frac{1}{2}$  | 0                      | -        | max.  | $\cap$ |
| $-1/\sqrt{3}$ | $\frac{5}{18}$ | $-\frac{4}{3}\sqrt{3}$ | 0        | decr. | infl.  |
| 0             | 0              | 0                      | +        | min.  | $\cup$ |
| $1/\sqrt{3}$  | $\frac{5}{18}$ | $\frac{4}{3}\sqrt{3}$  | 0        | incr. | infl.  |
| 1             | $\frac{1}{2}$  | 0                      | -        | max.  | $\cap$ |
| 2             | -4             | -12                    | -        | decr. | $\cap$ |



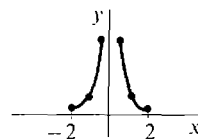
17

| $x$            | $f(x)$         | $f'(x)$         | $f''(x)$ |       |        |
|----------------|----------------|-----------------|----------|-------|--------|
| -4             | $-\frac{1}{4}$ | $-\frac{1}{16}$ | -        | decr. | $\cap$ |
| -1             | -1             | -1              | -        | decr. | $\cap$ |
| $-\frac{1}{4}$ | -4             | -16             | -        | decr. | $\cap$ |
| $\frac{1}{4}$  | 4              | -16             | +        | decr. | $\cup$ |
| 1              | 1              | -1              | +        | decr. | $\cup$ |
| 4              | $\frac{1}{4}$  | $-\frac{1}{16}$ | +        | decr. | $\cup$ |



19

| $x$            | $f(x)$        | $f'(x)$        | $f''(x)$ |       |        |
|----------------|---------------|----------------|----------|-------|--------|
| -2             | $\frac{1}{4}$ | $\frac{1}{4}$  | +        | incr. | $\cup$ |
| -1             | 1             | 2              | +        | incr. | $\cup$ |
| $-\frac{1}{2}$ | 4             | 16             | +        | incr. | $\cup$ |
| $\frac{1}{2}$  | 4             | -16            | +        | decr. | $\cup$ |
| 1              | 1             | -2             | +        | decr. | $\cup$ |
| 2              | $\frac{1}{4}$ | $-\frac{1}{4}$ | +        | decr. | $\cup$ |



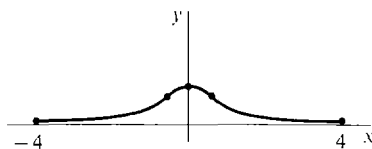
21

| $x$ | $f(x)$         | $f'(x)$         | $f''(x)$ |       |        |
|-----|----------------|-----------------|----------|-------|--------|
| 0   | -1             | 2               | -        | incr. | $\cap$ |
| 1   | 0              | $\frac{1}{2}$   | -        | incr. | $\cap$ |
| 10  | $\frac{9}{11}$ | $\frac{2}{121}$ | -        | incr. | $\cap$ |



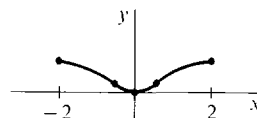
23

| $x$           | $f(x)$         | $f'(x)$ | $f''(x)$ |       |        |
|---------------|----------------|---------|----------|-------|--------|
| -4            | $\frac{1}{17}$ | .03     | +        | incr. | $\cup$ |
| $-1/\sqrt{3}$ | $\frac{3}{4}$  | 1.9     | 0        | incr. | infl.  |
| 0             | 1              | 0       | -        | max.  | $\cap$ |
| $1/\sqrt{3}$  | $\frac{3}{4}$  | -1.9    | 0        | decr. | infl.  |
| 4             | $\frac{1}{17}$ | -.03    | +        | decr. | $\cup$ |



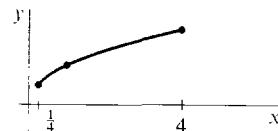
25

| $x$           | $f(x)$        | $f'(x)$         | $f''(x)$ |       |        |
|---------------|---------------|-----------------|----------|-------|--------|
| -2            | $\frac{4}{5}$ | $-\frac{2}{25}$ | -        | decr. | $\cap$ |
| $-1/\sqrt{3}$ | $\frac{1}{4}$ | $-\sqrt{3}/2$   | 0        | decr. | infl.  |
| 0             | 0             | 0               | +        | min.  | $\cup$ |
| $1/\sqrt{3}$  | $\frac{1}{4}$ | $\sqrt{3}/2$    | 0        | incr. | infl.  |
| 2             | $\frac{4}{5}$ | $\frac{2}{25}$  | -        | incr. | $\cap$ |



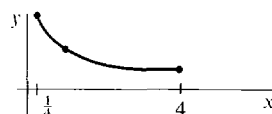
27

| $x$           | $f(x)$        | $f'(x)$       | $f''(x)$ |       |        |
|---------------|---------------|---------------|----------|-------|--------|
| $\frac{1}{4}$ | $\frac{1}{2}$ | 1             | -        | incr. | $\cap$ |
| 1             | 1             | $\frac{1}{2}$ | -        | incr. | $\cap$ |
| 4             | 2             | $\frac{1}{4}$ | -        | incr. | $\cap$ |



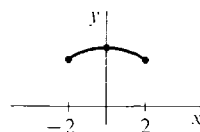
29

| $x$           | $f(x)$        | $f'(x)$         | $f''(x)$ |       |        |
|---------------|---------------|-----------------|----------|-------|--------|
| $\frac{1}{4}$ | 2             | -4              | +        | decr. | $\cup$ |
| 1             | 1             | $-\frac{1}{2}$  | +        | decr. | $\cup$ |
| 4             | $\frac{1}{2}$ | $-\frac{1}{16}$ | +        | decr. | $\cup$ |



31

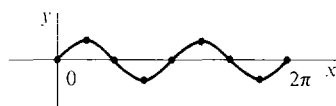
| $x$ | $f(x)$     | $f'(x)$       | $f''(x)$ |       |        |
|-----|------------|---------------|----------|-------|--------|
| -2  | $\sqrt{5}$ | $2/\sqrt{5}$  | -        | incr. | $\cap$ |
| 0   | 3          | 0             | -        | max.  | $\cap$ |
| 2   | $\sqrt{5}$ | $-2/\sqrt{5}$ | -        | decr. | $\cap$ |



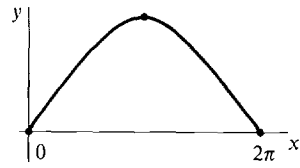
33

| $x$      | $f(x)$         | $f'(x)$ | $f''(x)$ |       |        |
|----------|----------------|---------|----------|-------|--------|
| 0        | 0              | 1       | 0        | incr. | infl.  |
| $\pi/4$  | $\frac{1}{2}$  | 0       | -        | max.  | $\cap$ |
| $\pi/2$  | 0              | -1      | 0        | decr. | infl.  |
| $3\pi/4$ | $-\frac{1}{2}$ | 0       | +        | min.  | $\cup$ |
| $\pi$    | 0              | 1       | 0        | incr. | infl.  |

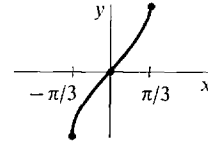
repeats



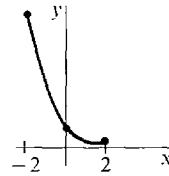
| 35 | $x$    | $f(x)$ | $f'(x)$        | $f''(x)$ |       |        |
|----|--------|--------|----------------|----------|-------|--------|
|    | 0      | 0      | $\frac{3}{2}$  | 0        | incr. | infl.  |
|    | $\pi$  | 3      | 0              | -        | max.  | $\cap$ |
|    | $2\pi$ | 0      | $-\frac{3}{2}$ | 0        | decr. | infl.  |



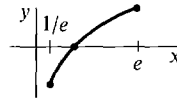
| 37 | $x$      | $f(x)$      | $f'(x)$ | $f''(x)$ |       |        |
|----|----------|-------------|---------|----------|-------|--------|
|    | $-\pi/3$ | $-\sqrt{3}$ | 4       | -        | incr. | $\cap$ |
|    | 0        | 0           | 1       | 0        | incr. | infl.  |
|    | $\pi/3$  | $\sqrt{3}$  | 4       | +        | incr. | $\cup$ |



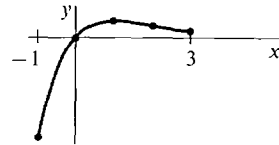
| 39 | $x$ | $f(x)$   | $f'(x)$   | $f''(x)$ |       |        |
|----|-----|----------|-----------|----------|-------|--------|
|    | -2  | $e^2$    | $-e^2$    | +        | decr. | $\cup$ |
|    | 0   | 1        | -1        | +        | decr. | $\cup$ |
|    | 2   | $e^{-2}$ | $-e^{-2}$ | +        | decr. | $\cup$ |



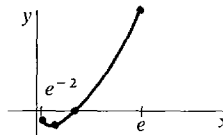
| 41 | $x$   | $f(x)$ | $f'(x)$ | $f''(x)$ |       |        |
|----|-------|--------|---------|----------|-------|--------|
|    | $1/e$ | -1     | $e$     | -        | incr. | $\cap$ |
|    | 1     | 0      | 1       | -        | incr. | $\cap$ |
|    | $e$   | 1      | $1/e$   | -        | incr. | $\cap$ |



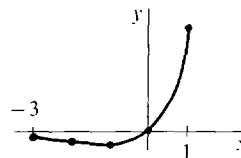
| 43 | $x$ | $f(x)$    | $f'(x)$    | $f''(x)$ |       |        |
|----|-----|-----------|------------|----------|-------|--------|
|    | -1  | $-e$      | $2e$       | -        | incr. | $\cap$ |
|    | 0   | 0         | 1          | -        | incr. | $\cap$ |
|    | 1   | $e^{-1}$  | 0          | -        | max.  | $\cap$ |
|    | 2   | $2e^{-2}$ | $-e^{-2}$  | 0        | decr. | infl.  |
|    | 3   | $3e^{-3}$ | $-2e^{-3}$ | +        | decr. | $\cup$ |



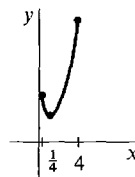
| 45 | $x$      | $f(x)$     | $f'(x)$ | $f''(x)$ |       |        |
|----|----------|------------|---------|----------|-------|--------|
|    | $e^{-2}$ | $-2e^{-2}$ | -1      | $e^2$    | decr. | $\cup$ |
|    | $e^{-1}$ | $-e^{-1}$  | 0       | $e$      | min.  | $\cup$ |
|    | 1        | 0          | 1       | 1        | incr. | $\cup$ |
|    | $e$      | $e$        | 2       | $e^{-1}$ | incr. | $\cup$ |



| 47 | $x$ | $f(x)$     | $f'(x)$    | $f''(x)$ |       |        |
|----|-----|------------|------------|----------|-------|--------|
|    | -3  | $-3e^{-3}$ | $-2e^{-3}$ | -        | decr. | $\cap$ |
|    | -2  | $-2e^{-2}$ | $-e^{-2}$  | 0        | decr. | infl.  |
|    | -1  | $-e^{-1}$  | 0          | +        | min.  | $\cup$ |
|    | 0   | 0          | 1          | +        | incr. | $\cup$ |
|    | 1   | $e$        | $2e$       | +        | incr. | $\cup$ |



| 49 | $x$           | $f(x)$             | $f'(x)$              | $f''(x)$ |       |        |
|----|---------------|--------------------|----------------------|----------|-------|--------|
|    | $\frac{1}{4}$ | $4e^{\frac{1}{4}}$ | $-12e^{\frac{1}{4}}$ | +        | decr. | $\cup$ |
|    | 1             | $e$                | 0                    | +        | min.  | $\cup$ |
|    | 4             | $e^4/4$            | $\frac{3}{16}e^4$    | +        | incr. | $\cup$ |



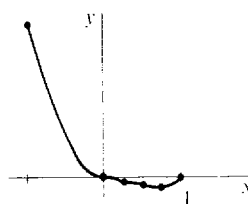
## Section 3.8

- 1  $f(0) = 1, f(1) = -1$     3  $f(4) = 0.236, f(9) = -2.838$   
 5  $f(0) = 0.586, f(1) = -0.732$     7  $f(0) = -1, f(1) = 1$     9  $f(0) = 1, f(1) = -1$   
 11  $f(0) = -1, f(1) = 1$     13  $f(0) = 0.9, f(\pi) = -1.1$     15  $f(1) = -1, f(e) = 0.632$   
 17 yes    19 yes    21 no    23 yes,  $f'(16/7) = 0$     25 no    27 yes    29 no  
 31 one    33 two    35  $1/2$     37  $8/27$     39 1    41  $\sqrt{3} - 1$   
 43  $f'(x) = 3x^2 - 3$  has no zeros in  $(-1, 1)$

## Extra Problems for Chapter 3

- 1  $A = 6V^{2/3}$     3  $x = 300t$     5  $s = 5x/4$     7  $16/\pi$  in./sec    9 0.01 in./sec  
 11 -3    13  $-1/4$     15  $-1 < x < 1$     17  $-2 < x < -1, 1 < x < 2$     19 none  
 21 max = 5 at  $x = 1$ , min = 3 at  $x = 2$     23 max = 4 at  $x = 0$ , min = 1 at  $x = \pm 1$   
 25 square of side  $\sqrt{2}$     27  $8b^3/27a^3$     29 min = -32 at  $x = -2$ , no max  
 31 base = height = 1    33 three zeros

| 33 | $x$           | $f(x)$            | $f'(x)$        | $f''(x)$ |        |
|----|---------------|-------------------|----------------|----------|--------|
|    | -1            | 2                 | -7             | +        | decr.  |
|    | 0             | 0                 | 0              | +        | horiz. |
|    | $\frac{1}{4}$ | $-\frac{3}{256}$  | $-\frac{1}{8}$ | -        | decr.  |
|    | $\frac{1}{2}$ | $-\frac{1}{16}$   | $-\frac{1}{4}$ | 0        | decr.  |
|    | $\frac{3}{4}$ | $-\frac{27}{256}$ | 0              | +        | min.   |
|    | 1             | 0                 | 1              | +        | incr.  |



- 37 none    39  $f(-1) = \sqrt[3]{7} - 2 < 0, f(0) = 1 > 0$

## Section 4.1

- 1 2    3  $5/4$     5  $11/8$     7 11    9  $-1/4$     11 136    13  $901/280 = 3.2$   
 15  $177/512 = 0.35$     17  $1 + \sqrt{2} + \sqrt{3}(\pi - 3) = 2.7$     19 1.90    21 1.55  
 23 3.18

## Section 4.2

- 1  $(16/3)x^{3/2}$     3  $t^3 + t$     5  $4t - t^3$     7  $-(7/2)s^{-2}$     9  $(1/3)(x - 6)^3$   
 11  $(2/5)y^{5/2}$     13  $x^2/2, x \geq 0; -x^2/2, x < 0$     15  $2/3$     17 12    19 0  
 21  $32/3$     23 14    25 12    27 9.9    29  $16/3$     31  $1/6$     33 2

## Section 4.3

- 1  $x + x^2 + x^3 + C$     3  $(3/2)t^8 - (1/2)t^6 + (2/3)t^3 + t + C$     5  $(2/3)t^{3/2} + 2t^{1/2} + C$   
 7  $(1/6)(2x - 3)^3 + C$     9  $(1/3)z^3 + 2z - z^{-1} + C$     11  $5 \sin x + C$   
 13  $x + \ln x + C$     15  $x + 2 \ln x - x^{-1} + C$     17  $12t - (4/3)t^{3/2} - t^2 + C$   
 19  $-4y^{-1} - 6y^{-1/2} + 2\sqrt{y} + C$     21  $(1/3)ax^3 + (1/2)bx^2 + cx + C$     23 0  
 25  $58/15$     27 0    29  $3 \ln 2$     31  $-\ln 3$     33  $(4/3)t^3 - t + 2$   
 35  $-2 \cos t + 12$     37  $v = (1/2)t^2, y = (1/6)t^3 + 1$   
 39  $v = t^3 + 1, y = (1/4)t^4 + t + 2$   
 41  $v = -(1/2)t^{-2} + 3/2, y = (1/2)t^{-1} + (3/2)t - 2$     43 (a), (d), (f), (g), (i), (l)  
 45  $y = t^3 + 2t + 1$

## Section 4.4

- 1  $-\frac{1}{2(2x+1)} + C$     3  $-\frac{1}{28}(3-4z)^7 + C$     5  $-(2/3)(1-t^2)^{3/2} + C$

- 7  $(1/30)(4 + 5x^2)^3 + C$     9  $-(1/3)\cos(3x) + C$     11  $-(3/2)\cos(4x - 1) + C$   
 13  $(1/2)\sin^2 \theta + C$     15  $-(1/4)\cos^4 \theta + C$     17  $-(1/2)\cos(x^2 + 1) + C$   
 19  $-\cos(\ln x) + C$     21  $(2/3)(\sin t)^{3/2} + C$     23  $(1/2)e^{2x} + C$   
 25  $ae^x - be^{-x} + C$     27  $\frac{1}{2}e^{x^2} + C$     29  $(b/a)e^{ax} + C$     31  $e^{\sin \theta} + C$   
 33  $\ln(x + 2) + C$     35  $\ln(e^x + 1) + C$     37  $x - \ln(x + 1) + C$   
 39  $2\ln(1 + \sqrt{x}) + C$     41  $-3t - 19\ln(5 - t) + C$     43  $(1/6)(x^4 + 5)^{3/2} + C$   
 45  $(1/3)(2 + y^2)^{3/2} + C$     47  $-(1 - u^2)^{1/2} + C$     49  $(2/3)(3s + 2)^{1/2} + C$   
 51  $-2(1 + x^{-1})^{1/2} + C$     53  $-(1/15)(3 + 5x^{-2})^{3/2} + C$     55  $-(2/3)(3 - \sqrt{x})^3 + C$   
 57  $-2z^{-1} - (1/2)z^{-2} + C$     59  $(2/3)(x^3 + 4)^{1/2} + C$     61  $(1/2)(1 + x^4)^{1/2} + C$   
 63  $(2/5)(t + 1)^{5/2} - (2/3)(t + 1)^{3/2} + C$     65  $(8/3)(1 - s)^{-3} - (1 - s)^{-2} + C$   
 67  $-(1/2)(y^2 + 1)^{-1} + (1/4)(y^2 + 1)^{-2} + C$   
 69  $(1/24)(4x + 1)^{3/2} - (1/8)(4x + 1)^{1/2} + C$  or  $(1/12)(2x - 1)(4x + 1)^{1/2} + C$   
 71  $-(2/27)(1 - 3u)^{3/2} + (2/45)(1 - 3u)^{5/2} + C$     73  $(1/6)(4x + 1)^{3/2} - (4x + 1)^{1/2} + C$   
 75  $-(1/4)\ln(1 - x^4) + C$     77  $(1/4)y^4 - (1/2)y^2 + (1/2)\ln(1 + y^2) + C$  or  
 $(1/4)(1 + y^2)^2 - (1 + y^2) + (1/2)\ln(1 + y^2) + C$   
 79  $(2/3)\ln(3u + 2) + 3/(3u + 2) + C$     81  $x - 4\sqrt{x} + 8\ln(2 + \sqrt{x}) + C$   
 83  $\ln(\sin \theta) + C$     85  $(1/b)\ln(a + bx) + C$     87  $-\ln(1 + \cos \theta) + C$   
 89  $(1/2)(\ln x)^2 + C$     91  $1/2$     93  $e - e^{-1}$     95  $\ln 2$  or  $(1/2)\ln 4$     97  $1/2$   
 99  $(2/3)(3\sqrt{3} - 1)$     101  $242/5$     103  $100$     105  $(1/6)(17\sqrt{17} - 27)$     107  $0$   
 109  $(\ln 7 - \ln 4)/3$     111  $2/3$     113  $93/35$     115  $0$     117  $f(g(x)) + C$

### Section 4.5

- 1  $56/3$     3  $2$     5  $(10\sqrt{5}/3) - 6$     7  $32/3$     9  $500/3$     11  $2$     13  $4$   
 15  $2$     17  $e^2 - 3$     19  $2(e - e^{-1})$     21  $2 - \ln 3$     23  $(3/2) - \ln 2$     25  $1/5$   
 27  $128\sqrt{6}/5$     29  $4/3$     31  $32/3$     33  $1/6$     35  $19/15$     37  $49/15$   
 39  $23/12$     41  $3/4$     43  $1 - (4/3) \cdot 2^{3/4} + (5/6) \cdot 2^{3/5} = 0.02$     45  $11/4$   
 47  $(3 + \sqrt{5})/2$     49  $\sqrt[3]{54}$  or  $3\sqrt[3]{2}$     51  $c = (1/2)^{2/3}$  or  $c = 2^{-2/3}$

### Section 4.6

- 1 In all cases, sum = 2, error = 0.  
 3 (a)  $\Delta x = 0.25$ : sum = 1.0426     $\Delta x = 0.1$ : sum = 1.0652  
 (b)  $\Delta x = 0.25$ : sum = 1.0594     $\Delta x = 0.1$ : sum = 1.0700  
 In all cases, there is no error estimate.  
 5 (a)  $\Delta x = 0.25$ : sum = 0.3229, error  $\leq 1/384$   
 $\Delta x = 0.1$ : sum = 0.3220, error  $\leq 1/2400$   
 (b)  $\Delta x = 0.25$ : sum = 0.3217108, error  $\leq 1/15,360$   
 $\Delta x = 0.1$ : sum = 0.3217505, error  $\leq 1/600,000$   
 7 (a)  $\Delta x = 0.5$ : sum = 2.8968, error  $\leq 1/48$   
 $\Delta x = 0.1$ : sum = 2.9012, error  $\leq 1/1200$   
 (b)  $\Delta x = 0.5$ : sum = 2.9013, error  $\leq 1/960$   
 $\Delta x = 0.1$ : sum = 2.901388, error  $\leq 1/600,000$   
 9 (a)  $\Delta x = 0.25$ : sum = 1.0968, error  $\leq \sqrt{2}/96$   
 $\Delta x = 0.1$ : sum = 1.0906, error  $\leq \sqrt{2}/600$   
 (b)  $\Delta x = 0.25$ : sum = 1.089413  
 $\Delta x = 0.1$ : sum = 1.089430  
 11 (a)  $\Delta x = 1$ : sum = 2.0214, error  $\leq 1$   
 $\Delta x = 0.1$ : sum = 1.9447, error  $\leq 0.01$   
 (b)  $\Delta x = 1$ : sum = 1.9587, error  $\leq 0.8$   
 $\Delta x = 0.1$ : sum = 1.945913, error  $\leq 0.0008$   
 13 (a)  $\Delta x = 3$ : sum = 1.8793, error  $\leq 45/8$   
 $\Delta x = 0.1$ : sum = 1.6685, error  $\leq 5/800$   
 (b)  $\Delta x = 3$ : sum = 1.738132  
 $\Delta x = 0.1$ : sum = 1.668231

- 15 (a)  $\Delta x = \pi/2$ : sum = 1.5708, error  $\leq \pi^3/48$  or 0.65  
 $\Delta x = \pi/10$ : sum = 1.9835, error  $\leq \pi^3/1200$  or 0.026  
 (b)  $\Delta x = \pi/2$ : sum = 2.0944, error  $\leq \pi^5/2880$  or 0.1  
 $\Delta x = \pi/10$ : sum = 2.00011, error  $\leq \pi^5/1,800,000$  or 0.00017  
 17 (a)  $\Delta x = 0.25$ : sum = 1.7272, error  $\leq e/192$  or 0.014  
 $\Delta x = 0.1$ : sum = 1.7197, error  $\leq e/1200$  or 0.002  
 (b)  $\Delta x = 0.25$ : sum = 1.718319, error  $\leq e/46,080$  or 0.00006  
 $\Delta x = 0.1$ : sum = 1.718283, error  $\leq e/1,800,000$  or 0.0000015  
 19 (a)  $\Delta x = 0.25$ : sum = 0.3837, error  $\leq 1/192$   
 $\Delta x = 0.1$ : sum = 0.3859, error  $\leq 1/1200$   
 (b)  $\Delta x = 0.25$ : sum = 0.386260, error  $\leq 1/46,080$   
 $\Delta x = 0.1$ : sum = 0.386293, error  $\leq 1/1,800,000$

### Extra Problems for Chapter 4

- 1 0.6025    3 7.875    5  $1/3$     7  $76 + 40\sqrt{5}$     9  $946\frac{2}{3}$     11 2  
 13  $2x + x^2/2 - x^3 + C$     15  $-(1/4)(x^2 - 1)^{-2} + C$     17  $-(1/3)(1 - 3u^2)^{1/2} + C$   
 19  $(1/3)[(2t + 1)^{3/2} - (2t - 1)^{3/2}] + C$     21  $(2/5)u^{5/2} - (4/3)u^{3/2} + C$   
 23  $2 \sin(x/2) + C$     25  $-e^{-t} + C$     27  $40/3$     29  $(e^4 - 1)/4$     31  $(x^3 + 2)^{1/2}$   
 33  $-\sqrt{u(u-1)^{1/2}}$     35  $(1/2)x^2 - x + b + 1/2$     37  $3x^2 - 6x + 5$   
 39  $F(x) = 1$  if  $x < 0$ ,  $(x^2/2) + 1$  if  $x \geq 0$     43  $\pi ab/2$     45  $xf(x) + \int_0^x f(t) dt$

### Section 5.1

- 1 3    3  $\infty$     5  $1/3$     7  $5/3$     9  $1/\sqrt{3}$     11  $\infty$     13  $\infty$     15 0  
 17  $-\infty$     19  $\infty$     21 0    23  $1/2$     25  $-1/4$     27 0    29  $-5/6$     31 0  
 33 does not exist    35  $\infty$     37 does not exist    39 does not exist    41  $\infty$   
 43 0    45  $-\infty$     47  $1/2$     49  $\infty$     51  $-3/2$     53 0    55 1    57 0  
 59 does not exist    61 0    63  $-\infty$     65 does not exist    67  $\infty$

### Section 5.2

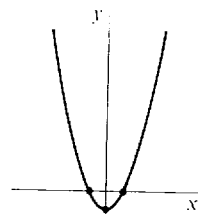
- 1  $1/6$     3  $1/16$     5 0    7  $-1/4$     9  $1/2$     11  $-1$     13 5    15  $\infty$   
 17 2    19 3    21 1    23 2    25 1    27 1    29 1    31 1    33  $1/2$   
 35 0    37  $1/2$     39  $1/2$     41  $4/\sqrt{2} = \sqrt{8}$     43 2    45  $\infty$     47  $3/2$   
 49 5    51 does not exist

### Section 5.3

- 1 (a) A, B, F, H (b) B, F, H (c) B, F (d) A, B, C, D, F, G, H  
 (e) B, D, F, G, H (f) B, D, F

3

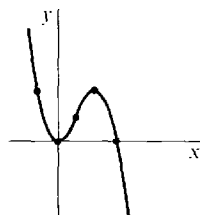
| $x$                            | $f(x)$   | $f'(x)$   | $f''(x)$ |       |        |
|--------------------------------|----------|-----------|----------|-------|--------|
| $\lim_{x \rightarrow -\infty}$ | $\infty$ | $-\infty$ |          |       |        |
| 0                              | 0        | -2        | +        | decr. | $\cup$ |
| 1                              | -1       | 0         | +        | min.  | $\cup$ |
| 2                              | 0        | 2         | +        | incr. | $\cup$ |
| $\lim_{x \rightarrow \infty}$  | $\infty$ | $\infty$  |          |       |        |





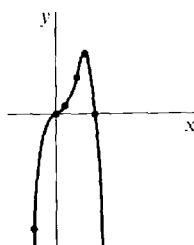
5

| $x$                            | $f(x)$         | $f'(x)$   | $f''(x)$ |       |        |
|--------------------------------|----------------|-----------|----------|-------|--------|
| $\lim_{x \rightarrow -\infty}$ | $\infty$       | $-\infty$ |          |       |        |
| -2                             | $5\frac{1}{3}$ | -6        | +        | decr. | $\cup$ |
| 0                              | 0              | 0         | +        | min.  | $\cup$ |
| 2                              | $2\frac{2}{3}$ | 2         | 0        | incr. | infl.  |
| 4                              | $5\frac{1}{3}$ | 0         | -        | max.  | $\cup$ |
| 6                              | 0              | -6        | -        | decr. | $\cup$ |
| $\lim_{x \rightarrow \infty}$  | $-\infty$      | $-\infty$ |          |       |        |



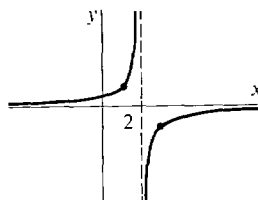
7

| $x$                            | $f(x)$         | $f'(x)$   | $f''(x)$ |        |        |
|--------------------------------|----------------|-----------|----------|--------|--------|
| $\lim_{x \rightarrow -\infty}$ | $-\infty$      | $\infty$  |          |        |        |
| -2                             | -12            | 20        | -        | incr.  | $\cup$ |
| 0                              | 0              | 0         | 0        | horiz. | infl.  |
| 1                              | $\frac{3}{4}$  | 2         | +        | incr.  | $\cup$ |
| 2                              | 4              | 4         | 0        | incr.  | infl.  |
| 3                              | $6\frac{3}{4}$ | 0         | -        | max.   | $\cup$ |
| 4                              | 0              | -16       | -        | decr.  | $\cup$ |
| $\lim_{x \rightarrow \infty}$  | $-\infty$      | $-\infty$ |          |        |        |



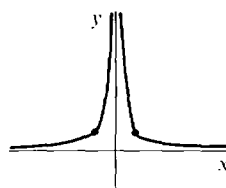
9

| $x$                            | $f(x)$    | $f'(x)$  | $f''(x)$ |       |        |
|--------------------------------|-----------|----------|----------|-------|--------|
| $\lim_{x \rightarrow -\infty}$ | 0         | 0        |          |       |        |
| 1                              | 1         | 1        | +        | incr. | $\cup$ |
| $\lim_{x \rightarrow 2^-}$     | $\infty$  | $\infty$ |          |       |        |
| $\lim_{x \rightarrow 2^+}$     | $-\infty$ | $\infty$ |          |       |        |
| 3                              | -1        | 1        | -        | incr. | $\cup$ |
| $\lim_{x \rightarrow \infty}$  | 0         | 0        |          |       |        |



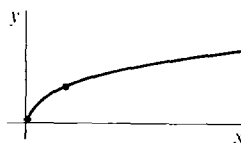
11

| $x$                            | $f(x)$   | $f'(x)$  | $f''(x)$ |       |        |
|--------------------------------|----------|----------|----------|-------|--------|
| $\lim_{x \rightarrow -\infty}$ | 0        | 0        |          |       |        |
| -1                             | 1        | 2        | +        | incr. | $\cup$ |
| $\lim_{x \rightarrow 0^-}$     | $\infty$ | $\infty$ |          |       |        |
| $\lim_{x \rightarrow 0^+}$     | $\infty$ | $\infty$ |          |       |        |
| 1                              | 1        | -2       | +        | decr. | $\cup$ |
| $\lim_{x \rightarrow \infty}$  | 0        | 0        |          |       |        |

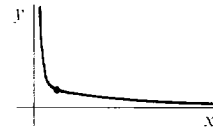


13

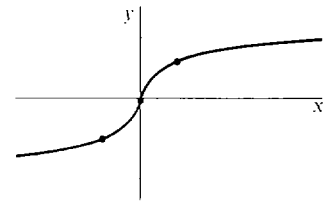
| $x$                           | $f(x)$   | $f'(x)$       | $f''(x)$ |       |        |
|-------------------------------|----------|---------------|----------|-------|--------|
| $\lim_{x \rightarrow 0^+}$    | 0        | $\infty$      |          |       |        |
| 1                             | 1        | $\frac{1}{2}$ | -        | incr. | $\cup$ |
| $\lim_{x \rightarrow \infty}$ | $\infty$ | 0             |          |       |        |



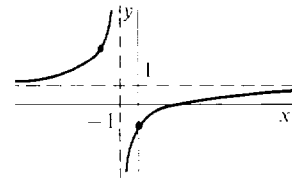
| 15 | $x$                           | $f(x)$   | $f'(x)$        | $f''(x)$ |       |        |
|----|-------------------------------|----------|----------------|----------|-------|--------|
|    | $\lim_{x \rightarrow 0^+}$    | $\infty$ | $-\infty$      |          |       |        |
|    | 1                             | 1        | $-\frac{1}{2}$ | +        | decr. | $\cap$ |
|    | $\lim_{x \rightarrow \infty}$ | 0        | 0              |          |       |        |



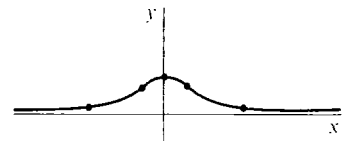
| 17 | $x$                            | $f(x)$    | $f'(x)$       | $f''(x)$ |       |        |
|----|--------------------------------|-----------|---------------|----------|-------|--------|
|    | $\lim_{x \rightarrow -\infty}$ | $-\infty$ | 0             |          |       |        |
|    | -1                             | -1        | $\frac{1}{3}$ | +        | incr. | $\cup$ |
|    | $\lim_{x \rightarrow 0^-}$     | 0         | $\infty$      |          |       |        |
|    | $\lim_{x \rightarrow 0^+}$     | 0         | $\infty$      |          |       |        |
|    | 1                              | 1         | $\frac{1}{3}$ | -        | incr. | $\cup$ |
|    | $\lim_{x \rightarrow \infty}$  | $\infty$  | 0             |          |       |        |



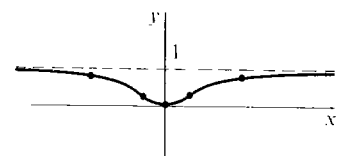
| 19 | $x$                            | $f(x)$    | $f'(x)$  | $f''(x)$ |       |        |
|----|--------------------------------|-----------|----------|----------|-------|--------|
|    | $\lim_{x \rightarrow -\infty}$ | 1         | 0        |          |       |        |
|    | -2                             | 3         | 2        | -        | incr. | $\cup$ |
|    | $\lim_{x \rightarrow -1^-}$    | $\infty$  | $\infty$ |          |       |        |
|    | $\lim_{x \rightarrow -1^+}$    | $-\infty$ | $\infty$ |          |       |        |
|    | 0                              | -1        | 2        | -        | incr. | $\cup$ |
|    | $\lim_{x \rightarrow \infty}$  | 1         | 0        |          |       |        |



| 21 | $x$                            | $f(x)$        | $f'(x)$         | $f''(x)$ |       |        |
|----|--------------------------------|---------------|-----------------|----------|-------|--------|
|    | $\lim_{x \rightarrow -\infty}$ | 0             | 0               |          |       |        |
|    | -2                             | $\frac{1}{5}$ | $\frac{4}{25}$  | +        | incr. | $\cup$ |
|    | $-1/\sqrt{3}$                  | $\frac{3}{4}$ | 1.9             | 0        | incr. | $\cup$ |
|    | 0                              | 1             | 0               | -        | max.  | $\cap$ |
|    | $1/\sqrt{3}$                   | $\frac{3}{4}$ | -1.9            | 0        | decr. | $\cap$ |
|    | 2                              | $\frac{1}{5}$ | $-\frac{4}{25}$ | +        | decr. | $\cup$ |
|    | $\lim_{x \rightarrow \infty}$  | 0             | 0               |          |       |        |

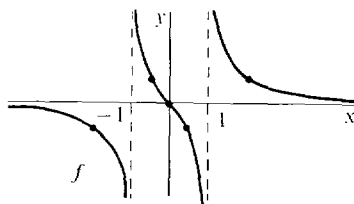


| 23 | $x$                            | $f(x)$        | $f'(x)$         | $f''(x)$ |       |        |
|----|--------------------------------|---------------|-----------------|----------|-------|--------|
|    | $\lim_{x \rightarrow -\infty}$ | 1             | 0               |          |       |        |
|    | -2                             | $\frac{4}{5}$ | $-\frac{2}{25}$ | -        | decr. | $\cap$ |
|    | $-1/\sqrt{3}$                  | $\frac{1}{4}$ | $-\sqrt{3}/2$   | 0        | decr. | $\cap$ |
|    | 0                              | 0             | 0               | +        | min.  | $\cup$ |
|    | $1/\sqrt{3}$                   | $\frac{1}{4}$ | $\sqrt{3}/2$    | 0        | incr. | $\cup$ |
|    | 2                              | $\frac{4}{5}$ | $\frac{2}{25}$  | -        | incr. | $\cap$ |
|    | $\lim_{x \rightarrow \infty}$  | 1             | 0               |          |       |        |



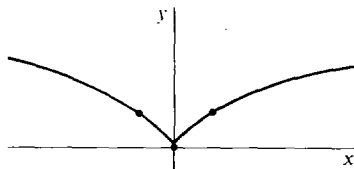
25

| $x$                            | $f(x)$         | $f'(x)$         | $f''(x)$ |       |        |
|--------------------------------|----------------|-----------------|----------|-------|--------|
| $\lim_{x \rightarrow -\infty}$ | 0              | 0               |          |       |        |
| -2                             | $-\frac{2}{3}$ | $-\frac{5}{9}$  | -        | decr. | $\cap$ |
| $\lim_{x \rightarrow -1^-}$    | $-\infty$      | $-\infty$       |          |       |        |
| $\lim_{x \rightarrow -1^+}$    | $\infty$       | $-\infty$       |          |       |        |
| $-\frac{1}{2}$                 | $\frac{2}{3}$  | $-\frac{20}{9}$ | +        | decr. | $\cup$ |
| 0                              | 0              | -1              | 0        | decr. | infl.  |
| $\frac{1}{2}$                  | $-\frac{2}{3}$ | $-\frac{20}{9}$ | -        | decr. | $\cap$ |
| $\lim_{x \rightarrow 1^-}$     | $-\infty$      | $-\infty$       |          |       |        |
| $\lim_{x \rightarrow 1^+}$     | $\infty$       | $-\infty$       |          |       |        |
| 2                              | $\frac{2}{3}$  | $-\frac{5}{9}$  | +        | decr. | $\cup$ |
| $\infty$                       | 0              | 0               |          |       |        |



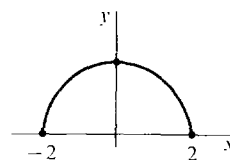
27

| $x$                            | $f(x)$   | $f'(x)$        | $f''(x)$ |                |        |
|--------------------------------|----------|----------------|----------|----------------|--------|
| $\lim_{x \rightarrow -\infty}$ | $\infty$ | 0              |          |                |        |
| -1                             | 1        | $-\frac{2}{3}$ | -        | decr.          | $\cap$ |
| $\lim_{x \rightarrow 0^-}$     | 0        | $-\infty$      |          | min.<br>(cusp) |        |
| $\lim_{x \rightarrow 0^+}$     | 0        | $\infty$       |          |                |        |
| 1                              | 1        | $\frac{2}{3}$  | -        | incr.          | $\cup$ |
| $\lim_{x \rightarrow \infty}$  | $\infty$ | 0              |          |                |        |



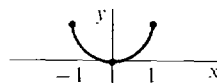
29

| $x$                         | $f(x)$ | $f'(x)$   | $f''(x)$ |      |        |
|-----------------------------|--------|-----------|----------|------|--------|
| $\lim_{x \rightarrow -2^+}$ | 0      | $\infty$  |          |      |        |
| 0                           | 2      | 0         | -        | max. | $\cap$ |
| $\lim_{x \rightarrow 2^-}$  | 0      | $-\infty$ |          |      |        |



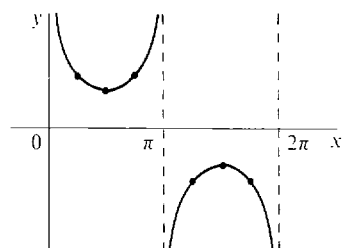
31

| $x$                         | $f(x)$ | $f'(x)$   | $f''(x)$ |      |        |
|-----------------------------|--------|-----------|----------|------|--------|
| $\lim_{x \rightarrow -1^+}$ | 1      | $-\infty$ |          |      |        |
| 0                           | 0      | 0         | +        | min. | $\cup$ |
| $\lim_{x \rightarrow 1^-}$  | 1      | $\infty$  |          |      |        |



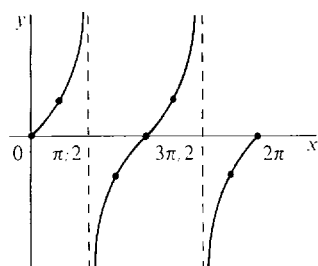
**33**

| $x$                           | $f(x)$      | $f'(x)$     | $f''(x)$ |       |        |
|-------------------------------|-------------|-------------|----------|-------|--------|
| $\lim_{x \rightarrow 0^+}$    | $\infty$    | $-\infty$   |          |       |        |
| $\pi/4$                       | $\sqrt{2}$  | $-\sqrt{2}$ | +        | decr. | $\cup$ |
| $\pi/2$                       | 1           | 0           | +        | min.  | $\cup$ |
| $3\pi/4$                      | $\sqrt{2}$  | $\sqrt{2}$  | +        | incr. | $\cup$ |
| $\lim_{x \rightarrow \pi^-}$  | $\infty$    | $\infty$    |          |       |        |
| $\lim_{x \rightarrow \pi^+}$  | $-\infty$   | $\infty$    |          |       |        |
| $5\pi/4$                      | $-\sqrt{2}$ | $\sqrt{2}$  | -        | incr. | $\cap$ |
| $3\pi/2$                      | -1          | 0           | -        | max.  | $\cap$ |
| $7\pi/4$                      | $-\sqrt{2}$ | $-\sqrt{2}$ | -        | decr. | $\cap$ |
| $\lim_{x \rightarrow 2\pi^-}$ | $-\infty$   | $-\infty$   |          |       |        |



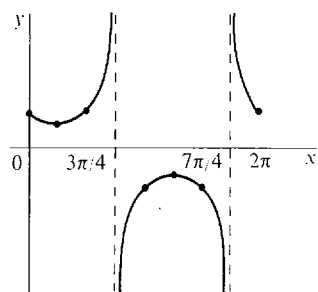
**35**

| $x$                             | $f(x)$    | $f'(x)$  | $f''(x)$ |       |        |
|---------------------------------|-----------|----------|----------|-------|--------|
| 0                               | 0         | 1        | 0        | incr. | infl.  |
| $\pi/4$                         | 1         | 2        | +        | incr. | $\cup$ |
| $\lim_{x \rightarrow \pi/2^-}$  | $\infty$  | $\infty$ |          |       |        |
| $\lim_{x \rightarrow \pi/2^+}$  | $-\infty$ | $\infty$ |          |       |        |
| $3\pi/4$                        | -1        | 2        | -        | incr. | $\cap$ |
| $\pi$                           | 0         | 1        | 0        | incr. | infl.  |
| $5\pi/4$                        | 1         | 2        | +        | incr. | $\cup$ |
| $\lim_{x \rightarrow 3\pi/2^-}$ | $\infty$  | $\infty$ |          |       |        |
| $\lim_{x \rightarrow 3\pi/2^+}$ | $-\infty$ | $\infty$ |          |       |        |
| $7\pi/4$                        | -1        | 2        | -        | incr. | $\cap$ |
| $2\pi$                          | 0         | 1        | 0        | incr. | infl.  |



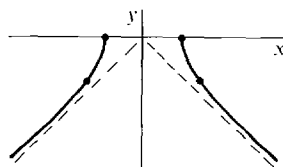
**37**

| $x$                             | $f(x)$        | $f'(x)$   | $f''(x)$ |       |        |
|---------------------------------|---------------|-----------|----------|-------|--------|
| 0                               | 1             | -1        | +        | decr. | $\cup$ |
| $\pi/4$                         | $\sqrt{2}/2$  | 0         | +        | min.  | $\cup$ |
| $\pi/2$                         | 1             | 1         | +        | incr. | $\cup$ |
| $\lim_{x \rightarrow 3\pi/4^-}$ | $\infty$      | $\infty$  |          |       |        |
| $\lim_{x \rightarrow 3\pi/4^+}$ | $-\infty$     | $\infty$  |          |       |        |
| $\pi$                           | -1            | 1         | -        | incr. | $\cap$ |
| $5\pi/4$                        | $-\sqrt{2}/2$ | 0         | -        | max.  | $\cap$ |
| $3\pi/2$                        | -1            | -1        | -        | decr. | $\cap$ |
| $\lim_{x \rightarrow 7\pi/4^-}$ | $-\infty$     | $-\infty$ |          |       |        |
| $\lim_{x \rightarrow 7\pi/4^+}$ | $\infty$      | $-\infty$ |          |       |        |
| $2\pi$                          | 1             | -1        | +        | decr. | $\cup$ |



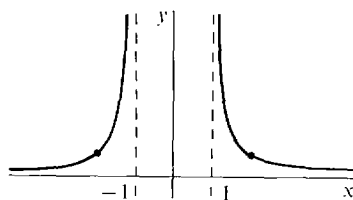
39

| $x$                            | $f(x)$      | $f'(x)$       | $f''(x)$ |       |        |
|--------------------------------|-------------|---------------|----------|-------|--------|
| $\lim_{x \rightarrow -\infty}$ | $-\infty$   | 1             |          |       |        |
| -3                             | $-\sqrt{5}$ | $3/\sqrt{5}$  | +        | incr. | $\cup$ |
| $\lim_{x \rightarrow -2^-}$    | 0           | $\infty$      |          |       |        |
| $\lim_{x \rightarrow 2^+}$     | 0           | $-\infty$     |          |       |        |
| 3                              | $-\sqrt{5}$ | $-3/\sqrt{5}$ | +        | decr. | $\cup$ |
| $\lim_{x \rightarrow \infty}$  | $-\infty$   | -1            |          |       |        |



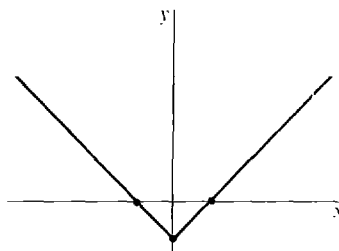
41

| $x$                            | $f(x)$       | $f(x)$           | $f''(x)$ |       |        |
|--------------------------------|--------------|------------------|----------|-------|--------|
| $\lim_{x \rightarrow -\infty}$ | 0            | 0                |          |       |        |
| -2                             | $1/\sqrt{3}$ | $2/(3\sqrt{3})$  | +        | incr. | $\cup$ |
| $\lim_{x \rightarrow -1^-}$    | $\infty$     | $\infty$         |          |       |        |
| $\lim_{x \rightarrow 1^+}$     | $\infty$     | $-\infty$        |          |       |        |
| 2                              | $1/\sqrt{3}$ | $-2/(3\sqrt{3})$ | +        | decr. | $\cup$ |
| $\lim_{x \rightarrow \infty}$  | 0            | 0                |          |       |        |



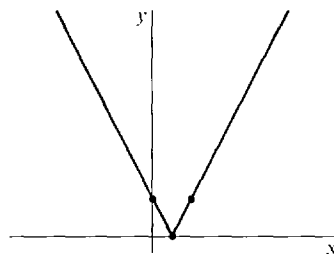
43

| $x$                            | $f(x)$   | $f'(x)$ | $f''(x)$ |                  |        |
|--------------------------------|----------|---------|----------|------------------|--------|
| $\lim_{x \rightarrow -\infty}$ | $\infty$ | -1      |          |                  |        |
| -1                             | 0        | -1      | 0        | decr.            | linear |
| $\lim_{x \rightarrow 0^-}$     | -1       | -1      |          | min.<br>(corner) |        |
| $\lim_{x \rightarrow 0^+}$     | -1       | 1       |          |                  |        |
| 1                              | 0        | 1       | 0        | incr.            | linear |
| $\lim_{x \rightarrow \infty}$  | $\infty$ | 1       |          |                  |        |



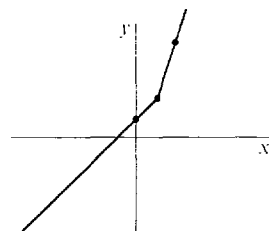
45

| $x$                                  | $f(x)$   | $f'(x)$ | $f''(x)$ |                  |        |
|--------------------------------------|----------|---------|----------|------------------|--------|
| $\lim_{x \rightarrow -\infty}$       | $\infty$ | -2      |          |                  |        |
| 0                                    | 1        | -2      | 0        | decr.            | linear |
| $\lim_{x \rightarrow \frac{1}{2}^-}$ | 0        | -2      |          | min.<br>(corner) |        |
| $\lim_{x \rightarrow \frac{1}{2}^+}$ | 0        | 2       |          |                  |        |
| 1                                    | 1        | 2       | 0        | incr.            | linear |
| $\lim_{x \rightarrow \infty}$        | $\infty$ | 2       |          |                  |        |



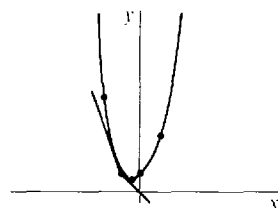
47

| $x$                            | $f(x)$    | $f'(x)$ | $f''(x)$ |        |        |
|--------------------------------|-----------|---------|----------|--------|--------|
| $\lim_{x \rightarrow -\infty}$ | $-\infty$ | 1       |          |        |        |
| 0                              | 2         | 1       | 0        | incr.  | linear |
| $\lim_{x \rightarrow 2^-}$     | 4         | 1       |          | corner |        |
| $\lim_{x \rightarrow 2^+}$     | 4         | 3       |          |        |        |
| 4                              | 10        | 3       | 0        | incr.  | linear |
| $\lim_{x \rightarrow \infty}$  | $\infty$  | 3       |          |        |        |



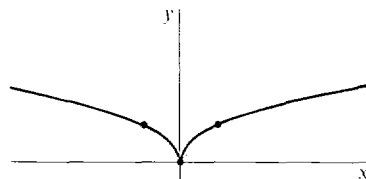
49

| $x$                            | $f(x)$        | $f'(x)$   | $f''(x)$ |        |        |
|--------------------------------|---------------|-----------|----------|--------|--------|
| $\lim_{x \rightarrow -\infty}$ | $\infty$      | $-\infty$ |          |        |        |
| -2                             | 5             | -5        | 2        | decr.  | $\cup$ |
| $\lim_{x \rightarrow -1^-}$    | 1             | -3        |          | corner |        |
| $\lim_{x \rightarrow -1^+}$    | 1             | -1        |          |        |        |
| $-\frac{1}{2}$                 | $\frac{3}{4}$ | 0         | 2        | min.   | $\cup$ |
| 0                              | 1             | 1         | 2        | incr.  | $\cup$ |
| 1                              | 3             | 3         | 2        | incr.  | $\cup$ |
| $\lim_{x \rightarrow \infty}$  | $\infty$      | $\infty$  |          |        |        |



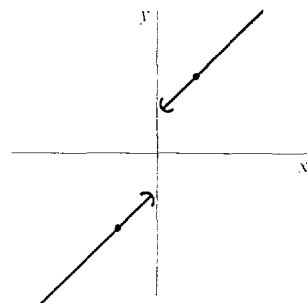
51

| $x$                            | $f(x)$   | $f'(x)$        | $f''(x)$ |                |        |
|--------------------------------|----------|----------------|----------|----------------|--------|
| $\lim_{x \rightarrow -\infty}$ | $\infty$ | 0              |          |                |        |
| -1                             | 1        | $-\frac{1}{2}$ | -        | decr.          | $\cap$ |
| $\lim_{x \rightarrow 0^-}$     | 0        | $-\infty$      |          | min.<br>(cusp) |        |
| $\lim_{x \rightarrow 0^+}$     | 0        | $\infty$       |          |                |        |
| 1                              | 1        | $\frac{1}{2}$  | -        | incr.          | $\cap$ |
| $\lim_{x \rightarrow \infty}$  | $\infty$ | 0              |          |                |        |

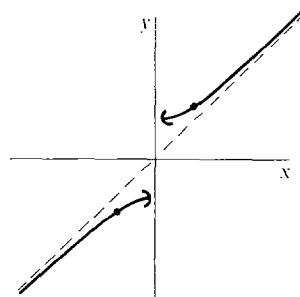


53

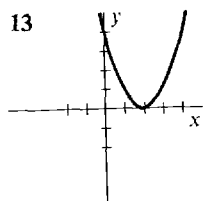
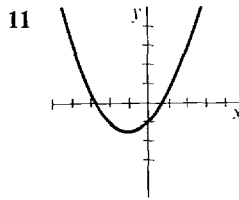
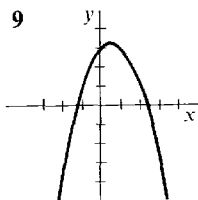
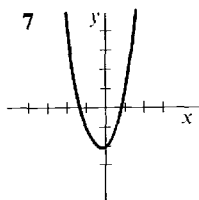
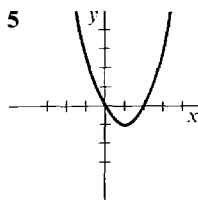
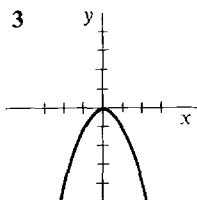
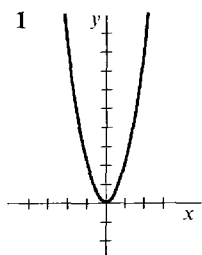
| $x$                            | $f(x)$    | $f'(x)$ | $f''(x)$ |       |        |
|--------------------------------|-----------|---------|----------|-------|--------|
| $\lim_{x \rightarrow -\infty}$ | $-\infty$ | 1       |          |       |        |
| -1                             | -2        | 1       | 0        | incr. | linear |
| $\lim_{x \rightarrow 0^-}$     | -1        | 1       |          | jump  |        |
| $\lim_{x \rightarrow 0^+}$     | 1         | 1       |          |       |        |
| 1                              | 2         | 1       | 0        | incr. | linear |
| $\lim_{x \rightarrow \infty}$  | $\infty$  | 1       |          |       |        |



| 55 | $x$                            | $f(x)$      | $f'(x)$      | $f''(x)$ |       |        |
|----|--------------------------------|-------------|--------------|----------|-------|--------|
|    | $\lim_{x \rightarrow -\infty}$ | $-\infty$   | 1            |          |       |        |
|    | $-1$                           | $-\sqrt{2}$ | $1/\sqrt{2}$ | $-$      | incr. | $\cap$ |
|    | $\lim_{x \rightarrow 0^-}$     | $-1$        | 0            |          | jump  |        |
|    | $\lim_{x \rightarrow 0^+}$     | 1           | 0            |          |       |        |
|    | 1                              | $\sqrt{2}$  | $1/\sqrt{2}$ | $+$      | incr. | $\cup$ |
|    | $\lim_{x \rightarrow \infty}$  | $\infty$    | 1            |          |       |        |

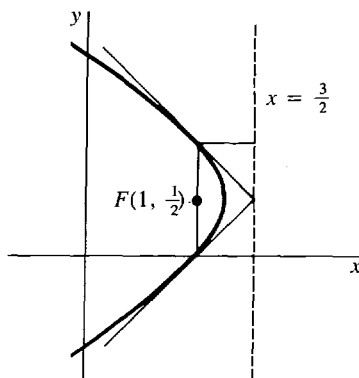
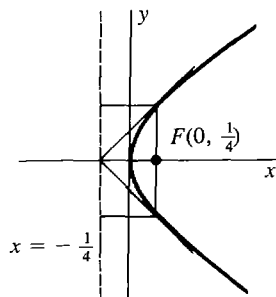


## Section 5.4



15 focus  $(1/4, 0)$ , directrix  $x = -1/4$

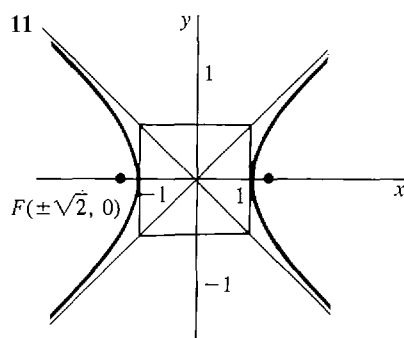
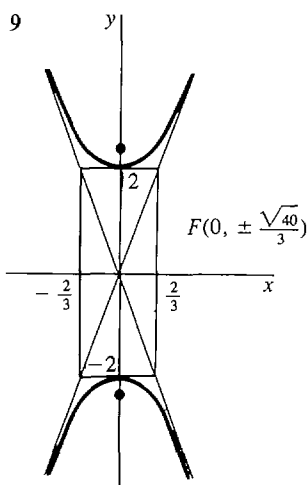
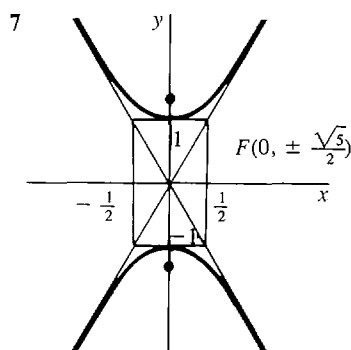
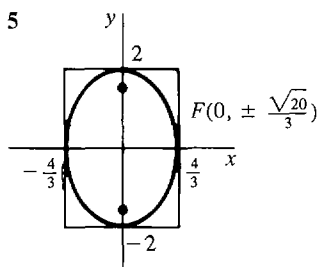
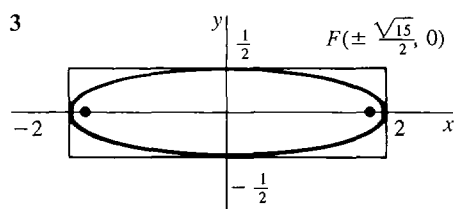
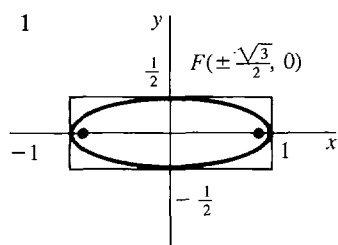
17 focus  $(3/2, 1/2)$ , directrix  $x = 2$



19  $y = (1/4)x^2 - x + 2$

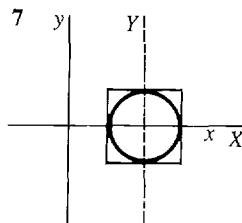
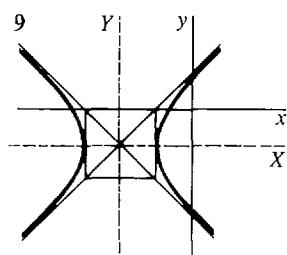
21  $(1, 3)$

## Section 5.5

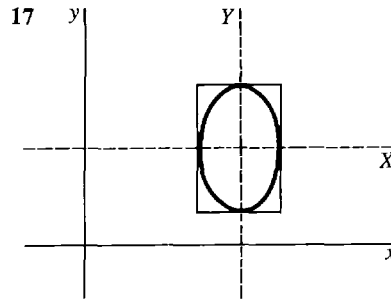
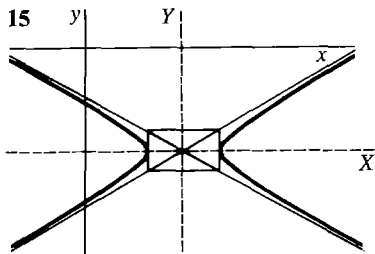
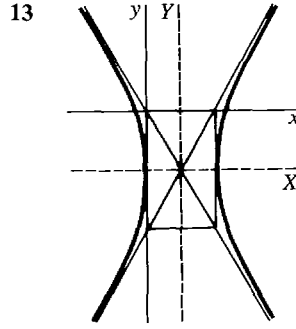
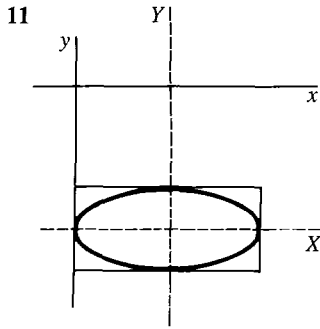


## Section 5.6

1 hyperbola    3 parabola    5 ellipse







### Section 5.7

- 1  $\alpha = 45^\circ, X^2 - Y^2 + 8 = 0$     3  $\alpha = 45^\circ, -2X^2 + 6Y^2 = 1$   
 5  $\alpha = 30^\circ, 2X^2 - 2Y^2 = 7$     7  $\alpha = 22.5^\circ, 1.207X^2 - 0.207Y^2 = 3$   
 9  $\alpha = -15^\circ, 4.232X^2 + 0.768Y^2 = 5$

### Section 5.8

Any smaller value of  $\delta > 0$  or larger value of  $B$  is also correct.

- 1  $\delta = 10^{-3}$     3  $\delta = 2 - (0.51)^{-1} \sim 0.039$     5  $\delta = 10^{-4}$   
 7  $\delta = 1 - \sqrt{1 - 10^{-6}} \sim 5 \times 10^{-7}$     9  $\delta = 10^{-2}$     11  $\delta = 10^{-3}$     13  $\delta = 10^{-4}$   
 15  $\delta = 1 - \sqrt{0.99} \sim 5 \times 10^{-3}$     17  $B = 99/4 = 24.75$     19  $B = \frac{5 + \sqrt{8025}}{4} \sim 23.7$   
 21  $B = \frac{10^4 - 1}{5} \sim 2 \times 10^3$   
 23 For all  $\varepsilon > 0$  there exists  $\delta > 0$  such that whenever  $c - \delta < x < c$ ,  $|f(x) - L| < \varepsilon$ .  
 25 For all  $A > 0$  there exists  $B > 0$  such that whenever  $x > B$ ,  $f(x) < -A$ .

### Section 5.9

- 1 1.42332    3 0.63688    5 1.22074    7 2.93923    9 0.38197    11 0.56714  
 13 1.10606    15 1.30633

### Section 5.10

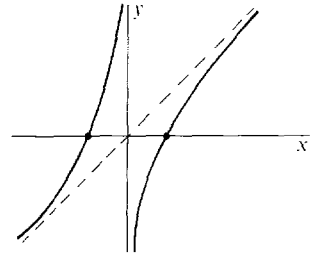
- 1  $f'(1) = 2$ , error  $\leq \Delta x$     3  $f'(4) = -(1/8)$ , error  $\leq (3/128)\Delta x$   
 5  $f'(3) = -1/9$ , error  $\leq (1/27)\Delta x$     7  $f'(0) = 1$ , error  $\leq 0.5\Delta x$   
 9  $f'(\pi/3) = -\sqrt{3}$ , error  $\leq 2\Delta x$     11  $f'(1) = 1$ , error  $\leq \Delta x/2$

- 13  $f'(1) = e$ , error  $\leq (1/2)e \Delta x$   
 15  $f'(100) = 1/20$ , error  $\leq (1/8) \cdot 99^{-3/2} \Delta x$  or  $0.000127 \Delta x$   
 17  $f'(2) = (2/\sqrt{5})$ , error  $\leq 1/(4\sqrt{2}) \Delta x$     19  $f'(1) = \sqrt{2} + 2^{-3/2}$ , error  $\leq \Delta x/2$   
 21  $\sqrt[3]{65} \sim 8^{1/6}$ , error  $\leq 1/4096$     23  $(0.301)^4$  or  $0.008208$ , error  $\leq 6 \times 10^{-7}$   
 25  $1/97$  or  $10^{-2} + 3 \times 10^{-4} = 0.0103$ , error  $\leq 10^{-5}$   
 27  $\sqrt[3]{1.02} + \sqrt[3]{1.02} \sim 2^{1/60}$ , error  $\leq \frac{17}{8} \times 10^{-4}$     29  $(1.003)^5$  or  $1.015$ , error  $\leq 10^{-4}$   
 31  $\sin(\pi/3 + 0.004)$  or  $\sqrt{3}/2 + 0.002$ , error  $\leq 8 \times 10^{-6}$   
 33  $\tan(0.005)$  or  $0.005$ , error  $\leq 10^{-6}$     35  $e^{0.002}$  or  $1.002$ , error  $\leq 3 \times 10^{-6}$   
 37  $\ln(1.006)$  or  $0.006$ , error  $\leq 18 \times 10^{-6}$

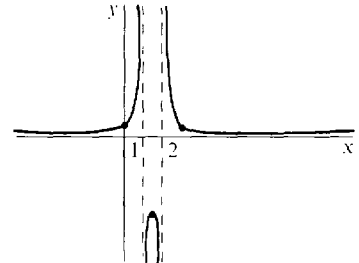
## Extra Problems for Chapter 5

1 0 3 0 5  $\infty$  7 0 9 3/2

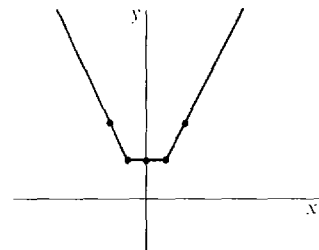
| 11 | $x$                            | $f(x)$    | $f'(x)$  | $f''(x)$ |       |        |
|----|--------------------------------|-----------|----------|----------|-------|--------|
|    | $\lim_{x \rightarrow -\infty}$ | $-\infty$ | 1        |          |       |        |
|    | -1                             | 0         | 2        | +        | incr. | $\cup$ |
|    | $\lim_{x \rightarrow 0^-}$     | $\infty$  | $\infty$ |          |       |        |
|    | $\lim_{x \rightarrow 0^+}$     | $-\infty$ | $\infty$ |          |       |        |
|    | 1                              | 0         | 2        | -        | incr. | $\cap$ |
|    | $\lim_{x \rightarrow \infty}$  | $\infty$  | 1        |          |       |        |

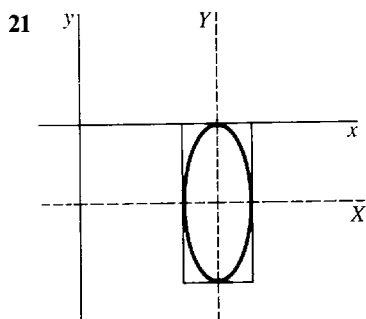
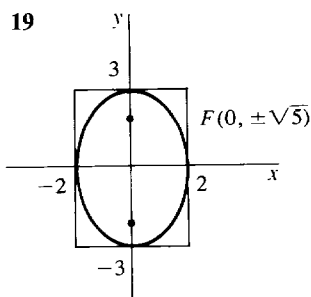
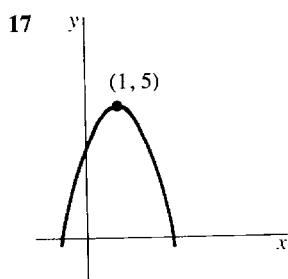


| 13 | $x$                            | $f(x)$        | $f'(x)$        | $f''(x)$ |       |        |
|----|--------------------------------|---------------|----------------|----------|-------|--------|
|    | $\lim_{x \rightarrow -\infty}$ | 0             | 0              |          |       |        |
|    | 0                              | $\frac{1}{2}$ | $\frac{3}{4}$  | +        | incr. | $\cup$ |
|    | $\lim_{x \rightarrow 1^-}$     | $\infty$      | $\infty$       |          |       |        |
|    | $\lim_{x \rightarrow 1^+}$     | $-\infty$     | $\infty$       |          |       |        |
|    | $\frac{3}{2}$                  | -4            | 0              | -        | max.  | $\cap$ |
|    | $\lim_{x \rightarrow 2^-}$     | $-\infty$     | $-\infty$      |          |       |        |
|    | $\lim_{x \rightarrow 2^+}$     | $\infty$      | $-\infty$      |          |       |        |
|    | 3                              | $\frac{1}{2}$ | $-\frac{3}{4}$ | +        | decr. | $\cup$ |
|    | $\lim_{x \rightarrow \infty}$  | 0             | 0              |          |       |        |



| 15 | $x$                            | $f(x)$   | $f'(x)$ | $f''(x)$ |          |        |
|----|--------------------------------|----------|---------|----------|----------|--------|
|    | $\lim_{x \rightarrow -\infty}$ | $\infty$ | -2      |          |          |        |
|    | -2                             | 4        | -2      | 0        | decr.    | linear |
|    | $\lim_{x \rightarrow -1^-}$    | 2        | -2      |          | corner   |        |
|    | $\lim_{x \rightarrow -1^+}$    | 2        | 0       |          |          |        |
|    | 0                              | 2        | 0       | 0        | constant | linear |
|    | $\lim_{x \rightarrow 1^-}$     | 2        | 0       |          | corner   |        |
|    | $\lim_{x \rightarrow 1^+}$     | 2        | 2       |          |          |        |
|    | 2                              | 4        | 2       | 0        | incr.    | linear |
|    | $\lim_{x \rightarrow \infty}$  | $\infty$ | 2       |          |          |        |





- 23  $\alpha = 45^\circ$ ,  $X^2 - Y^2 = 18$     25  $\delta \leq 0.155$     27 1.7866    29  $(3/4096) \Delta x$   
 31  $25 - 2/15 = 24.8666$ , error  $\leq 1/(9 \cdot (124)^{4/3})$

### Section 6.1

- 1  $1/3$     3  $\sqrt{3}/12$     5  $\pi/24$     7  $2cr^2/3$

### Section 6.2

- 1 (a)  $\pi/5$  (b)  $\pi/2$     3 (a)  $8\pi$  (b)  $128\pi/5$     5 (a)  $\pi/3$  (b)  $\pi/3$   
 7 (a)  $4\pi/3$  (b)  $(2/3)\pi(2\sqrt{2} - 1)$     9 (a)  $31\pi/160$  (b)  $\pi$     11 (a)  $4\pi/45$  (b)  $\pi/3$   
 13 (a)  $27\pi$  (b)  $18\pi$     15 (a)  $2\pi/35$  (b)  $\pi/10$     17 (a)  $\pi/3$  (b)  $\pi/9$   
 19  $20\pi/3$  (b)  $29\pi/6$     21 (a)  $16\pi/3$  (b)  $184\pi/15$     23  $2\pi$     25  $(\pi^2/4) - (\pi/2)$   
 27  $(\pi/2)(e^2 - 1)$     29  $(\pi/6)(e^2 - 1)$     31  $\pi \ln 2$     33  $\pi(3 - \ln 4)$     35  $2\pi$   
 37  $2\pi$     39  $\pi(e - 1)$     41  $2\pi(e^{-1} - e^{-4})$     43  $2\pi \ln 2$     45  $(\pi/2) \ln 7$   
 47  $\frac{4}{3}\pi(r^2 - a^2)^{3/2}$     49  $\frac{1}{3}\pi r^2 h - \pi a^2 h + \frac{2\pi h a^3}{3r}$   
 51 (a)  $\pi\left(\frac{b^2 a}{3} + \frac{2}{3}r^3 - r^2 a + \frac{a^3}{3}\right)$  or  $\frac{2}{3}\pi r^2(r - a)$   
 (b)  $\frac{2}{3}\pi(ba^2 + b^3)$  or  $\frac{2}{3}\pi[a^2\sqrt{r^2 - a^2} + (r^2 - a^2)^{3/2}]$     53  $2\pi^2 cr^2$

### Section 6.3

- 1  $\frac{2}{3}(6\sqrt{6} - 3\sqrt{3})$     3  $\frac{8}{3}(3\sqrt{3}/2\sqrt{2} - 1)$     5  $\frac{8}{27}(10\sqrt{10} - 1)$     7  $379/12$   
 9  $387/20$     11  $3/2$     13  $36$     15  $\frac{4}{3}(2^{5/2} - 1)$     17  $2\sqrt{3}/3$   
 19  $8 \cdot 10\sqrt{10} - 37\sqrt{37}$     21  $2\pi$     23  $4\sqrt{2}$     25  $\frac{t^2}{\sqrt{1 + 4t^2}}$   
 27  $\int_0^4 \sqrt{1 + (4x - 1)^2} dx$     29  $\int_1^2 \sqrt{4 + \frac{1}{4t}} dt$

$$31 \quad \int_1^2 \sqrt{1 + \frac{1}{4x}} dx \sim \frac{1}{4} \left( \frac{\sqrt{5}}{4} + \sqrt{\frac{3}{2}} + \sqrt{\frac{7}{4}} + \sqrt{2} + \frac{3}{4} \right) \sim 1.32$$

$$33 \quad \int_1^5 \sqrt{1 + x^{-4}} dx \sim \frac{\sqrt{2}}{2} + \frac{\sqrt{17}}{4} + \frac{\sqrt{82}}{9} + \frac{\sqrt{257}}{16} + \frac{\sqrt{626}}{50} \sim 4.246$$

## Section 6.4

$$1 \quad \frac{\pi}{6} (17\sqrt{17} - 1) \quad 3 \quad \frac{2\pi}{81} \left( \frac{2}{5} \cdot 10^{5/2} - \frac{2}{3} \cdot 10^{3/2} + \frac{4}{15} \right) \sim 8.2 \quad 5 \quad 256\pi/15$$

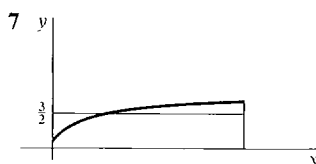
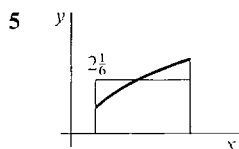
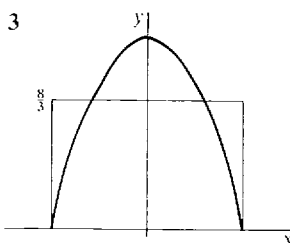
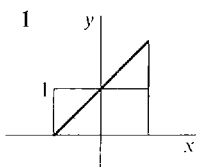
$$7 \quad 1575\pi/8 \quad 9 \quad \frac{2\pi}{3} (10\sqrt{10} - 2\sqrt{2}) \quad 11 \quad \pi(2\sqrt{2} - 1) \quad 13 \quad \frac{\pi}{9} (2\sqrt{2} - 1)$$

$$15 \quad 47\pi/16 \quad 17 \quad 7\pi/9 \quad 19 \quad 16\pi\sqrt{5} \quad 21 \quad 2\pi ra \quad 23 \quad (a) \int_0^1 2\pi x \sqrt{1 + 25x^8} dx$$

$$(b) \int_0^1 2\pi x^5 \sqrt{1 + 25x^8} dx \quad 25 \quad (a) \int_1^{10} 2\pi(t^2 + t)\sqrt{8t^2 + 4t + 1} dt$$

$$(b) \int_1^{10} 2\pi(t^2 - 1)\sqrt{8t^2 + 4t + 1} dt \quad 27 \quad \int_0^1 \pi x^2 \sqrt{1 + x^2} dx \sim 1.357$$

## Section 6.5



$$9 \quad 3 - (2/\sqrt{3}) \quad 11 \quad -6 \quad 13 \quad 0 \quad 15 \quad 2/\pi \quad 17 \quad 1/\pi \quad 19 \quad (1/2)(e - e^{-1})$$

$$21 \quad (1/3)\ln 4 \quad 23 \quad 1 \quad 25 \quad 8/9 \quad 27 \quad 6\sqrt{15}/25 \quad 29 \quad 13/2 \quad 31 \quad 116/45$$

$$33 \quad (a) 2 \quad (b) 32/7$$

## Section 6.6

$$1 \quad (a) 4 \quad (b) M_x = 0, M_y = 4 \quad (c) (1, 0) \quad 3 \quad (a) 128/3 \quad (b) M_x = 512/5, M_y = 0$$

$$(c) (0, 12/5) \quad 5 \quad (a) 6k \quad (b) M_x = 8k, M_y = 2k \quad (c) (\frac{1}{3}, \frac{4}{3}) \quad 7 \quad (a) 128/15$$

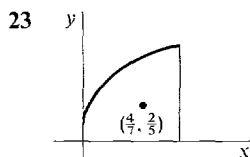
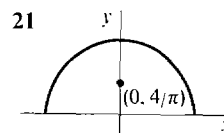
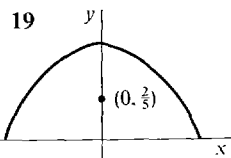
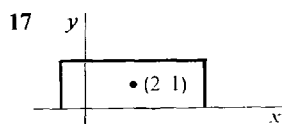
$$(b) M_x = 1024/105, M_y = 0 \quad (c) (0, \frac{8}{9}) \quad 9 \quad (a) \frac{1}{2} \quad (b) M_x = \frac{1}{8}, M_y = \frac{1}{3} \quad (c) (\frac{2}{3}, \frac{1}{4})$$

$$11 \quad (a) 2\sqrt{2} - 2 \quad (b) M_x = 1 - \frac{\sqrt{2}}{2}, M_y = \frac{4\sqrt{2} - 2}{3}$$

$$(c) \left( \frac{2\sqrt{2} - 1}{3\sqrt{2} - 3}, \frac{2 - \sqrt{2}}{4\sqrt{2} - 4} \right) \text{ or } \left( 1 + \frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{4} \right)$$

$$13 \quad (a) 4/3 \quad (b) M_x = 1/2, M_y = 0 \quad (c) (0, 3/8)$$

$$15 \quad (a) 128/3 \quad (b) M_x = 1024/15, M_y = 512/5 \quad (c) (\frac{12}{5}, \frac{8}{3})$$



- 25  $2/3$     27  $e^2 - 1$     29  $(2 \ln 2, 7/24)$     31  $(8/15, 16/105)$     33  $(1/2, 2/5)$   
 35 80 ft lbs    37  $320/3$  ft lbs    39 50 ft lbs    41 215 ft lbs    43  $9k/400$

### Section 6.7

- 1  $1/2$     3 diverges to  $\infty$     5 diverges to  $-\infty$     7 diverges to  $\infty$     9  $1/2$   
 11 diverges to  $\infty$     13 3    15 6    17  $-3/2$     19 3    21 diverges to  $\infty$   
 23 diverges to  $\infty$     25 diverges    27  $1/2$     29 diverges    31  $4\sqrt{2}$     33  $\infty$   
 37 4    39  $2/3$     43 (a)  $\pi$  (b)  $\infty$     45 (a)  $\infty$  (b)  $8\pi$     47  $4/3$   
 49 (a)  $11\pi/9$  (b)  $16\pi/15$     51 (a)  $\frac{\pi}{6}(5\sqrt{5} - 1)$  (b)  $\int_0^1 2\pi x \sqrt{1 + \frac{1}{4x}} dx$   
 53 (a)  $2\pi r a$  (b)  $2\pi r^2 - 2\pi r \sqrt{r^2 - a^2}$

### Extra Problems for Chapter 6

- 1  $8r^3/3$     3  $2r^3/3$     5 (a)  $11\pi/3$  (b)  $(16\pi/3) - 2\pi\sqrt{3}$     7 (a)  $16\pi$  (b)  $16\pi/3$   
 9  $\pi p/(p+1)$     11  $\frac{1}{27}(46\sqrt{46} - 10\sqrt{10})$     13 5    15  $(b-a)\sqrt{A^2 + C^2}$   
 17  $\frac{\pi}{3A}[(8A^2 + 4AB + B^2)^{3/2} - (4A^2 + B^2)^{3/2}]$     19  $\frac{b^{p+1} - 1}{(b-1)(p+1)}$     21  $(p+1)^{-1/p}$   
 23  $(\frac{1}{2}, \frac{2}{3})$     25  $(\frac{5}{2}, 0)$     27 15 ft lbs    29 diverges to  $\infty$     31  $-5/4$   
 33 diverges to  $\infty$     35 8    37  $a^2 h/3$     39  $m = \pi\rho, M_x = 2\rho, M_y = 0, (\bar{x}, \bar{y}) = (0, 2/\pi)$   
 43  $\pi\rho r^4/4$     45  $\omega\rho b^2/2$

### Section 7.1

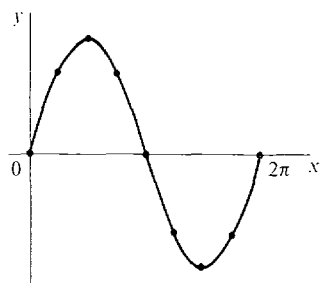
- 7  $(\pi/4) + k\pi$     9 none    13 0    15 0    17 0    19  $k\pi$

### Section 7.2

- 1  $5 \cos(5x)$     3  $6\theta \cos(3\theta^2)$     5  $4 \sec^2(4\theta - 3)$     7  $a \cos \theta - b \sin \theta$   
 9  $-\frac{1}{2\sqrt{x}} \sin(\sqrt{x})$     11  $\cos \theta \sec^2(\sin \theta)$     13  $\frac{3 \csc(3t) \cot(3t)}{(2 + \csc(3t))^2}$     15  $\frac{1}{2 \sin y \cos y}$   
 17  $3/2$     19  $\infty$     21 2    23 1

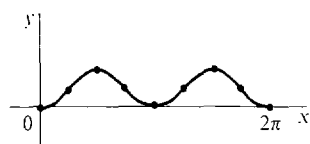
25

| $x$      | $f(x)$        | $f'(x)$       | $f''(x)$ |       |        |
|----------|---------------|---------------|----------|-------|--------|
| 0        | 0             | 3             | 0        | incr. | infl.  |
| $\pi/4$  | $3/\sqrt{2}$  | $3/\sqrt{2}$  | —        | incr. | $\cap$ |
| $\pi/2$  | 3             | 0             | —        | max.  | $\cap$ |
| $3\pi/4$ | $3/\sqrt{2}$  | $-3/\sqrt{2}$ | —        | decr. | $\cap$ |
| $\pi$    | 0             | -3            | 0        | decr. | infl.  |
| $5\pi/4$ | $-3/\sqrt{2}$ | $-3/\sqrt{2}$ | +        | decr. | $\cup$ |
| $3\pi/2$ | -3            | 0             | +        | min.  | $\cup$ |
| $7\pi/4$ | $-3/\sqrt{2}$ | $3/\sqrt{2}$  | +        | incr. | $\cup$ |
| $2\pi$   | 0             | 3             | 0        | incr. | infl.  |



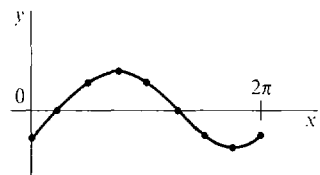
27

| $x$      | $f(x)$ | $f'(x)$ | $f''(x)$ |       |        |
|----------|--------|---------|----------|-------|--------|
| 0        | 0      | 0       | +        | min.  | $\cup$ |
| $\pi/4$  | 1/2    | 1       | 0        | incr. | infl.  |
| $\pi/2$  | 1      | 0       | —        | max.  | $\cap$ |
| $3\pi/4$ | 1/2    | -1      | 0        | decr. | infl.  |
| $\pi$    | 0      | 0       | +        | min.  | $\cup$ |
| $5\pi/4$ | 1/2    | 1       | 0        | incr. | infl.  |
| $3\pi/2$ | 1      | 0       | —        | max.  | $\cap$ |
| $7\pi/4$ | 1/2    | -1      | 0        | decr. | infl.  |
| $2\pi$   | 0      | 0       | +        | min.  | $\cup$ |



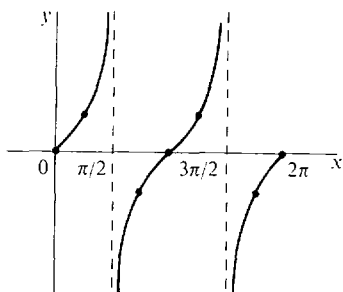
29

| $x$      | $f(x)$        | $f'(x)$       | $f''(x)$ |       |        |
|----------|---------------|---------------|----------|-------|--------|
| 0        | $-1/\sqrt{2}$ | $1/\sqrt{2}$  | +        | incr. | $\cup$ |
| $\pi/4$  | 0             | 1             | 0        | incr. | infl.  |
| $\pi/2$  | $1/\sqrt{2}$  | $1/\sqrt{2}$  | —        | incr. | $\cap$ |
| $3\pi/4$ | 1             | 0             | —        | max.  | $\cap$ |
| $\pi$    | $1/\sqrt{2}$  | $-1/\sqrt{2}$ | —        | decr. | $\cap$ |
| $5\pi/4$ | 0             | -1            | 0        | decr. | infl.  |
| $3\pi/2$ | $-1/\sqrt{2}$ | $-1/\sqrt{2}$ | +        | decr. | $\cup$ |
| $7\pi/4$ | -1            | 0             | +        | min.  | $\cup$ |
| $2\pi$   | $-1/\sqrt{2}$ | $1/\sqrt{2}$  | +        | incr. | $\cup$ |

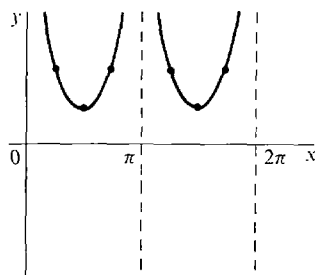


31

| $x$                             | $f(x)$    | $f'(x)$  | $f''(x)$ |       |        |
|---------------------------------|-----------|----------|----------|-------|--------|
| 0                               | 0         | 1        | 0        | incr. | infl.  |
| $\pi/4$                         | 1         | 2        | +        | incr. | $\cup$ |
| $\lim_{x \rightarrow \pi/2^-}$  | $\infty$  | $\infty$ |          |       |        |
| $\lim_{x \rightarrow \pi/2^+}$  | $-\infty$ | $\infty$ |          |       |        |
| $3\pi/4$                        | -1        | 2        | —        | incr. | $\cap$ |
| $\pi$                           | 0         | 1        | 0        | incr. | infl.  |
| $5\pi/4$                        | 1         | 2        | +        | incr. | $\cup$ |
| $\lim_{x \rightarrow 3\pi/2^-}$ | $\infty$  | $\infty$ |          |       |        |
| $\lim_{x \rightarrow 3\pi/2^+}$ | $-\infty$ | $\infty$ |          |       |        |
| $7\pi/4$                        | -1        | 2        | —        | incr. | $\cap$ |
| $2\pi$                          | 0         | 1        | 0        | incr. | infl.  |



| 33 | $x$                           | $f(x)$   | $f'(x)$   | $f''(x)$ |       |        |
|----|-------------------------------|----------|-----------|----------|-------|--------|
|    | $\lim_{x \rightarrow 0^+}$    | $\infty$ | $-\infty$ |          |       |        |
|    | $\pi/4$                       | 2        | -4        | +        | decr. | $\cup$ |
|    | $\pi/2$                       | 1        | 0         | +        | min.  | $\cup$ |
|    | $3\pi/4$                      | 2        | 4         | +        | incr. | $\cup$ |
|    | $\lim_{x \rightarrow \pi^-}$  | $\infty$ | $\infty$  |          |       |        |
|    | $\lim_{x \rightarrow \pi^+}$  | $\infty$ | $-\infty$ |          |       |        |
|    | $5\pi/4$                      | 2        | -4        | +        | decr. | $\cup$ |
|    | $3\pi/2$                      | 1        | 0         | +        | min.  | $\cup$ |
|    | $7\pi/4$                      | 2        | 4         | +        | incr. | $\cup$ |
|    | $\lim_{x \rightarrow 2\pi^-}$ | $\infty$ | $\infty$  |          |       |        |



- 37  $-\frac{1}{2}\cos(2t) + C$     39  $\frac{1}{3}\sec^3 x + C$     41  $2\sin(\sqrt{x}) + C$     43  $-\frac{1}{5}\csc(5\theta) + C$   
 45  $2\sqrt{\sec x + 1} + C$     47  $\tan \theta - \sec \theta + C$     49 2    51  $(3 - \sqrt{3})/2$     53  $\infty$   
 55  $-\frac{3}{10}$  radians/sec    57 6    59  $1/2$     61  $2\pi$     63  $\sqrt{2}$     65  $\pi\sqrt{2}$

## Section 7.3

- 1  $\pi/3$     3  $-\pi/4$     5  $\pi/3$     7  $\sqrt{1-x^2}$     9  $\frac{\pi}{2} - x$   
 15  $\frac{5}{|5x-2|\sqrt{(5x-2)^2-1}}$     17  $\frac{2x}{\sqrt{1-x^4}}$     19  $\arcsin t + \frac{t}{\sqrt{1-t^2}}$     21  $\frac{1-x}{\sqrt{1-x^2}}$   
 23  $\arcsin x$     25  $-\frac{1}{2\sqrt{x}(x+1)}$     27  $-\pi/2$     29 1    31  $\frac{1}{3}\arctan(x/3) + C$   
 33  $\arcsin(2x-1) + C$     35  $\operatorname{arcsec}(2x) + C$     37  $\frac{1}{2}\arcsin(x^2) + C$   
 39  $2\operatorname{arcsec}(\sqrt{x}) + C$  or  $2\arctan(\sqrt{x-1}) + C$     41  $\frac{1}{2}(\arcsin x)^2 + C$     43  $\pi/3$   
 45  $\pi/6$     47  $\pi/a$     49  $\pi/2$

## Section 7.4

- 1  $x \sin x + \cos x + C$     3  $t^2 \sin t + 2t \cos t - 2 \sin t + C$   
 5  $-\frac{1}{2}t \cos(2t-1) + \frac{1}{4} \sin(2t-1) + C$   
 7  $-\frac{x^2}{4} \cos(4x) + \frac{x}{8} \sin(4x) + \frac{1}{32} \cos(4x) + C$   
 9  $\frac{1}{4}x^4 \operatorname{arcsec} x - \frac{1}{12}(x^2-1)^{3/2} - \frac{1}{4}\sqrt{x^2-1} + C$   
 11  $-2\sqrt{x} \cos(\sqrt{x}) + 2 \sin(\sqrt{x}) + C$     13  $(x+1) \arctan(\sqrt{x}) - \sqrt{x} + C$   
 15  $x^2\sqrt{x^2-1} - \frac{2}{3}(x^2-1)^{3/2} + C$  or  $\frac{1}{3}(x^2-1)^{3/2} + \sqrt{x^2-1} + C$  or  $(\frac{1}{3}x^2 + \frac{2}{3})\sqrt{x^2-1} + C$   
 17  $-\frac{1}{4}x \cos(2x) + \frac{1}{8} \sin(2x) + C$  or  $\frac{1}{2}x \sin^2 x - \frac{1}{4}x + \frac{1}{4} \sin x \cos x + C$   
 19  $\frac{2}{3} \sin^3 \theta + C$  or  $\frac{1}{3}[\cos \theta \sin(2\theta) - 2 \sin \theta (\cos(2\theta))] + C$   
 21  $\frac{1}{8} \cos(4x) - \frac{1}{12} \cos(6x) + C$  or  $\frac{5}{24} \sin x \sin(5x) + \frac{1}{24} \cos x \cos(5x) + C$   
 23  $\frac{1}{2} \sin(t^2) - \frac{1}{2}t^2 \cos(t^2) + C$     25  $\frac{1}{x} \cos\left(\frac{1}{x}\right) - \sin\left(\frac{1}{x}\right) + C$   
 27  $(t^2+4)^{3/2} \left(\frac{t^2}{5} - \frac{8}{15}\right) + C$  or  $\frac{1}{3}t^2(t^2+4)^{3/2} - \frac{2}{15}(t^2+4)^{5/2} + C$  or  $-\frac{4}{3}(t^2+4)^{3/2} + \frac{1}{5}(t^2+4)^{5/2} + C$     29  $(\pi/2) - 1$     31  $\pi/2$     33  $1/2$   
 35  $\frac{2}{3}\pi - \frac{\sqrt{3}}{2}$

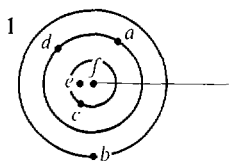
## Section 7.5

- 1  $(1/\cos t) + \cos t + C$     3  $-\cot x - x + C$     5  $\frac{1}{4} \cos^3 x \sin x + \frac{3}{8} \sin x \cos x + \frac{3}{8} x + C$   
 or  $\frac{3}{8} x + \frac{1}{4} \sin(2x) + \frac{1}{32} \sin(4x) + C$     7  $\frac{1}{6} \tan^6 x + \frac{1}{4} \tan^4 x + C$  or  $\frac{1}{6} \sec^6 x - \frac{1}{4} \sec^4 x + C$     9  $\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$     11  $-\frac{1}{3} \cot^3 \theta + C$   
 13  $\frac{2}{9} (\tan x)^{9/2} + \frac{2}{5} (\tan x)^{5/2} + C$     15  $\tan \theta - \cot \theta + C$  or  $-2 \cot(2\theta) + C$   
 17  $-\cot \theta + \csc \theta + C$     19  $4/3$     21  $\frac{\pi}{4} - \frac{2}{3}$     23  $3\pi/8$   
 25  $\cos(\sqrt{x}) \sin(\sqrt{x}) + \sqrt{x} + C$  or  $\frac{1}{2} \sin(2\sqrt{x}) + \sqrt{x} + C$   
 27  $-x \cos x + \frac{1}{3} x \cos^3 x + \frac{2}{3} \sin x + \frac{1}{9} \sin^3 x + C$   
 29  $\frac{1}{3} x \sin^3 x + \frac{1}{3} \cos x - \frac{1}{9} \cos^3 x + C$     31  $\frac{1}{9} \tan^9 \theta + \frac{2}{7} \tan^7 \theta + \frac{1}{5} \tan^5 \theta + C$   
 33  $\frac{1}{2} \sec x \tan x + \frac{1}{2} \int \sec x dx$     35  $\frac{1}{2} \sec x \tan x - \frac{1}{2} \int \sec x dx$   
 37  $-\frac{1}{4} \csc^4 x \cos x + \frac{1}{8} \csc^2 x \cos x - \frac{1}{8} \int \csc x dx$     39  $\int \sec x dx + \int \csc x dx$   
 41  $\int x^n \sin x dx = -x^n \cos x + n \int x^{n-1} \cos x dx = -x^n \cos x + nx^{n-1} \sin x - n(n-1) \int x^{n-2} \sin x dx$   
 43 (a)  $\pi^2/16$  (b)  $\pi^2/4$

## Section 7.6

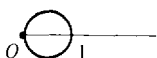
- 1  $\frac{1}{2} \arcsin(2x) + C$     3  $\frac{1}{3}(9+x^2)^{3/2} - 9(9+x^2)^{1/2} + C$  or  $\sqrt{9+x^2}(\frac{1}{3}x^2 - 6) + C$   
 5  $\frac{x}{4\sqrt{4-x^2}} + C$     7  $\arccos\left(\frac{\cos \theta}{\sqrt{2}}\right) + C$  or  $-\arcsin\left(\frac{\cos \theta}{\sqrt{2}}\right) + C$   
 9  $-\frac{1}{x} - \arctan x + C$     11  $2 \arcsin(x/2) - \frac{1}{2}x\sqrt{4-x^2} + C$   
 13  $\frac{1}{8} \arcsin x - \frac{1}{4}x(1-x^2)^{3/2} + \frac{1}{8}x\sqrt{1-x^2} + C$   
 15  $2 \arcsin\left(\frac{x-2}{2}\right) + \frac{1}{2}(x-2)\sqrt{4x-x^2} + C$     17  $\frac{1}{48}(4x^2-1)^{3/2} + \frac{1}{16}\sqrt{4x^2-1} + C$   
 or  $\sqrt{4x^2-1}(\frac{1}{24} - \frac{1}{12}x^2) + C$     19  $\frac{1}{3}(a^2-x^2)^{3/2} - a^2\sqrt{a^2-x^2} + C$   
 or  $-\frac{1}{3}\sqrt{a^2-x^2}(2a^2+x^2) + C$     21  $\frac{1}{2}\sqrt{x^4-1} - \frac{1}{2} \arccsc(x^2) + C$   
 23  $\sqrt{a^2+x^2} + \frac{a^2}{\sqrt{a^2+x^2}} + C$     25  $\pi/2$     27  $1/9$     29  $\infty$   
 31  $(\frac{1}{2}x^2 - \frac{1}{4}) \arcsin x + \frac{1}{4}x\sqrt{1-x^2} + C$     33  $\frac{1}{3}x^3 \arcsin x + \frac{1}{3}\sqrt{1-x^2} - \frac{1}{9}(1-x^2)^{3/2} + C$   
 35  $-\frac{\arcsin x}{2x^2} - \frac{\sqrt{1-x^2}}{2x} + C$     37  $\frac{2\pi^2}{3\sqrt{3}} + \frac{\pi}{2}$

## Section 7.7

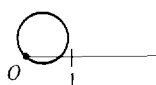


- 3  $r = \frac{2}{\sin \theta - 5 \cos \theta}$     5  $r = 2 \sec \theta$     7  $r \sin \theta = r^2 \cos^2 \theta + 1$   
 9  $r \sin \theta = 3r^2 \cos^2 \theta - 2r \cos \theta$  or  $r = \frac{\sin \theta + 2 \cos \theta}{3 \cos^2 \theta}$     11  $r = \sin \theta$

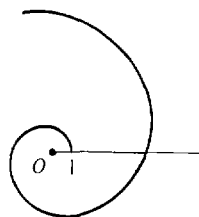
13



15

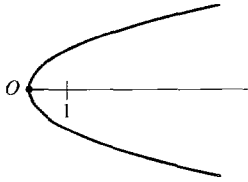


17





19

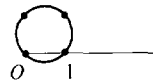


21  $x = \sin(3\theta) \cos \theta$ ,  $y = \sin(3\theta) \sin \theta$       23  $x = \theta^2 \cos \theta$ ,  $y = \theta^2 \sin \theta$

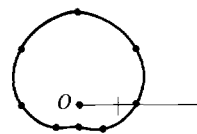
## Section 7.8

1  $\theta$       3  $-\cot \theta$       5  $-\frac{1 + \cot \theta}{\sin \theta}$

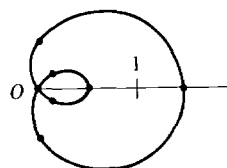
| 7 | $\theta$ | $r$         | $dr/d\theta$ | $\tan \psi$ | $ r $     |
|---|----------|-------------|--------------|-------------|-----------|
|   | 0        | 1           | 1            | 1           | incr.     |
|   | $\pi/4$  | $\sqrt{2}$  | 0            |             | max.      |
|   | $\pi/2$  | 1           | -1           | -1          | decr.     |
|   | $3\pi/4$ | 0           | $-\sqrt{2}$  |             | crosses O |
|   | $\pi$    | -1          | -1           | 1           | incr.     |
|   | $5\pi/4$ | $-\sqrt{2}$ | 0            |             | max.      |
|   | $3\pi/2$ | -1          | 1            | -1          | decr.     |
|   | $7\pi/4$ | 0           | $\sqrt{2}$   |             | crosses O |
|   | $2\pi$   | 1           | 1            | 1           | incr.     |



| 9 | $\theta$ | $r$ | $dr/d\theta$ | $\tan \psi$ | $ r $ |
|---|----------|-----|--------------|-------------|-------|
|   | 0        | 1.5 | 1            | 1.5         | incr. |
|   | $\pi/4$  | 2.2 | 0.7          | 3.1         | incr. |
|   | $\pi/2$  | 2.5 | 0            |             | max.  |
|   | $3\pi/4$ | 2.2 | -0.7         | -3.1        | decr. |
|   | $\pi$    | 1.5 | -1           | -1.5        | decr. |
|   | $5\pi/4$ | 0.8 | -0.7         | -1.1        | decr. |
|   | $3\pi/2$ | 0.5 | 0            |             | min.  |
|   | $7\pi/4$ | 0.8 | 0.7          | 1.1         | incr. |
|   | $2\pi$   | 1.5 | 1            | 1.5         | incr. |

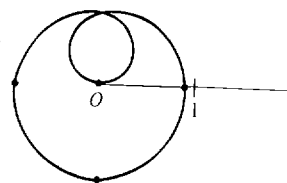


| 11 | $\theta$ | $r$  | $dr/d\theta$ | $\tan \psi$ | $ r $     |
|----|----------|------|--------------|-------------|-----------|
|    | 0        | 1.5  | 0            |             | max.      |
|    | $\pi/2$  | 0.5  | -1           | -0.5        | decr.     |
|    | $2\pi/3$ | 0    | -0.9         |             | crosses O |
|    | $3\pi/4$ | -0.2 | -0.7         | 0.3         | incr.     |
|    | $\pi$    | -0.5 | 0            |             | max.      |
|    | $5\pi/4$ | -0.2 | 0.7          | -0.3        | decr.     |
|    | $4\pi/3$ | 0    | 0.9          |             | crosses O |
|    | $3\pi/2$ | 0.5  | 1            | 0.5         | incr.     |
|    | $2\pi$   | 1.5  | 0            |             | max.      |



13

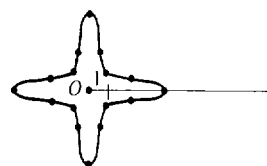
| $\theta$ | $r$  | $dr/d\theta$   | $\tan \psi$ | $ r $       |
|----------|------|----------------|-------------|-------------|
| 0        | 0    | $\frac{1}{3}$  |             | crosses $O$ |
| $\pi$    | 0.9  | $\frac{1}{6}$  | 5           | incr.       |
| $1.5\pi$ | 1    | 0              |             | max.        |
| $2\pi$   | 0.9  | $-\frac{1}{6}$ | -5          | decr.       |
| $3\pi$   | 0    | $-\frac{1}{3}$ |             | crosses $O$ |
| $4\pi$   | -0.9 | $-\frac{1}{6}$ | 5           | incr.       |
| $4.5\pi$ | -1   | 0              |             | max.        |
| $5\pi$   | -0.9 | $\frac{1}{6}$  | -5          | decr.       |
| $6\pi$   | 0    | $\frac{1}{3}$  |             | crosses $O$ |



15

| $\theta$ | $r$ | $dr/d\theta$ | $\tan \psi$ | $ r $ |
|----------|-----|--------------|-------------|-------|
| 0        | 4   | 0            |             | max.  |
| $\pi/8$  | 2.5 | -6           | -0.4        | decr. |
| $2\pi/8$ | 1   | 0            |             | min.  |
| $3\pi/8$ | 2.5 | 6            | 0.4         | incr. |
| $4\pi/8$ | 4   | 0            |             | max.  |
| $5\pi/8$ | 2.5 | -6           | -0.4        | decr. |
| $6\pi/8$ | 1   | 0            |             | min.  |
| $7\pi/8$ | 2.5 | 6            | 0.4         | incr. |
| $\pi$    | 4   | 0            |             | max.  |

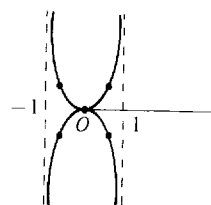
repeats



17

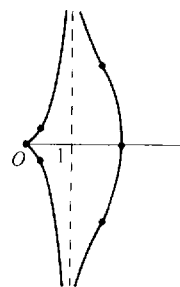
| $\theta$                            | $r$       | $dr/d\theta$ | $\tan \psi$    | $ r $             |
|-------------------------------------|-----------|--------------|----------------|-------------------|
| 0                                   | 0         | 1            |                | crosses $O$       |
| $\pi/4$                             | 1         | 2            | $\frac{1}{2}$  | incr.             |
| $\lim_{\theta \rightarrow \pi/2^-}$ | $\infty$  |              |                | $x \rightarrow 1$ |
| $\lim_{\theta \rightarrow \pi/2^+}$ | $-\infty$ |              |                | $x \rightarrow 1$ |
| $3\pi/4$                            | -1        | 2            | $-\frac{1}{2}$ | decr.             |
| $\pi$                               | 0         | 1            |                | crosses $O$       |

repeats, with  $x \rightarrow -1$  as  $\theta \rightarrow 3\pi/2$

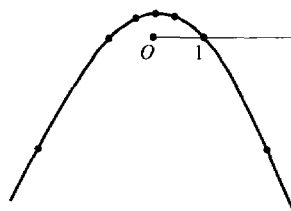


19

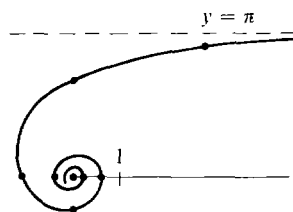
| $\theta$                             | $r$       | $dr/d\theta$ | $\tan \psi$ | $ r $             |
|--------------------------------------|-----------|--------------|-------------|-------------------|
| 0                                    | 2         | 0            |             | min.              |
| $\pi/4$                              | 2.4       | 2.4          | 1           | incr.             |
| $\lim_{\theta \rightarrow \pi/2^-}$  | $\infty$  |              |             | $x \rightarrow 1$ |
| $\lim_{\theta \rightarrow \pi/2^+}$  | $-\infty$ |              |             | $x \rightarrow 1$ |
| $3\pi/4$                             | -0.4      | 0.4          | -1          | decr.             |
| $\pi$                                | 0         | 0            |             | cusp at $O$       |
| $5\pi/4$                             | -0.4      | -0.4         | 1           | incr.             |
| $\lim_{\theta \rightarrow 3\pi/2^-}$ | $-\infty$ |              |             | $x \rightarrow 1$ |
| $\lim_{\theta \rightarrow 3\pi/2^+}$ | $\infty$  |              |             | $x \rightarrow 1$ |
| $7\pi/4$                             | 2.4       | -2.4         | -1          | decr.             |
| $2\pi$                               | 2         | 0            |             | min.              |



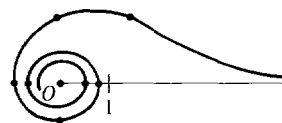
| 21 | $\theta$                             | $r$      | $dr/d\theta$ | $\tan \psi$ | $ r $                   |
|----|--------------------------------------|----------|--------------|-------------|-------------------------|
|    | 0                                    | 1        | -1           | -1          | decr.                   |
|    | $\pi/4$                              | 0.6      | -0.3         | -2          | decr.                   |
|    | $\pi/2$                              | 0.5      | 0            |             | min.                    |
|    | $3\pi/4$                             | 0.6      | 0.3          | 2           | incr.                   |
|    | $\pi$                                | 1        | 1            | 1           | incr.                   |
|    | $5\pi/4$                             | 3.4      | 8            | 0.4         | incr.                   |
|    | $\lim_{\theta \rightarrow 3\pi/2^-}$ | $\infty$ |              |             | $x \rightarrow -\infty$ |
|    | $\lim_{\theta \rightarrow 3\pi/2^+}$ | $\infty$ |              |             | $x \rightarrow \infty$  |
|    | $7\pi/4$                             | 3.4      | -8           | -0.4        | decr.                   |
|    | $2\pi$                               | 1        | -1           | -1          | decr.                   |



| 23 | $\theta$                        | $r$           | $dr/d\theta$       | $\tan \psi$ | $ r $               |
|----|---------------------------------|---------------|--------------------|-------------|---------------------|
|    | $\lim_{\theta \rightarrow 0^+}$ | $\infty$      |                    |             | $y \rightarrow \pi$ |
|    | $\pi/4$                         | 4             | $-16/\pi$          | $-\pi/4$    | decr.               |
|    | $\pi/2$                         | 2             | $-4/\pi$           | $-\pi/2$    | decr.               |
|    | $\pi$                           | 1             | $-1/\pi$           | $-\pi$      | decr.               |
|    | $3\pi/2$                        | $\frac{2}{3}$ | $-\frac{4}{3}\pi$  | $-3\pi/2$   | decr.               |
|    | $2\pi$                          | $\frac{1}{2}$ | $-\frac{1}{4}\pi$  | $-2\pi$     | decr.               |
|    | $3\pi$                          | $\frac{1}{3}$ | $-\frac{1}{6}\pi$  | $-3\pi$     | decr.               |
|    | $4\pi$                          | $\frac{1}{4}$ | $-\frac{1}{16}\pi$ | $-4\pi$     | decr.               |



| 25 | $\theta$                        | $r$                  | $dr/d\theta$      | $\tan \psi$ | $ r $             |
|----|---------------------------------|----------------------|-------------------|-------------|-------------------|
|    | $\lim_{\theta \rightarrow 0^+}$ | $\infty$             |                   |             | $y \rightarrow O$ |
|    | $\pi/4$                         | 2                    | $-4/\pi$          | $-\pi/2$    | decr.             |
|    | $\pi/2$                         | $\sqrt{2}$           | $-\sqrt{2}/\pi$   | $-\pi$      | decr.             |
|    | $\pi$                           | 1                    | $-\frac{1}{2}\pi$ | $-2\pi$     | decr.             |
|    | $3\pi/2$                        | $\sqrt{\frac{1}{3}}$ | -                 | -           | decr.             |
|    | $2\pi$                          | $1/\sqrt{2}$         | -                 | -           | decr.             |
|    | $3\pi$                          | $1/\sqrt{3}$         | -                 | -           | decr.             |
|    | $4\pi$                          | $\frac{1}{2}$        | -                 | -           | decr.             |



27  $x$ : max of  $\sqrt{32/27}$  at  $\theta = \arcsin(1/\sqrt{3})$ ,  $2\pi - \arcsin(1/\sqrt{3})$ ; min of  $-\sqrt{32/27}$  at  $\theta = \pi + \arcsin(1/\sqrt{3})$ ,  $\pi - \arcsin(1/\sqrt{3})$

$y$ : max of 2 at  $\theta = \pi/2$ ; min of -2 at  $\theta = 3\pi/2$

29  $x$ : max of  $\frac{5}{2}$  at  $\theta = 0$ ; min of  $-\frac{9}{16}$  at  $\theta = \arccos(-\frac{3}{4})$ ,  $2\pi - \arccos(-\frac{3}{4})$

$y$ : max of  $\sin \theta_0 (\frac{3}{2} + \cos \theta_0)$  at  $\theta = \theta_0 = \arccos(\frac{\sqrt{41}-3}{8})$ ; min of  $-\sin \theta_0 (\frac{3}{2} + \cos \theta_0)$

at  $\theta = 2\pi - \arccos(\frac{\sqrt{41}-3}{8})$

31  $(\frac{1}{2}, \frac{\pi}{12})$ ,  $(\frac{1}{2}, \frac{5\pi}{12})$ ,  $(\frac{1}{2}, \frac{13\pi}{12})$ ,  $(\frac{1}{2}, \frac{17\pi}{12})$ ,  $(\frac{1}{2}, \frac{7\pi}{12})$ ,  $(\frac{1}{2}, \frac{11\pi}{12})$ ,  $(\frac{1}{2}, \frac{19\pi}{12})$ ,  $(\frac{1}{2}, \frac{23\pi}{12})$

## Section 7.9

1  $\pi a^2$     3 1    5  $2 - \frac{\pi}{2}$     7  $\frac{3\pi}{16}$     9  $\frac{\pi}{4} - \frac{3\sqrt{3}}{8}$     11  $\frac{\pi}{12} + \frac{\sqrt{3}}{16}$     13  $\frac{3}{2} - \frac{\pi}{4}$

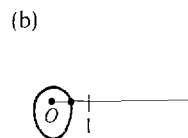
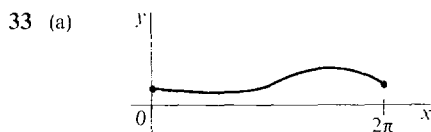
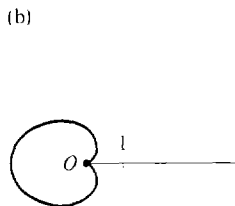
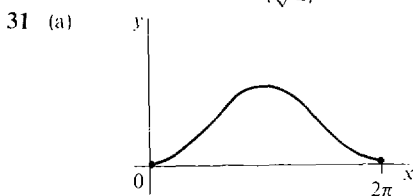
15  $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$     17  $\pi^2$     19  $\frac{7\pi}{12} - \sqrt{3}$     21  $\frac{1}{2} \int_a^b (g(\theta))^2 - (f(\theta))^2 d\theta$

## Section 7.10

$$\begin{array}{llll}
 1 & 14\pi & 3 & 2 \\
 5 & \frac{2048 - 493\sqrt{17}}{15} \sim 1.02 & 7 & 4 \\
 9 & 4 & & \\
 11 & \int_0^{2\pi} \sqrt{\sin^2(2\theta) + 4\cos^2(2\theta)} d\theta & 13 & \int_0^b \sqrt{\theta^2 + 1} d\theta \\
 17 & \pi\sqrt{a^2 + b^2} \left( a + \frac{b\pi}{2} \right) & & \\
 19 & \pi\sqrt{2} & 21 & \frac{\pi}{5}(8 - \sqrt{2})
 \end{array}$$

## Extra Problems for Chapter 7

$$\begin{array}{llll}
 1 & 1 + \cos x & 3 & \frac{\cos \theta}{2\sqrt{\theta}} - \sqrt{\theta} \sin \theta \\
 5 & 4/3 & 7 & -\sin(\cos \theta) + C \\
 9 & 8 & & \\
 11 & -600/29 \text{ radians/hr} & 13 & -\frac{1}{2\sqrt{x}\sqrt{1-x}} \\
 15 & \frac{1}{1+t^2} - 1 \text{ or } -\frac{t^2}{1+t^2} & 17 & 1 \\
 19 & \operatorname{arcsec}(x-1) + C & 21 & \frac{1}{3}(x^2+1)^{3/2} - (x^2+1)^{1/2} + C = x^2(x^2+1)^{1/2} \\
 & -\frac{2}{3}(x^2+1)^{3/2} + C = (x^2+1)^{1/2}(\frac{1}{3}x^2 - \frac{2}{3}) + C & 23 & 2\sin 1 + 2\cos 1 - 2 \\
 25 & \pi\left(1 - \frac{\pi}{4}\right) & 27 & \frac{\sec^9 \theta}{9} - \frac{2\sec^7 \theta}{7} + \frac{\sec^5 \theta}{5} + C \\
 29 & -\frac{\sqrt{2-x^2}}{x} - \arcsin\left(\frac{x}{\sqrt{2}}\right) + C
 \end{array}$$



$$35 \ 19\pi/8 \quad 37 \ (3\pi/2) - 4 \quad 39 \ 4\sqrt{2}\pi/5$$

## Section 8.1

$$9 \ 0 \quad 11 \ \infty \quad 13 \ 0 \quad 15 \ \infty \quad 17 \ 27 \quad 19 \ \infty \quad 21 \ 1/6 \quad 23 \ 1/(2\pi)$$

## Section 8.2

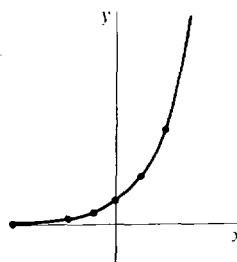
$$\begin{array}{llll}
 1 & x & 3 & -x^2 \\
 5 & x/y^2 & 7 & \frac{2}{3} \log_b x \\
 9 & \frac{1}{2}(\log_b x + \log_b y) & 11 & 2 \\
 13 & 1/2 & & \\
 15 & x = \log_5 3 & 17 & x = -\frac{2}{3} \\
 19 & x = 5^{1/3} & 21 & x = 2, x = 3 \\
 23 & x = 2^{1/(1-\log_3 2)} = 3^{1/(\log_2 3 - 1)} & 25 & -\infty \\
 27 & -\infty & 29 & -\infty
 \end{array}$$

## Section 8.3

$$\begin{array}{llll}
 1 & 3e^{3x+4} & 3 & -4^{-x} \ln 4 \\
 5 & e^t \cos(e^t) & 7 & 2^{t^2+1} t \ln 2 \\
 9 & e^t e^{(e^t)} & & \\
 11 & \frac{\ln 3}{2\sqrt{x}} 3^{\sqrt{x}} & 13 & -\frac{e^{x+y}}{e^{x+y} + \sin y} \\
 15 & \frac{1}{2\sqrt{e^t(t^{-1} - t^{-2})}} \text{ or } \frac{t^2}{2\sqrt{e^t(t-1)}} & 17 & \infty \\
 19 & 1 & 21 & -\infty \\
 23 & e^c & 25 & e^c
 \end{array}$$

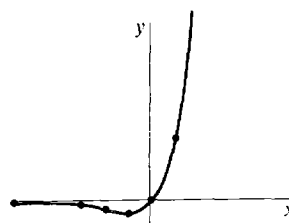
27

| $x$                            | $f(x)$   | $f'(x)$     | $f''(x)$ |       |   |
|--------------------------------|----------|-------------|----------|-------|---|
| $\lim_{x \rightarrow -\infty}$ | 0        | 0           |          |       |   |
| -2                             | 1/4      | $(\ln 2)/4$ | +        | incr. | ∪ |
| -1                             | 1/2      | $(\ln 2)/2$ | +        | incr. | ∪ |
| 0                              | 1        | $\ln 2$     | +        | incr. | ∪ |
| 1                              | 2        | $2 \ln 2$   | +        | incr. | ∪ |
| 2                              | 4        | $4 \ln 2$   | +        | incr. | ∪ |
| $\lim_{x \rightarrow \infty}$  | $\infty$ | $\infty$    |          |       |   |



29

| $x$                            | $f(x)$     | $f'(x)$    | $f''(x)$ |       |       |
|--------------------------------|------------|------------|----------|-------|-------|
| $\lim_{x \rightarrow -\infty}$ | 0          | 0          |          |       |       |
| -3                             | $-3e^{-3}$ | $-2e^{-3}$ | -        | decr. | ∩     |
| -2                             | $-2e^{-2}$ | $-e^{-2}$  | 0        | decr. | infl. |
| -1                             | $-e^{-1}$  | 0          | +        | min.  | ∪     |
| 0                              | 0          | 1          | +        | incr. | ∪     |
| 1                              | $e$        | $2e$       | +        | incr. | ∪     |
| $\lim_{x \rightarrow \infty}$  | $\infty$   | $\infty$   |          |       |       |



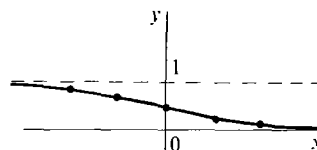
31

| $x$                            | $f(x)$     | $f'(x)$               | $f''(x)$ |        |       |
|--------------------------------|------------|-----------------------|----------|--------|-------|
| $\lim_{x \rightarrow -\infty}$ | 0          | 0                     |          |        |       |
| -2                             | $e^{-8}$   | $12e^{-8}$            | +        | incr.  | ∪     |
| -1                             | $e^{-1}$   | $3e^{-1}$             | +        | incr.  | ∪     |
| $-(\frac{2}{3})^{1/3}$         | $e^{-2/3}$ | +                     | 0        | incr.  | infl. |
| $-\frac{1}{2}$                 | $e^{-1/8}$ | $\frac{3}{4}e^{-1/8}$ | -        | incr.  | ∩     |
| 0                              | 1          | 0                     | 0        | horiz. | infl. |
| 1                              | $e$        | $3e$                  | +        | incr.  | ∪     |
| $\lim_{x \rightarrow \infty}$  | $\infty$   | $\infty$              |          |        |       |



33

| $x$                            | $f(x)$ | $f'(x)$ | $f''(x)$ |       |       |
|--------------------------------|--------|---------|----------|-------|-------|
| $\lim_{x \rightarrow -\infty}$ | 1      | 0       |          |       |       |
| -2                             | 0.88   | -0.1    | -        | decr. | ∩     |
| -1                             | 0.73   | -0.2    | -        | decr. | ∩     |
| 0                              | 0.5    | -0.25   | 0        | decr. | infl. |
| 1                              | 0.27   | -0.2    | +        | decr. | ∪     |
| 2                              | 0.12   | -0.1    | +        | decr. | ∪     |
| $\lim_{x \rightarrow \infty}$  | 0      | 0       |          |       |       |



35  $\frac{1}{2}e^{2x} + C$     37  $-\frac{1}{2}e^{-x^2} + C$     39  $\frac{1}{3}(1 + e^{2x})^{3/2} + C$     41  $xe^x - e^x + C$

43  $\frac{1}{2}e^x(\sin x - \cos x) + C$     45  $\frac{e^{10} - 1}{5}$     47  $\infty$

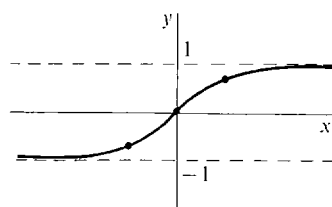
49  $\frac{1}{r^2}$  if  $r > 0$ ,  $\infty$  if  $r \leq 0$     51 (a)  $\frac{\pi}{2}(e^2 - 1)$  (b)  $2\pi$     53  $\sqrt{2}(e^{2\pi} - 1)$

## Section 8.4

1  $3 \cosh(3x)$     3  $-\tanh x \operatorname{sech} x$     5 1    7 0

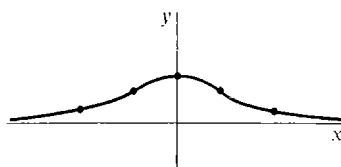
9

| $x$                            | $f(x)$ | $f'(x)$ | $f''(x)$ |       |        |
|--------------------------------|--------|---------|----------|-------|--------|
| $\lim_{x \rightarrow -\infty}$ | -1     | 0       |          |       |        |
| -1                             | -0.76  | 0.4     | +        | incr. | $\cup$ |
| 0                              | 0      | 1       | 0        | incr. | infl.  |
| 1                              | 0.76   | 0.4     | -        | incr. | $\cap$ |
| $\lim_{x \rightarrow \infty}$  | 1      | 0       |          |       |        |



11

| $x$                            | $f(x)$ | $f'(x)$ | $f''(x)$ |       |        |
|--------------------------------|--------|---------|----------|-------|--------|
| $\lim_{x \rightarrow -\infty}$ | 0      | 0       |          |       |        |
| -2                             | 0.27   | 0.26    | +        | incr. | $\cup$ |
| -0.88                          | 0.71   | 0.5     | 0        | incr. | infl.  |
| 0                              | 1      | 0       | -        | max.  | $\cap$ |
| 0.88                           | 0.71   | -0.5    | 0        | decr. | infl.  |
| 2                              | 0.27   | -0.26   | +        | decr. | $\cup$ |
| $\lim_{x \rightarrow \infty}$  | 0      | 0       |          |       |        |



13  $\frac{\sinh^2 x}{2} + C$  or  $\frac{\cosh^2 x}{2} + C$     15  $x \cosh x - \sinh x + C$

17  $\frac{1}{2}x \sinh x \cosh x + \frac{1}{4}x^2 - \frac{1}{4}\sinh^2 x + C$  or  $\frac{1}{4}x \sinh(2x) - \frac{1}{8}\cosh(2x) + \frac{1}{4}x^2 + C$     19  $\infty$

23 (a)  $\frac{\pi}{8}(4 + e^2 - e^{-2})$  (b)  $2\pi(1 - e^{-1})$     25  $50,000(e^2 - 1) \sim 320,000$  dollars

27 (a)  $10^6(90e^{0.2} - 110) \sim -74,000$  dollars (b)  $10^6(90e^{0.3} - 120) \sim 1,490,000$  dollars

## Section 8.5

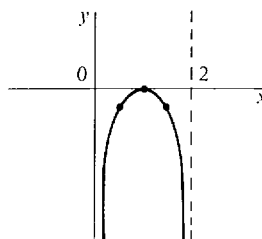
1  $(3/x)(\ln x)^2$     3  $-\tan x$     5  $\ln t$     7  $t^{-2}(1 - \ln t)$     9  $\frac{1}{x \ln 2}$

11  $\frac{1}{y} + \frac{3}{2(3y+1)}$     13  $y - \frac{y}{x}$     15  $\frac{1}{x+y-1}$     17 0    19 0    21  $\ln a$

23 1    25 1

27

| $x$                        | $f(x)$    | $f'(x)$        | $f''(x)$ |       |        |
|----------------------------|-----------|----------------|----------|-------|--------|
| $\lim_{x \rightarrow 0^+}$ | $-\infty$ | $\infty$       |          |       |        |
| $\frac{1}{2}$              | -0.3      | $\frac{4}{3}$  | -        | incr. | $\cup$ |
| 1                          | 0         | 0              | -        | max.  | $\cap$ |
| $\frac{3}{2}$              | 0.3       | $-\frac{4}{3}$ | -        | decr. | $\cap$ |
| $\lim_{x \rightarrow 2^-}$ | $-\infty$ | $-\infty$      |          |       |        |



29  $\frac{1}{2} \ln |2x + 3| + C$     31  $2x + 2 \ln |x - 1| + C$     33  $\frac{1}{2}(\ln x)^2 + C$     35  $\ln |\ln t| + C$

37  $\ln x(\ln(\ln x) - 1) + C$     39  $\frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^2} + C$

41  $x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x - 6x + C$     43  $\sin(\ln x) + C$     45  $10 - \ln(11)$

47 1    49  $-\infty$     51  $\infty$     53 (a)  $\pi(e-2)$  (b)  $\frac{\pi}{2}(e^2+1)$

55  $\sqrt{e^2+1} - \sqrt{2} + 1 + \ln\left(\frac{1+\sqrt{2}}{1+\sqrt{e^2+1}}\right)$

## Section 8.6

1  $y = -\frac{2}{x^2+C}$     3  $y = \pm\sqrt{\frac{2}{3}x^3+C}$     5  $y = -\ln\left(\frac{C-x^2}{2}\right)$

7  $y = \ln(e^x+C)$     9  $y = \frac{1}{\pm\sqrt{2\cos x+C}}$     11  $y = \frac{x^3}{3} + \frac{x^2}{2} + Ax + B$

13  $y = Ax + B$     15  $y = A \sinh(\sqrt{3}x) + B \cosh(\sqrt{3}x)$  or  $y = Ae^{\sqrt{3}x} + Be^{-\sqrt{3}x}$

17  $y = 10^7 e^{(0.01)t}$     19  $\frac{6 \ln 10}{\ln 2} \text{ min} \sim 19.93 \text{ min}$     21  $10^5 \times (1.15)^4 \sim 174,900$

23  $t = \frac{\ln(0.5)}{\ln(0.9)} \sim 6.576 \text{ years}$     25  $y = e^t + e^{-t}$     27  $y = \cos t - 2 \sin t$

29  $y = 10^{6.5} \sqrt{t}$     31  $y = \frac{10^8}{1+9e^{-t}}$     33 (b)  $y_0 = L/2$

35 (a)  $y = 10^7 - 6 \times 10^6 2^{-t}$  (b)  $y = 10^7 + 3 \times 10^6 3^{-t}$

37  $y^2 - x^2 = C$  or  $y = \pm\sqrt{x^2+C}$

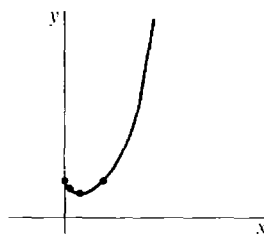
## Section 8.7

1  $\left(\frac{3x-2}{4x+3}\right) \left[\frac{3}{3x-2} - \frac{4}{4x+3}\right]$

3  $\frac{(x^2+1)\sqrt{3x+4}}{(2x-3)\sqrt{x^2-4}} \left[\frac{2x}{x^2+1} + \frac{3}{2(3x+4)} - \frac{2}{2x-3} - \frac{x}{x^2-4}\right]$

5  $(x-1)^{x^2+1} \left[2x \ln(x-1) + \frac{x^2+1}{x-1}\right]$     7  $e^{(e^x)}e^x$     9  $\sqrt[3]{t} \left[\frac{1}{t^2} - \frac{1}{t^2} \ln t\right]$

| 11 | $x$                           | $f(x)$      | $f'(x)$   | $f''(x)$ |       |   |
|----|-------------------------------|-------------|-----------|----------|-------|---|
|    | $\lim_{x \rightarrow 0^+}$    | 1           | $-\infty$ |          |       |   |
|    | 1/10                          | $10^{-0.1}$ | -         | +        | decr. | ∪ |
|    | 1/e                           | $e^{-1/e}$  | 0         | +        | min.  | ∪ |
|    | 1                             | 1           | 1         | +        | incr. | ∪ |
|    | $\lim_{x \rightarrow \infty}$ | $\infty$    | $\infty$  |          |       |   |



15  $\ln|\sin \theta| + C$     17  $\frac{1}{3} \ln|\sec(3\theta)| + C$

19  $\frac{1}{4} \sec^4 \theta \sin \theta + \frac{3}{8} \sec^2 \theta \sin \theta + \frac{3}{8} \ln|\sec \theta + \tan \theta| + C$     21  $\ln|\tan x| + C$

23  $\ln|\sec x + \tan x| - \sin x + C$     25  $\frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2} \ln|x + \sqrt{x^2-1}| + C$

27  $\frac{1}{2} \ln \left| \frac{x}{\sqrt{4-x^2}+2} \right| + C$  or  $\frac{1}{4} \ln \left| \frac{2-\sqrt{4-x^2}}{2+\sqrt{4-x^2}} \right| + C$     29  $\frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + C$

31  $\sqrt{x^2-1} - \operatorname{arcsec} x + C$  or  $\sqrt{x^2-1} - \arctan \sqrt{x^2-1} + C$

33  $\frac{x}{4}(x^2+1)^{3/2} - \frac{x}{8}(x^2+1)^{1/2} - \frac{1}{8} \ln(x + \sqrt{x^2+1}) + C$  or

$x(x^2+1)^{1/2} \left( \frac{1}{4}x^2 + \frac{1}{8} \right) - \frac{1}{8} \ln(x + \sqrt{x^2+1}) + C$

- 35  $-\frac{\arcsin x}{x} - \ln \left| \frac{1 + \sqrt{1 - x^2}}{x} \right| + C$  or  $-\frac{\arcsin x}{x} + \ln \left| \frac{1 - \sqrt{1 - x^2}}{x} \right| + C$   
 37  $x \operatorname{arccsc} x + \ln(x + \sqrt{x^2 - 1}) + C, x \geq 1; x \operatorname{arccsc} x - \ln|x + \sqrt{x^2 - 1}| + C, x \leq -1$   
 39  $\sqrt{5} + \frac{1}{2} \ln(2 + \sqrt{5}) \sim 2.96$     41  $\frac{1}{2}a\sqrt{1 + a^2} + \frac{1}{2} \ln(a + \sqrt{1 + a^2})$

## Section 8.8

- 1  $\frac{1}{2} \ln|2x - 7| + C$     3  $\frac{1}{12} \ln|x - 4| - \frac{1}{12} \ln|x| + C$     5  $-\frac{1}{5} \ln|x - 1| + \frac{1}{5} \ln|x + 4| + C$   
 7  $\frac{x^2}{2} - 3x + \frac{1}{4} \ln|x| + \frac{5}{4} \ln|x + 4| + C$     9  $-\frac{1}{2(x + 1)^2} + C$   
 11  $\ln|x - 1| - \frac{1}{x - 1} - \frac{1}{2(x - 1)^2} + C$     13  $-\ln|x| + \frac{1}{2} \ln|x + 1| + \frac{1}{2} \ln|x - 1| + C$   
 15  $x + \ln|x| - \frac{1}{x} + C$     17  $x - 4 \arctan(x/4) + C$     19  $\frac{5}{4} \ln|x - 3| - \frac{1}{4} \ln|x + 1| + C$   
 21  $\frac{1}{3}x^3 - x + \arctan x + C$     23  $\arctan x - (1/\sqrt{2}) \arctan(x/\sqrt{2}) + C$   
 25  $-x^{-1} - \arctan x + C$     27  $\frac{1}{3} \ln|x + 1| - \frac{1}{6} \ln|x^2 - x + 1| + \frac{1}{\sqrt{3}} \arctan\left(\frac{2x - 1}{\sqrt{3}}\right) + C$   
 29  $\frac{x^2}{2} - \frac{3}{4} \ln|x + 1| + \frac{5}{4} \ln|x - 1| - \frac{1}{4} \ln|x^2 + 1| + \arctan x + C$   
 31  $\ln|x| - \frac{1}{2} \ln|x^2 + 1| - \frac{\arctan x}{x} + C$

## Section 8.9

- 1  $-3 \cos x + 4 \sin x + C$     3  $\frac{3}{4}(x^2 - 1)^{2/3} + C$     5  $\frac{1}{3}[(x + 2)^{3/2} + x^{3/2}] + C$   
 7  $2(1 + \sqrt{x}) - 2 \ln(1 + \sqrt{x}) + C$  or  $2\sqrt{x} - 2 \ln(1 + \sqrt{x}) + C$   
 9  $-(\sqrt{4x + 1}/x) + C$     11  $-2 \cos(\sqrt{x}) + C$   
 13  $\frac{2}{3}(x - 3)^{7/2} + \frac{12}{5}(x - 3)^{5/2} + 6(x - 3)^{3/2} + C$   
 15  $\frac{1}{3}[(3x + 4) \ln(3x + 4) - (3x + 4)] + C$  or  $\frac{1}{3}(3x + 4) \ln(3x + 4) - x + C$   
 17  $x \tan x - \ln|\sec x| - (x^2/2) + C$     19  $-\frac{1}{6} \cos(3x^2 + 1) + C$   
 21  $2x \ln|x| - 3x + (x + 1) \ln(x + 1) + C$     23  $\cos \theta - \cos \theta \ln(\cos \theta) + C$   
 25  $\ln(2 - \cos \theta) + C$     27  $\frac{1}{4}(e^x + 1)^4 + C$     29  $2 \sin(\sqrt{x}) - 2\sqrt{x} \cos(\sqrt{x}) + C$   
 31  $\ln(1 + \cosh x) + C$     33  $-\frac{\ln x}{1 + x} + \ln\left(\frac{x}{1 + x}\right) + C$   
 35  $\frac{4}{5}(1 - \cos x)^{5/2} - \frac{2}{7}(1 - \cos x)^{7/2} + C$     37  $\frac{1}{\sqrt{2}} \ln \left| \frac{x}{\sqrt{2} + \sqrt{2 + x^2}} \right| + C$   
 39  $4 \arcsin\left(\frac{\sqrt{x}}{2}\right) + \sqrt{x}\sqrt{4 - x} - \frac{1}{2}\sqrt{x}(4 - x)^{3/2} + C$  or  $\frac{1}{2}(x - 2)\sqrt{x}\sqrt{4 - x} + 2 \arcsin\left(\frac{x - 2}{2}\right) + C$   
 41  $\frac{1}{5}[(5x - 2) \arcsin(5x - 2) + \sqrt{1 - (5x - 2)^2}] + C$     43  $\frac{1}{2}e^x(\sin x + \cos x) + C$   
 45  $(x^3/3) - x + 2 \arctan x + C$     47  $\frac{1}{2}x^2 \operatorname{arcsec}(x^2) - \frac{1}{2} \ln(x^2 + \sqrt{x^4 - 1}) + C$   
 49  $2x \ln x + \frac{4x - 1}{8} \ln(4x - 1) - \frac{5}{2}x + C$     51  $2 \ln|2x + \sqrt{4x^2 - 1}| - \frac{\sqrt{4x^2 - 1}}{x} + C$   
 53  $(x + 1) \arctan(\sqrt{x}) - \sqrt{x} + C$     55  $\frac{1}{8} \ln|\sec(4x^2 + 7) + \tan(4x^2 + 7)| + C$   
 57  $-\frac{1}{243}\sqrt{1 - 9x^2}(2 + 9x^2) + C$     59  $(\sqrt{x^2 - 3}/3x) + C$   
 61  $3x^{2/3} \sin(\sqrt[3]{x}) + 6(\sqrt[3]{x}) \cos(\sqrt[3]{x}) - 6 \sin(\sqrt[3]{x}) + C$   
 63  $\frac{1}{3}x^3(1 - x^2)^{-3/2} + x(1 - x^2)^{-1/2} + C$     65  $\frac{x}{5}(\cos^2(\ln x) + \sin(2 \ln x) + 2) + C$



**Extra Problems for Chapter 8**

- 1  $-\infty$     3  $-e^{\cos \theta} \sin \theta$     5  $-3 \operatorname{csch}^3 x \coth x$     7  $-\frac{1}{\ln 3} \cos(3^x) + C$   
 9  $\frac{1}{2} e^x \sqrt{1 - e^{2x}} + \frac{1}{2} \arcsin(e^x) + C$  or  $\frac{1}{2} e^x \sqrt{1 - e^{2x}} - \frac{1}{2} \arccos(e^x) + C$   
 11  $\frac{1}{4} e^{2x} - \frac{1}{2} x + C$     13  $\frac{8x}{x^2 - 1}$     15  $\frac{3}{3x + 2} + \frac{5}{5x - 4} - \frac{2}{2x - 1} - \frac{2x}{x^2 + 1}$     17  $e^2$   
 19  $(1/a) \ln |x| - (b/a) \ln |a + bx| + C$     21  $\infty$     23  $y = \pm \sqrt{ax^2 + C}$   
 25  $P = CV^{-1/k}$     27 air temp.  $= 60^\circ$ ,  $y = 60 + 80 \cdot 2^{-t/10}$   
 29  $(4t + 1)^4 (t - 3)^{2t+1} \left\{ \ln [(t - 3)^2 (4t + 1)] + \frac{4t}{4t + 1} + \frac{2t + 1}{t - 3} \right\}$     31  $\ln |\cosh x| + C$   
 33  $\frac{2}{3}(x + 1)^{5/2} + C$     35  $\pi(2\sqrt{2} + \ln |\sqrt{2} + 1| - \ln |\sqrt{2} - 1|)$  or  $2\pi(\sqrt{2} + \ln |\sqrt{2} + 1|)$   
 37  $e^{0.03} \sim 1.03$ , error  $\leq 0.0005$     39  $\ln 6 \sim \frac{1}{2} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{12} = \frac{28}{15}$ , error  $\leq \frac{1}{2}$   
 41  $\bar{x} = \frac{2 \ln 2 - \frac{3}{4}}{2 \ln 2 - 1}$ ,  $\bar{y} = \frac{(\ln 2 - 1)^2}{2 \ln 2 - 1}$   
 43  $\pi[e\sqrt{1 + e^2} - \sqrt{2} + \ln(e + \sqrt{1 + e^2}) - \ln(1 + \sqrt{2})]$

**Section 9.1**

- 1  $2^{-n}$     3  $(-1)^n n$     5  $2 - (1/2^{n-1})$     7  $2^{2^{(n-1)}}$     9 diverges    11 1  
 13 0    15 diverges    17 diverges    19  $1/e$     21 0    23 -1    25 diverges  
 27 diverges    29 0

**Section 9.2**

- 1  $\frac{3}{2}(1 - (1/3)^n)$ , converges to  $\frac{3}{2}$     3  $4(1 - (3/4)^n)$ , converges to 4  
 5  $1 - \frac{1}{(n+1)!}$ , converges to 1    7  $1 - \frac{1}{n+1}$ , converges to 1  
 9  $n$  even:  $-\frac{n}{2}$ ,  $n$  odd:  $\frac{n}{2} + \frac{1}{2}$ , diverges    11  $1 - (n+1)^{-2}$ , converges to 1  
 13  $\frac{1}{2} \left( 1 - \frac{1}{2n+1} \right)$ , converges to  $\frac{1}{2}$     15 diverges because  $\lim_{n \rightarrow \infty} a_n = \frac{1}{2}$   
 17 diverges because for infinite  $H$ ,  $S_{2H} \not\approx S_{2 \cdot 2H}$   
 19 diverges because for infinite  $H$ ,  $S_{2H} \not\approx S_H$     21  $A: 2/3$ ,  $B: 1/3$   
 23  $A: 9/19$ ,  $B: 6/19$ ;  $C: 4/19$

**Section 9.3**

- 1  $1/42$     3  $11/4$     5  $5/48$     7  $216/5$     9  $80/9$     11  $\frac{543}{99}$   
 13  $492.315 + (41/999,000)$     15  $1/7$

**Section 9.4**

- 1 diverges    3 converges    5 converges    7 converges    9 diverges  
 11 diverges    13 converges    15 diverges    17 converges    19 converges  
 21 diverges    23 converges    25 converges    27 diverges    29 diverges  
 31 converges    33 diverges    35 converges    37 converges

**Section 9.5**

- 1 diverges    3 diverges    5 converges    7 diverges    9 diverges  
 11 diverges    13 converges    15 converges    17 converges    19 diverges  
 21 0.90    23 0.37

## Section 9.6

- 1 diverges    3 diverges    5 absolutely converges    7 diverges    9 diverges  
 11 diverges    13 conditionally converges    15 absolutely converges  
 17 conditionally converges    19 diverges    21 diverges    23 no information  
 25 converges    27 diverges    29 converges    31 converges    33 no information  
 35 converges    37 converges    39 converges    41 no information    43 converges

## Section 9.7

- 1 1    3 0    5  $e$     7  $4/e^2$     9 1    11 1    13 0    15  $1/3$     17  $\infty$   
 19  $\infty$     21 0    23  $\sqrt[3]{5}$     25 1    27  $(-1, 1)$     29  $(-1, 1]$     31  $[-1/2, 1/2)$   
 33  $(-\infty, \infty)$     35  $(-3, -1]$     37  $(-\infty, \infty)$     39  $(-1, 1)$     41  $(-5/4, 5/4)$   
 43  $(-\sqrt{5}, \sqrt{5})$     45  $(-\infty, \infty)$

## Section 9.8

- 1  $f'(x) = \sum_{n=1}^{\infty} n 10^n x^{n-1}, \int_0^x f(t) dt = \sum_{n=0}^{\infty} \frac{10^n}{n+1} x^{n+1}$   
 3  $f'(x) = \sum_{n=1}^{\infty} n^{-2} x^{n-1}, \int_0^x f(t) dt = \sum_{n=1}^{\infty} \frac{n^{-3}}{n+1} x^{n+1}$   
 5  $f'(x) = \sum_{n=1}^{\infty} (n+1)x^{n-1}, \int_0^x f(t) dt = \sum_{n=1}^{\infty} \frac{1}{n} x^{n+1}$   
 7  $f'(x) = \sum_{n=1}^{\infty} \frac{n!}{n^{n-1}} x^{n-1}, \int_0^x f(t) dt = \sum_{n=1}^{\infty} \frac{n!}{n^n(n+1)} x^{n+1}$   
 9  $f'(x) = \sum_{n=1}^{\infty} 2n x^{2n-1}, \int_0^x f(t) dt = \sum_{n=0}^{\infty} \frac{1}{2n+1} x^{2n+1}$     11  $\sum_{n=0}^{\infty} (-1)^n 3^n x^n, r = \frac{1}{3}$   
 13  $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n+1} x^{4n+2}}{2n+1}, r = \frac{1}{2}$     15  $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+1} x^{n+2}}{n+1}, r = \frac{1}{2}$   
 17  $\sum_{n=0}^{\infty} \frac{(-1)^n 4^n x^n}{n!}, r = \infty$     19  $\sum_{n=0}^{\infty} \frac{3^{2n+1} x^{2n+1}}{(2n+1)!}, r = \infty$   
 21  $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+1} x^{2n+3}}{(n+1)(2n+3)}, r = \frac{1}{\sqrt{2}}$     23  $\sum_{n=0}^{\infty} \frac{x^{3n+1}}{n!(3n+1)}, r = \infty$   
 25  $\sum_{n=0}^{\infty} \frac{x^{n+3}}{(n+1)(n+3)}, r = 1$     27  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(n+1)^2}, r = 1$   
 29  $\sum_{n=1}^{\infty} (-1)^{n+1} 2n x^{2n-1}, r = 1$     31  $\sum_{n=0}^{\infty} (-1)^n 2 \cdot x^{4n+1}, r = 1$   
 33  $\sum_{n=0}^{\infty} \frac{(-1)^n (1 + 2^{2n+1}) x^{2n+1}}{2n+1}, r = \frac{1}{2}$

## Section 9.9

- 1 0.18232    3 0.7788008    5 0.4854019    7 0.3293740    9 1.003009  
 11 1.098614    13  $f(x) = x + x^2 + \cdots + x^{n+1} + E, |E| \leq 2|x|^{n+2}, f(\frac{1}{2}) = 1$   
 15  $f(x) = 1 + x^2 + \cdots + x^{2n} + E, |E| \leq \frac{4}{3}x^{2n+2}, f(\frac{1}{2}) = \frac{4}{3}$   
 17  $f(x) = -x - (x^2/2^2) - \cdots - (x^n/n^2) + E, |E| \leq 2|x|^{n+1}/(n+1)^2, f(\frac{1}{2}) \sim 0.58$

## Section 9.10

- 1  $f(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \frac{(-1)^{n+1} (\cos t) x^{2n+2}}{(2n+2)!}, f\left(\frac{1}{2}\right) \sim 0.8776$

- 3  $f(x) = 2x - \frac{2^3 x^3}{3!} + \cdots + \frac{(-1)^{n+1} 2^{2n+1} x^{2n+1}}{(2n+1)!} + \frac{(-1)^n \cos(2t) (2x)^{2n+3}}{(2n+3)!}, f\left(\frac{1}{2}\right) \sim 0.8415$
- 5  $f(x) = x - \frac{4x^3}{3!} + \frac{4^2 x^5}{5!} - \cdots + \frac{(-1)^n 4^n x^{2n+1}}{(2n+1)!} + \frac{(-1)^{n+1} 4^{n+1} (\cos^2 t - \sin^2 t) x^{2n+3}}{(2n+3)!},$   
 $f\left(\frac{1}{2}\right) \sim 0.4207$
- 7  $f(x) = 2^{-3} + 3 \cdot 2^{-6} x + \cdots + \frac{(-1)^n (1 \cdot 3 \cdots (2n+1)) 2^{-3(n+1)} x^n}{n!}$   
 $+ \frac{(-1)^{n+1} (1 \cdot 3 \cdots (2n+3)) (4+t)^{(-5/2-n)} x^{n+1}}{2^{n+1} (n+1)!}, f\left(\frac{1}{2}\right) \sim 0.1048$
- 9  $x + \frac{x^3}{3}, 0.346$     11  $x + \frac{x^3}{6}, 0.340$     13  $x - \frac{x^2}{2}, 0.28$     15  $x + \frac{x^3}{3}, 0.346$
- 17  $\frac{x^2}{2} - \frac{x^3}{6}, 0.049$
- 19  $e^x = e^2 + e^2(x-2) + \frac{e^2(x-2)^2}{2!} + \cdots + \frac{e^2(x-2)^n}{n!} + \frac{e^t(x-2)^{n+1}}{(n+1)!}$
- 21  $x^p = 1 + p(x-1) + \frac{p(p-1)(x-1)^2}{2} + \cdots + \frac{p(p-1) \cdots (p-n+1)(x-1)^n}{n!}$   
 $+ \frac{p(p-1) \cdots (p-n)t^{p-n-1}(x-1)^{n+1}}{(n+1)!}$

## Section 9.11

- 1 96    3 6    5 1008    7  $\sum_{n=0}^{\infty} x^n / (2^n n!), r = \infty$
- 9  $1 + x + \sum_{n=2}^{\infty} \frac{(-1)^{n+1} (1 \cdot 3 \cdot 5 \cdots (2n-3)) x^n}{n!}, r = \frac{1}{2}$
- 11  $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}$ , converges to  $\cos \sqrt{x}$  for  $x \geq 0$     13  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}, r = \infty$
- 15  $1 - \frac{x^2}{2} + \sum_{n=2}^{\infty} -\frac{(1 \cdot 3 \cdot 5 \cdots (2n-3)) x^{2n}}{2^n n!}, r = 1$     17  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+4}}{(6n+4)(2n+1)!}, r = \infty$
- 19  $\sum_{n=0}^{\infty} \frac{x^{4n+1}}{(4n+1)(2n+1)!}, r = \infty$     21  $x + \sum_{n=1}^{\infty} \frac{\frac{1}{3}(\frac{1}{3}-1) \cdots (\frac{1}{3}-n+1) x^{2n+1}}{n!(2n+1)}, r = 1$
- 23  $x + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdots (2n-1) x^{2n+1}}{2^n n! (2n+1)^2}, r = 1$     25  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-1)^n}{n}, r = 1$

## Extra Problems for Chapter 9

- 1 converges to 1    3 converges to 1    5 diverges to  $\infty$     7 diverges
- 9 converges to  $28\frac{4}{15}$     11 converges to 42    13 diverges    15 diverges
- 17 converges    19 converges    21 diverges    23 converges    25 converges
- 27 converges    29 diverges    31  $1/2$     33 1    35  $\sqrt{e}$
- 37  $f'(x) = \sum_{n=1}^{\infty} n^{a+1} (n+1) 2^n x^{n-1}, \int_0^x f(t) dt = \sum_{n=1}^{\infty} n^a (n+1) 2^{n-1} x^{n+1}, r = \frac{1}{2}$
- 39  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(2n+1)(4n+1)!}, r = 1$     41 -0.006    43 0.646
- 45  $x + \sum_{n=1}^{\infty} \frac{2^n (-1)^n (2 \cdot 5 \cdot 8 \cdots (3n-1)) x^{2n+1}}{3^n (2n+1) n!}, r = \frac{1}{\sqrt{2}}$     47  $e^{50}$

## Section 10.1

- 1  $i + 2j$     3  $-3i - 4j$     5  $(2, -4)$     7  $(-1, 12)$     9  $A + B = -3i + j$
- 11  $A + B + C = j$     13  $3A = 3i - 6j$     15  $B - A = -5i + 5j$

17  $A - 2B + 3C = 18i - 8j$     19  $|B| = 5$     21  $|A - B| = 5\sqrt{2}$     23  $|6A| = 6\sqrt{5}$   
 25  $\frac{13}{4}i - \frac{5}{4}j$     27  $-\frac{4}{3}i + \frac{2}{3}j, (-\frac{4}{3}, \frac{2}{3})$     29  $\arccos(-(2\sqrt{5})/5)$     31  $\arccos(-\frac{4}{5})$   
 33  $7i + 3j$     35  $8i - 2j$     37  $-4i - 6j$     39 5    41  $4i + 3j$     43  $10i + 2j$   
 45  $-2\sqrt{2}i + 2\sqrt{2}j$     47  $6i + 8j$     49  $2i - j, 2i + 2j, -5i + 2j, i - 3j$

## Section 10.2

1  $X = 3i - j + t(-i + j)$     3  $X = 3i + 4j + t(-2i + 5j)$     5  $X = i + 4j + t(i - 5j)$   
 7  $X = 2i + 5j + tj$     9  $X = 2j + t(i + 5j)$     11  $X = 3j + ti$   
 13  $X = 6i + 5j + t(i - 3j)$     15  $y = -2x + 10$     17  $y = 3$     19 no    21 yes  
 23 yes    25  $(-2, 3)$     27  $(2, 10)$     29  $(4, 3)$

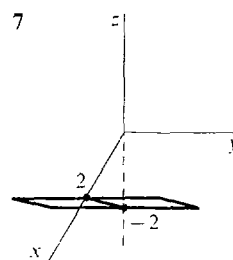
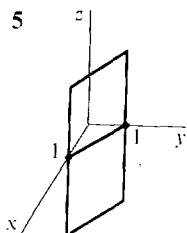
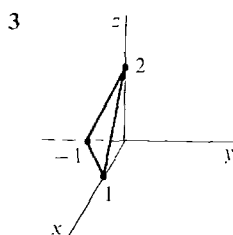
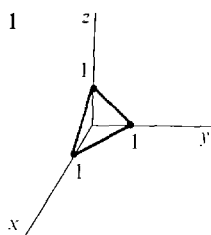
## Section 10.3

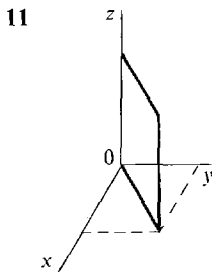
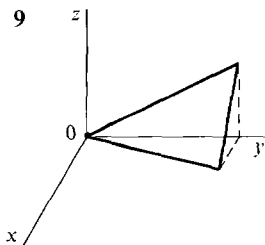
1  $5i - j + 7k$     3  $-k$     5  $(0, 0, 0)$     7  $3i + j - 4k$     9  $i + 5j - 8k$   
 11  $-2i - 3j + 6k$     13 3    15  $\sqrt{26}$     17  $\arccos(\frac{8}{21})$     19 0  
 21  $\frac{1}{3}i - \frac{2}{3}j + \frac{2}{3}k, (\frac{1}{3}, -\frac{2}{3}, \frac{2}{3})$     23  $-3i + 3j + 3\sqrt{2}k$     25  $\frac{2}{3}, -\frac{2}{3}$     27  $\frac{3}{10}i - \frac{1}{2}j + \frac{1}{10}k$   
 29  $125i + 250j + 625k$     31  $X = i + 3j + k + t(i - k)$   
 33  $X = -i + 4j + 3k + t(-i - 7j + 3k)$     35  $X = t(-3i + 4k)$

## Section 10.4

1 (a) 20 (b)  $A \parallel B$  (c) 1    3 (a) 0 (b)  $A \perp B$  (c) 0  
 5 (a) 24 (b) neither (c)  $12/37$     7 (a)  $-92$  (b) neither (c)  $-92/117$   
 9 (a) 0 (b)  $A \perp B$  (c) 0    11 (a)  $8\sqrt{5}$  (b)  $A \parallel B$  (c) 1    13 41  
 15  $3i + 3j + 3k$     17  $200\sqrt{2}$     19  $i - j$     21  $-2i - 2j - 4k$     23  $-i + j + k$   
 23  $-i - j + k$     27  $\frac{4}{3}i + \frac{2}{3}j$     29  $i - j$     31  $\arccos(\frac{1}{3})$     33  $\pi/3$

## Section 10.5





- 13 (a)  $\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$  (b)  $\mathbf{i} + 2\mathbf{j}$  (c)  $-3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$  (d)  $\mathbf{i} + 6\mathbf{k}$  (e)  $\mathbf{j}$  (f)  $-\mathbf{j} + \mathbf{k}$   
 15  $y + 2z = -9$  17  $x + y + 2z = 0$  19  $y - z = -1$   
 21  $4x + 20y - 5z = 20$  23  $x + 2y + z = 9$  25  $2x + y + 3z = 13$   
 27  $x + y - 2z = 20$  29  $x - y - 3z = 3$  31  $24x - 19y + 4z = 57$  33 parallel  
 35 neither 37  $\mathbf{X} = 5\mathbf{i} + 3\mathbf{j} - \mathbf{k} + t(\mathbf{i} - \mathbf{j} + 3\mathbf{k})$  39  $\mathbf{X} = -\mathbf{i} + \mathbf{k} + t(3\mathbf{i} - \mathbf{j} - 2\mathbf{k})$   
 41  $\mathbf{X} = \mathbf{j} - \mathbf{k} + t(-\mathbf{i} + \mathbf{j} + \mathbf{k})$  43  $(3, 1, 1)$  45  $(-\frac{4}{3}, 0, \frac{8}{3})$  47  $(-2, 1, 2)$   
 49  $(1, 1, 3)$

## Section 10.6

- 1  $\mathbf{X} = 3t\mathbf{i} + 9t^2\mathbf{j}$  3  $\mathbf{X} = \frac{1}{\sqrt{5}}t^3\mathbf{i} + \frac{2}{\sqrt{5}}t^3\mathbf{j}$  5  $\mathbf{X} = (1/\sqrt{t^2 + 1})(t\mathbf{i} + \mathbf{j})$   
 7  $\mathbf{X} = \frac{\cos t + \cos(3t)}{2}\mathbf{i} + \frac{\sin t + \sin(3t)}{2}\mathbf{j}$   
 9  $\mathbf{X} = (4 \cos(t/3) - \cos(4t/3))\mathbf{i} + (4 \sin(t/3) - \sin(4t/3))\mathbf{j}$   
 11  $\mathbf{X} = (\cos t + t \sin t)\mathbf{i} + (\sin t - t \cos t)\mathbf{j}$  13  $\mathbf{X} = \cos t\mathbf{i} + \sin t\mathbf{j} + t^2\mathbf{k}$   
 15  $\mathbf{X} = \mathbf{i} + \left(2 + \frac{t^2}{\sqrt{2t^4 - 2t^2 + 1}}\right)\mathbf{j} + \left(1 + \frac{t^2 - 1}{\sqrt{2t^4 - 2t^2 + 1}}\right)\mathbf{k}$   
 17  $\mathbf{X} = \frac{1}{2}(\cos t + \cos(2t))\mathbf{i} + \frac{1}{2}\sin t\mathbf{j} + \frac{1}{2}\sin(2t)\mathbf{k}$   
 19  $\mathbf{X} = (-\frac{5}{3} + \frac{2}{3}\cos t - \frac{1}{3}\sin t)\mathbf{i} + (-\frac{5}{3} - \frac{1}{3}\cos t + \frac{2}{3}\sin t)\mathbf{j} + (\frac{1}{3} - \frac{1}{3}\cos t - \frac{1}{3}\sin t)\mathbf{k}$   
 21  $\mathbf{X} = (t - t \cos^2 t - 2t \sin t \cos t)\mathbf{i} + (2t - t \sin t \cos t - 2t \sin^2 t)\mathbf{j} + 3t\mathbf{k}$   
 23  $\mathbf{P}(t) = 2t\mathbf{i} + \frac{1}{2t(t+1)}\mathbf{j} + (t+1)\mathbf{k}$

## Section 10.7

- 1  $5 \cos t\mathbf{i} - 5 \sin t\mathbf{j}$  3  $-e^t \sin(e^t)\mathbf{i} + e^t \cos(e^t)\mathbf{j}$  5  $-6\mathbf{i} + (6/t)\mathbf{j} - 6e^t\mathbf{k}$   
 7  $2 \cos 2t$  9  $t/\sqrt{1+t^2}$  11  $\mathbf{i} + \mathbf{j} + \mathbf{k} + t(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$   
 13  $\mathbf{V} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ ,  $|\mathbf{V}| = \sqrt{29}$ ,  $\mathbf{A} = \mathbf{0}$   
 15  $\mathbf{V} = -\sin t\mathbf{i} + \cos t\mathbf{j} + \mathbf{k}$ ,  $|\mathbf{V}| = \sqrt{2}$ ,  $\mathbf{A} = -\cos t\mathbf{i} - \sin t\mathbf{j}$   
 17  $\mathbf{V} = -e^t \sin(e^t)\mathbf{i} + e^t \cos(e^t)\mathbf{j}$ ,  $|\mathbf{V}| = e^t$ ,  
 $\mathbf{A} = (-e^t \sin e^t - e^{2t} \cos e^t)\mathbf{i} + (e^t \cos e^t - e^{2t} \sin e^t)\mathbf{j}$   
 19  $\mathbf{V} = 2t\mathbf{i} + 4t\mathbf{j} - 2t\mathbf{k}$ ,  $|\mathbf{V}| = 2\sqrt{6}|t|$ ,  $\mathbf{A} = 2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$   
 21  $\mathbf{S} = t \cos t\mathbf{i} + t \sin t\mathbf{j}$ ,  $\mathbf{V} = (-t \sin t + \cos t)\mathbf{i} + (t \cos t + \sin t)\mathbf{j}$ ,  $|\mathbf{V}| = \sqrt{1+t^2}$ ,  
 $\mathbf{A} = (-t \cos t - 2 \sin t)\mathbf{i} + (-t \sin t + 2 \cos t)\mathbf{j}$   
 23  $\mathbf{S} = \cos(t^2)\mathbf{i} + \sin(t^2)\mathbf{j}$ ,  $\mathbf{V} = -2t \sin(t^2)\mathbf{i} + 2t \cos(t^2)\mathbf{j}$ ,  $|\mathbf{V}| = |2t|$ ,  
 $\mathbf{A} = (-2 \sin(t^2) - 4t^2 \cos(t^2))\mathbf{i} + (2 \cos(t^2) - 4t^2 \sin(t^2))\mathbf{j}$   
 25  $\mathbf{V} = (e^x/\sqrt{1+e^{2x}})\mathbf{i} + (e^{2x}/\sqrt{1+e^{2x}})\mathbf{j}$ ,  $|\mathbf{V}| = e^x$ ,  
 $\mathbf{A} = [-e^{3x}(1+e^{2x})^{-3/2} + e^x(1+e^{2x})^{-1/2}]\mathbf{i} + [-e^{4x}(1+e^{2x})^{-3/2} + 2e^{2x}(1+e^{2x})^{-1/2}]\mathbf{j}$   
 27  $2\pi$  29 5 31  $2\sqrt{3} + \sqrt{2} \ln(\sqrt{3} + \sqrt{2})$  33  $\frac{3}{2} + \ln 2$   
 35  $\mathbf{F}(t) = (\frac{1}{2}t^2 + \frac{1}{2})\mathbf{i} + (\frac{1}{3}t^3 + \frac{5}{3})\mathbf{j} + (\frac{1}{4}t^4 + \frac{11}{4})\mathbf{k}$

$$37 \quad \mathbf{F}(t) = \ln|t-1|\mathbf{i} + (\ln|t-2| - \ln 2)\mathbf{j} + (\ln|t-3| - \ln 3)\mathbf{k}$$

$$39 \quad \mathbf{S} = (t - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$$

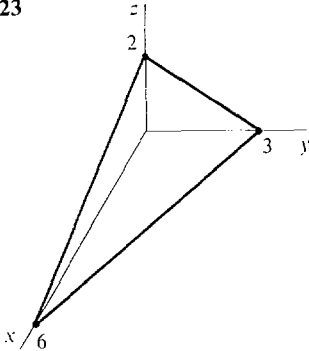
### Section 10.8

- 1 infinitesimal    3 infinite    5 finite    7 infinite    9 infinite  
 11 infinitesimal (zero)    13 infinite    15 infinite    17 infinitesimal    19 infinite  
 21  $-\sin x\mathbf{i} + \cos x\mathbf{j}$     23  $10\mathbf{i} - 20\mathbf{j} + 5\mathbf{k}$     25  $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$     27 0  
 29  $(1/|\mathbf{A}|)\mathbf{A} \cdot \mathbf{U}$  if  $\mathbf{A} \neq \mathbf{0}$ , 1 if  $\mathbf{A} = \mathbf{0}$     31 (a) no (b) no    33 (a) yes (b) no  
 35 (a) yes (b) no    37 (a) no (b) no    39 (a) yes (b) no

### Extra Problems for Chapter 10

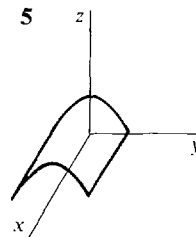
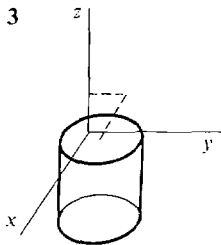
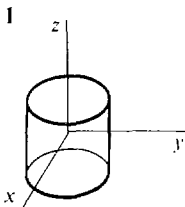
- 1  $5\mathbf{i} - 12\mathbf{j}$     3  $-3\mathbf{i} - 3\mathbf{j}$     5  $9\mathbf{i} + 11\mathbf{j}$     7  $\mathbf{X} = -\mathbf{i} + \mathbf{j} + t(4\mathbf{i} - 3\mathbf{j})$     9  $(2, 5/2)$   
 11  $(1/\sqrt{105}, -10/\sqrt{105}, 2/\sqrt{105})$     13  $100\mathbf{i} - 500\mathbf{j} + 300\mathbf{k}$   
 15  $\mathbf{X} = \mathbf{i} + 4\mathbf{j} + 3\mathbf{k} + t\mathbf{k}$     17  $\mathbf{A} \perp \mathbf{B}$     19 -20    21  $-\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

23

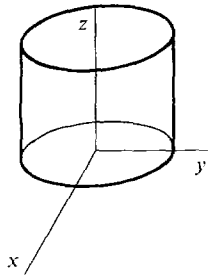


- 25  $-x - y + 2z = 0$     27  $(-1, -2, 4)$   
 29  $\mathbf{S} = \sin t \cos t \mathbf{i} + \sin^2 t \mathbf{j} + \cos t \mathbf{k}$ ,  $\mathbf{S} = -\sin t \cos t \mathbf{i} - \sin^2 t \mathbf{j} + \cos t \mathbf{k}$   
 31  $\mathbf{V} = \cos(2t)\mathbf{i} + \sin(2t)\mathbf{j} - \sin t \mathbf{k}$  or  $\mathbf{V} = -\cos(2t)\mathbf{i} - \sin(2t)\mathbf{j} - \sin t \mathbf{k}$ ,  $|\mathbf{V}| = \sqrt{1 + \sin^2 t}$ ,  
 $\mathbf{A} = -2 \sin(2t)\mathbf{i} + 2 \cos(2t)\mathbf{j} - \cos t \mathbf{k}$  or  $\mathbf{A} = 2 \sin(2t)\mathbf{i} - 2 \cos(2t)\mathbf{j} - \cos t \mathbf{k}$   
 33  $\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{1}{3}\mathbf{k} + t(-\mathbf{i} - \frac{1}{2}\mathbf{j} - \frac{1}{3}\mathbf{k})$   
 35  $\mathbf{S} = (-\cos e^t + \cos 1)\mathbf{i} + (\sin e^t - \sin 1)\mathbf{j} + (e^t - 1)\mathbf{k}$

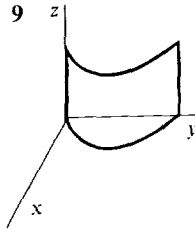
### Section 11.1



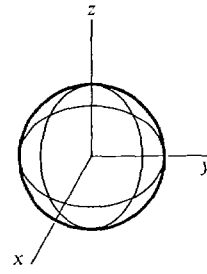
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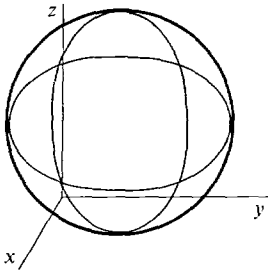
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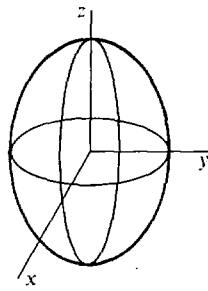
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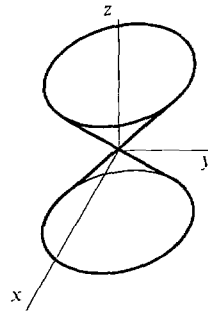
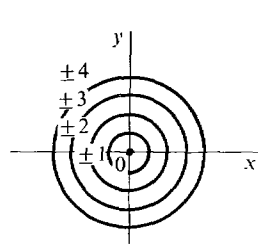
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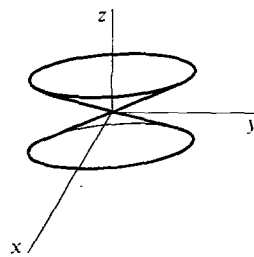
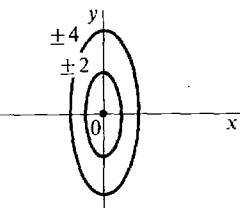
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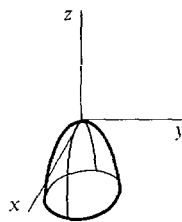
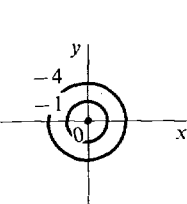
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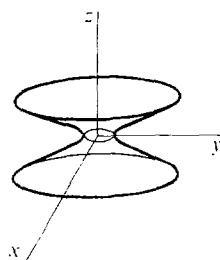
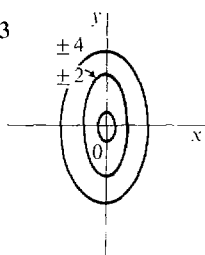
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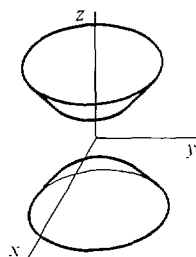
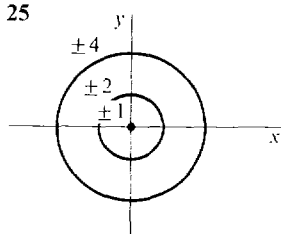
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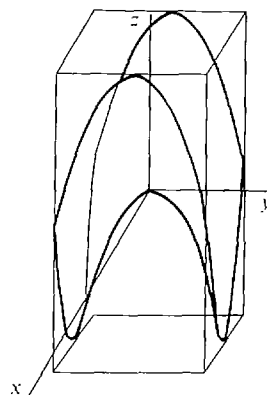
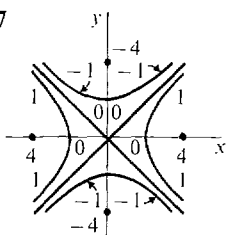
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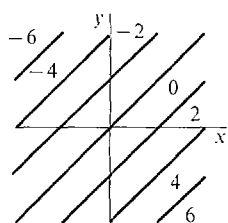
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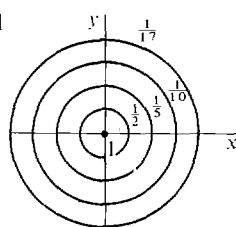
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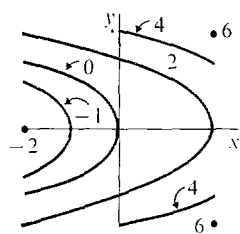
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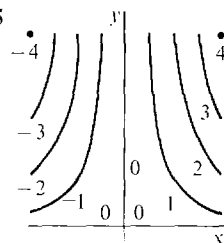
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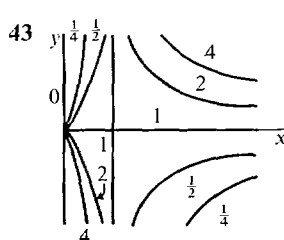
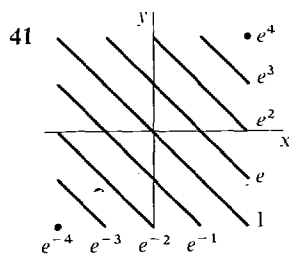
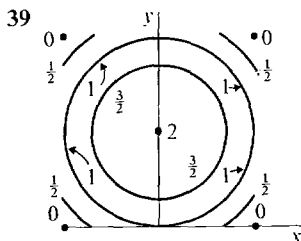
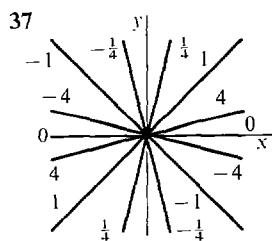
33



35







### Section 11.2

- 1 all  $(x, y)$     3 all  $(x, y)$     5  $y \neq -x$     7  $y \neq -2$     9  $x \neq 2$  and  $y \neq -1$   
 11  $x + y > 0$     13  $x > y$     15  $x > 0$     17  $x^2 > y$     19  $x > 0$  and  $y > 0$   
 21  $x^2 > y$     23  $x > 0$  and  $y > 0$  and  $x \neq 1/y$     25  $x > 0$  and  $y > 0$   
 27  $x \neq 4y$  and  $y^2 > x$     29  $\cos x + y > 0$     31  $x \neq y$     33  $x > y$  and  $y \neq z$   
 35  $(x, y, z) \neq (0, 0, 0)$     37  $x + y > 0$  and  $z \neq 0$     43  $e$     45  $\infty$

### Section 11.3

- 1  $\partial z/\partial x = 4, \partial z/\partial y = -3$     3  $\partial z/\partial x = y^2 + 3x^2y, \partial z/\partial y = 2xy + x^3$   
 5  $\frac{\partial z}{\partial x} = -\frac{2x}{(x^2 + y^2)^2}, \frac{\partial z}{\partial y} = -\frac{2y}{(x^2 + y^2)^2}$     7  $f_x(x, y) = y, f_y(x, y) = x$   
 9  $f_x(x, y) = a, f_y(x, y) = b$     11  $f_x(x, y) = 2xe^{x^2-y^2}, f_y(x, y) = -2ye^{x^2-y^2}$   
 13  $f_x(x, y) = \frac{1}{2\sqrt{x+2y}}, f_y(x, y) = \frac{1}{\sqrt{x+2y}}$     15  $\partial z/\partial x = yx^{y-1}, \partial z/\partial y = x^y \ln x$   
 17  $\partial z/\partial x = 1/x, \partial z/\partial y = 1/y$   
 19  $\frac{\partial z}{\partial x} = -\frac{\ln y}{x(\ln x)^2}, \frac{\partial z}{\partial y} = \frac{1}{y \ln x}$     21  $\frac{\partial z}{\partial x} = \frac{2xy}{\sqrt{1-x^4y^2}}, \frac{\partial z}{\partial y} = \frac{x^2}{\sqrt{1-x^4y^2}}$   
 23  $\frac{\partial w}{\partial x} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}, \frac{\partial w}{\partial y} = \frac{y}{\sqrt{x^2 + y^2 + z^2}}, \frac{\partial w}{\partial z} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$   
 25  $f_x(x, y, z) = a, f_y(x, y, z) = b, f_z(x, y, z) = c$   
 27  $\partial w/\partial x = -z \sin x, \partial w/\partial y = z \cos y, \partial w/\partial z = \cos x + \sin y$     29  $f_x(1, 2) = 4, f_y(1, 2) = 4$   
 31  $f_x(-1, 1) = -1, f_y(-1, 1) = 1$     33  $\frac{\partial z}{\partial x}(0, 2) = 2, \frac{\partial z}{\partial y}(0, 2) = 0$   
 35  $\frac{\partial z}{\partial x}(1, 0) = e, \frac{\partial z}{\partial y}(1, 0) = 0$     37  $\frac{\partial z}{\partial x}(2, 3) = -\frac{4}{961}, \frac{\partial z}{\partial y}(2, 3) = -\frac{27}{961}$   
 39  $f_x(1, 2, 3) = 2, f_y(1, 2, 3) = 4, f_z(1, 2, 3) = 6$     41  $1/\sqrt{5}$     43  $1/3$     45  $b = 2, c = 1$   
 47  $C_x(x, y) = 3 - \frac{1}{2}\sqrt{y/x}, C_y(x, y) = 4 - \frac{1}{2}\sqrt{x/y}$

## Section 11.4

- 1  $\Delta z = 3 \Delta x - 2 \Delta y, dz = 3 dx - 2 dy$   
 3  $\Delta z = 2xy^2 \Delta x + 2x^2y \Delta y + y^2 \Delta x^2 + 4xy \Delta x \Delta y + x^2 \Delta y^2 + 2x \Delta x \Delta y^2 + 2y \Delta x^2 \Delta y + \Delta x^2 \Delta y^2, dz = 2xy^2 dx + 2x^2y dy$   
 5  $\Delta z = -\frac{x \Delta y + y \Delta x + \Delta x \Delta y}{(x + \Delta x)(y + \Delta y)xy}, dz = -\frac{dx}{x^2y} - \frac{dy}{xy^2}$   
 7  $\Delta z = e^{3x-4y}(e^{3\Delta x-4\Delta y} - 1), dz = e^{3x-4y}(3 dx - 4 dy)$   
 9  $\Delta z = \cos(x + \Delta x) \sin(y + \Delta y) - \cos x \sin y, dz = -\sin x \sin y dx + \cos x \cos y dy$   
 11  $\Delta z = x \ln(1 + (\Delta y/y)) + \Delta x \ln(y + \Delta y), dz = \ln y dx + (x/y) dy$   
 13  $\Delta w = \Delta x + 2 \Delta y + 3 \Delta z, dw = dx + 2 dy + 3 dz$   
 15  $\Delta w = y \Delta x + (x + z) \Delta y + y \Delta z + \Delta x \Delta y + \Delta y \Delta z, dw = y dx + (x + z) dy + y dz$   
 17  $\Delta z = dz + \Delta x \Delta x + \Delta y \Delta y$     19  $\Delta z = dz + (y \Delta x + 2x \Delta y + \Delta x \Delta y) \Delta x + 0 \Delta y$   
 21  $\Delta z = dz + \frac{\Delta y}{y(y + \Delta y)} \Delta x + \frac{x \Delta y}{y^2(y + \Delta y)} \Delta y$     23  $z = 4x + 4y - 6$   
 25  $z = 4x + 4y - 2$     27  $z = \frac{1}{2}x + \frac{1}{2}y + 1$     29  $z = 6e^3x + e^3y - 8e^3$   
 31  $z = \frac{\sqrt{2}}{4}x + \frac{\sqrt{6}}{4}y + \frac{\sqrt{6}}{4} - \pi\left(\frac{\sqrt{2}}{12} + \frac{\sqrt{6}}{16}\right)$     33  $z = x - 2$     35  $z = 1$   
 37  $x - 2y + 2z = 9$     39  $x + y - z = 1$

## Section 11.5

- 1  $2e^{2t} + 2e^{-2t}$     3  $-\frac{\cos(t/a) + \cos(t/b)}{[a \sin(t/a) + b \sin(t/b)]^2}$     5  $\frac{2y}{x \ln y} - \frac{2x \ln x}{y(\ln y)^2}$   
 7  $(t+1)^{1/t} \left[ \frac{1}{t(t+1)} - \frac{\ln(t+1)}{t^2} \right]$     9  $(\sin t)^{\cos t} \left[ \frac{\cos^2 t}{\sin t} - \sin t \ln(\sin t) \right]$   
 11  $\frac{2t}{(t^2-1)\ln(t^2+1)} - \frac{2t \ln(t^2-1)}{(t^2+1)[\ln(t^2+1)]^2}$     13  $3\sqrt{1-t^4} - 2\sqrt{1-t^3}$   
 15  $\frac{\partial z}{\partial s} = 3s^2 \cos^3 t, \frac{\partial z}{\partial t} = -3s^2 \sin t \cos^2 t$     17  $\frac{\partial z}{\partial s} = \frac{2s}{s^2-t^2}, \frac{\partial z}{\partial t} = -\frac{2t}{s^2-t^2}$   
 19  $\frac{\partial z}{\partial s} = -\frac{a}{(s+t)^2} - \frac{b}{(s-t)^2}, \frac{\partial z}{\partial t} = -\frac{a}{(s+t)^2} + \frac{b}{(s-t)^2}$     21  $b \frac{\partial z}{\partial x} = abf'(ax+by) = a \frac{\partial z}{\partial y}$   
 25  $\frac{dw}{dt} = e^t \cos(\sqrt{t}) - e^{-t} \sin(\sqrt{t}) + \frac{1}{2\sqrt{t}}(-e^t \sin(\sqrt{t}) + e^{-t} \cos(\sqrt{t}))$   
 27  $\frac{dz}{dt} = \frac{x(dx/dt) + y(dy/dt)}{\sqrt{x^2 + y^2}}$  or  $\frac{dz}{dt} = \frac{x}{z} \frac{dx}{dt} + \frac{y}{z} \frac{dy}{dt}$   
 29  $\frac{dz}{dt} = x^y \left( \frac{y}{x} \frac{dx}{dt} + \ln x \frac{dy}{dt} \right)$  or  $\frac{dz}{dt} = \frac{zy}{x} \frac{dx}{dt} + z \ln x \frac{dy}{dt}$   
 31  $\frac{\partial z}{\partial s} = af'(u), \frac{\partial z}{\partial t} = bf'(u)$     33  $\frac{\partial z}{\partial s} = z \frac{\partial u}{\partial s}, \frac{\partial z}{\partial t} = z \frac{\partial u}{\partial t}$   
 35  $\frac{\partial z}{\partial s} = g'(s)h(t), \frac{\partial z}{\partial t} = g(s)h'(t)$     37  $-2$     39  $-3$     41  $2$     43  $1/25$

## Section 11.6

- 1  $\frac{\partial z}{\partial x} = 3, \frac{dz}{dx} = 3 - 4e^x$     3  $\frac{\partial z}{\partial x} = -\sin x, \frac{dz}{dx} = -\sin x + 3 \cos(3x)$   
 5  $\frac{\partial z}{\partial x} = yx^{y-1}, \frac{dz}{dx} = x^y(1 + \ln x)$     7  $\frac{\partial z}{\partial x} = \frac{y}{1+x^2y^2}, \frac{dz}{dx} = \frac{e^{-x}(1-x)}{1+x^2e^{-2x}}$     9  $\frac{y+x}{y-x}$   
 11  $-\frac{2x+2y^3}{6xy^2+1}$     13  $-\frac{1+y \cos(xy)}{x \cos(xy)}$     15 slope  $= -\frac{3}{4}, 4y = -3x + 14$   
 17 slope  $= 5, y = 5x - 6$     19 slope  $= -1, y = -x + 2$     21 slope  $= 1, y = x$   
 23  $\frac{\partial w}{\partial x}(x, y) = 15, \frac{\partial w}{\partial y}(x, y) = -34$

$$\begin{aligned}
 25 \quad \frac{\partial w}{\partial x}(x, y) &= \frac{10x - 6y}{\sqrt{10x^2 - 12xy + 5y^2}}, \quad \frac{\partial w}{\partial y}(x, y) = \frac{-6x + 5y}{\sqrt{10x^2 - 12xy + 5y^2}} \\
 27 \quad 6x - 5y + 4z &= -21 \quad 29 \quad x + y + z = 3 \quad 31 \quad x + y + z = 0 \\
 33 \quad \frac{\partial z}{\partial x} &= -\frac{xy}{z}, \quad \frac{\partial z}{\partial y} = -\frac{x^2}{2z} \quad 35 \quad \frac{\partial z}{\partial x} = \frac{\cos(xy)}{\sin(yz)}, \quad \frac{\partial z}{\partial y} = \frac{x \cos(xy) - z \sin(yz)}{y \sin(yz)} \\
 37 \quad \frac{\partial z}{\partial x} &= -\frac{z}{3x}, \quad \frac{\partial z}{\partial y} = -\frac{2z}{3y} \quad 39 \quad \frac{dz}{dx} = y - \frac{x}{1 + \sqrt{x}} \\
 41 \quad \frac{dw}{dx} &= \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} \frac{dy}{dx} + \frac{\partial w}{\partial z} \frac{dz}{dx}
 \end{aligned}$$

## Section 11.7

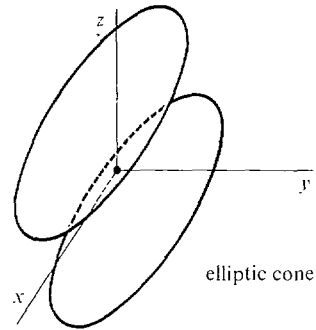
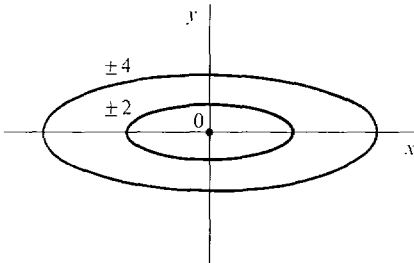
- 1 max = 3 at  $(-1, -1)$  and  $(1, 1)$ ; min = 0 at  $(0, 0)$   
 3 max = 48 at  $(3, 3)$ ; min = -6 at  $(1, -2)$     5 max = 4 at  $(2, 1)$ ; min = -5 at  $(2, 4)$   
 7 max = 2 at  $(\pi/2, \pi/2)$ ; min = 0 at  $(0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$   
 9 max = 1 at  $(-1, 1)$  and  $(1, 1)$ ; min =  $-\frac{1}{4}$  at  $(0, \frac{1}{2})$   
 11 max =  $\frac{1}{4}$  at  $(\frac{1}{2}, 0, \frac{1}{2})$  and  $(\frac{1}{2}, 1, -\frac{1}{2})$ ; min =  $-\frac{1}{4}$  at  $(0, \frac{1}{2}, \frac{1}{2})$  and  $(1, \frac{1}{2}, -\frac{1}{2})$   
 13 max =  $1 + \sqrt{2}$  at  $(\sqrt{2}/2, \sqrt{2}/2, 1)$ ; min =  $-\frac{1}{2}$  at  $(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$   
 15 max = 3 at  $(-1, -1, 1), (-1, 1, 1), (1, -1, 1), (1, 1, 1)$ ; min = 0 at  $(0, 0, 0)$   
 17 no max; min = -4 at  $(-2, 0)$     19 no max; no min  
 21 no max; min = 6 at  $(\frac{1}{2}, 4)$     23 max = 1 at  $(0, 0)$ ; no min    25 no max, no min  
 27  $x = 4, y = 2, z = 2$     29 6 in.  $\times$  6 in.  $\times$  12 in.    31  $2\sqrt{3}/3$     33  $(\frac{5}{3}, \frac{10}{3}, -\frac{5}{3})$   
 35  $(1, 1, 1), (1, -1, -1), (-1, 1, -1), (-1, -1, 1)$   
 39  $\frac{2}{3}(\frac{2}{3}V)^{1/3}$  in.  $\times$   $\frac{2}{3}(\frac{2}{3}V)^{1/3}$  in.  $\times$   $(\frac{2}{3}V)^{1/3}$  in.    41  $x = 100, y = 200, P(100, 200) = 50,000$

## Section 11.8

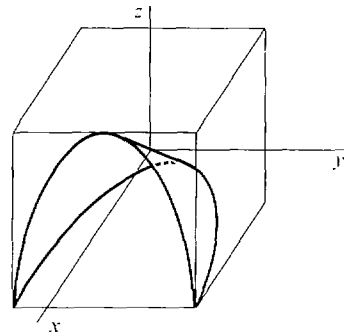
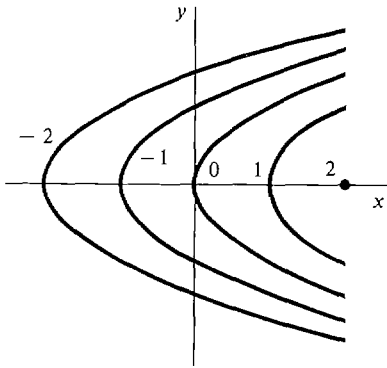
$$\begin{aligned}
 1 \quad \frac{\partial^2 z}{\partial x^2} &= 2, \quad \frac{\partial^2 z}{\partial y^2} = 4, \quad \frac{\partial^2 z}{\partial x \partial y} = 0 \quad 3 \quad \frac{\partial^2 z}{\partial x^2} = 2a, \quad \frac{\partial^2 z}{\partial y^2} = 2c, \quad \frac{\partial^2 z}{\partial x \partial y} = b \\
 5 \quad \frac{\partial^2 z}{\partial x^2} &= e^{x+y}(x+2), \quad \frac{\partial^2 z}{\partial y^2} = xe^{x+y}, \quad \frac{\partial^2 z}{\partial x \partial y} = e^{x+y}(x+1) \\
 7 \quad \frac{\partial^2 z}{\partial x^2} &= -\frac{a^2}{(ax+by)^2}, \quad \frac{\partial^2 z}{\partial y^2} = -\frac{b^2}{(ax+by)^2}, \quad \frac{\partial^2 z}{\partial x \partial y} = -\frac{ab}{(ax+by)^2} \\
 9 \quad \frac{\partial^2 z}{\partial x^2} &= a(a-1)x^{a-2}y^b, \quad \frac{\partial^2 z}{\partial y^2} = b(b-1)x^ay^{b-2}, \quad \frac{\partial^2 z}{\partial x \partial y} = abx^{a-1}y^{b-1} \\
 11 \quad \frac{\partial^2 w}{\partial x^2} &= \frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 w}{\partial z^2} = \frac{\partial^2 w}{\partial x \partial y} = \frac{\partial^2 w}{\partial x \partial z} = \frac{\partial^2 w}{\partial y \partial z} = -\frac{1}{4}(x+y+z)^{-3/2} \\
 13 \quad \frac{\partial^3 z}{\partial x^3} &= 12y^3, \quad \frac{\partial^3 z}{\partial y^3} = -36x^2, \quad \frac{\partial^3 z}{\partial x^2 \partial y} = 24xy - 36y^2, \quad \frac{\partial^3 z}{\partial x \partial y^2} = 12x^2 - 72xy \\
 15 \quad \frac{\partial^3 z}{\partial x^3} &= a^3e^{ax+by}, \quad \frac{\partial^3 z}{\partial y^3} = b^3e^{ax+by}, \quad \frac{\partial^3 z}{\partial x^2 \partial y} = a^2be^{ax+by}, \quad \frac{\partial^3 z}{\partial x \partial y^2} = ab^2e^{ax+by} \\
 17 \quad \frac{\partial^2 z}{\partial \theta^2} &= r^2 \sin^2 \theta \frac{\partial^2 f}{\partial x^2} - 2r^2 \sin \theta \frac{\partial^2 f}{\partial x \partial y} + r^2 \cos^2 \theta \frac{\partial^2 f}{\partial y^2} - r \cos \theta \frac{\partial f}{\partial x} - r \sin \theta \frac{\partial f}{\partial y} \\
 19 \quad \frac{\partial^2 z}{\partial x^2} &= a^2 f''(u), \quad \frac{\partial^2 z}{\partial y^2} = b^2 f''(u), \quad \frac{\partial^2 z}{\partial x \partial y} = ab f''(u) \quad 21 \quad \frac{\partial^2 z}{\partial x^2} = g''(x), \quad \frac{\partial^2 z}{\partial y^2} = h''(y), \quad \frac{\partial^2 z}{\partial x \partial y} = 0 \\
 23 \quad \frac{\partial^2 z}{\partial x^2} &= n(n-1)u^{n-2} \left( \frac{\partial u}{\partial x} \right)^2 + nu^{n-1} \frac{\partial^2 u}{\partial x^2}, \quad \frac{\partial^2 z}{\partial y^2} = n(n-1)u^{n-2} \left( \frac{\partial u}{\partial y} \right)^2 + nu^{n-1} \frac{\partial^2 u}{\partial y^2}, \\
 &\quad \frac{\partial^2 z}{\partial x \partial y} = n(n-1)u^{n-2} \left( \frac{\partial u}{\partial x} \right) \left( \frac{\partial u}{\partial y} \right) + nu^{n-1} \frac{\partial^2 u}{\partial x \partial y} \\
 25 \quad \frac{\partial^2 z}{\partial s^2} &= a \frac{\partial^2 x}{\partial s^2} + b \frac{\partial^2 y}{\partial s^2}, \quad \frac{\partial^2 z}{\partial t^2} = a \frac{\partial^2 x}{\partial t^2} + b \frac{\partial^2 y}{\partial t^2}, \quad \frac{\partial^2 z}{\partial s \partial t} = a \frac{\partial^2 x}{\partial s \partial t} + b \frac{\partial^2 y}{\partial s \partial t} \\
 27 \quad \frac{\partial^2 z}{\partial s^2} &= f'(x)[(g'(s))^2 + g'(s)g''(s)], \quad \frac{\partial^2 z}{\partial t^2} = f'(x)[(h'(t))^2 + h'(t)h''(t)], \quad \frac{\partial^2 z}{\partial s \partial t} = f'(x)g'(s)h'(t)
 \end{aligned}$$

## Extra Problems for Chapter 11

1



3



- 5  $x \neq 0$     7  $1/x + 1/y > 0$     9  $f_x(x, y) = a, f_y(x, y) = b$   
 11  $\frac{\partial z}{\partial x} = \frac{1}{x \ln y}, \frac{\partial z}{\partial y} = -\frac{\ln x}{y(\ln y)^2}$     13  $\Delta z = -\frac{\Delta x}{x(x + \Delta x)} - \frac{2 \Delta y}{y(y + \Delta y)}, dz = -\frac{dx}{x^2} - \frac{2 dy}{y^2}$   
 15  $-8y + z = 4$     17  $\frac{dz}{dt} = -\frac{2 \log_{(2t+1)}(3t+2)}{(2t+1) \ln(2t+1)} + \frac{3}{(3t+2) \ln(2t+1)}$     19 1  
 21  $y = -8x + 17$ , slope =  $-8$     23 max = 4 at  $(0, 0)$  and  $(3, 3)$ ; min =  $-1$  at  $(1, 2)$   
 25 no max; no min    27  $\frac{\partial^2 z}{\partial x^2} = -\frac{2xy^3}{(1+x^2y^2)^2}, \frac{\partial^2 z}{\partial y^2} = -\frac{2x^3y}{(1+x^2y^2)^2}, \frac{\partial^2 z}{\partial x \partial y} = -\frac{1-x^2y^2}{(1+x^2y^2)^2}$   
 29  $\frac{\partial^2 z}{\partial t^2} = \cosh^2 \theta \frac{\partial^2 f}{\partial x^2} + 2 \sinh \theta \cosh \theta \frac{\partial^2 f}{\partial x \partial y} + \sinh^2 \theta \frac{\partial^2 f}{\partial y^2}$

## Section 12.1

- 1  $168/64 = 2.625$     3 44    5 1.025    7  $(\pi/6)^2(7 + 4\sqrt{3}) \sim 3.8$   
 9  $e^{-5} + e^{-4} + 2e^{-3} + 2e^{-2} + 2e^{-1} + 2 + 2e + 2e^2 + e^3 + e^4 \sim 97.52$   
 11  $150/64 \sim 2.34$     13  $((11 + 7\sqrt{2})\pi)/32 \sim 2.05$     15 108

## Section 12.2

- 1  $7/2 = 3.5$     3  $128/3 = 42\frac{2}{3}$     5  $\frac{3}{2} \ln 2 \sim 1.04$     7 4  
 9  $\frac{1}{2}(e^6 - e^2 - e^{-2} + e^{-6}) \sim 197.9$     11  $13/6 \sim 2.17$     13  $32/15 \sim 2.13$   
 15  $36\pi \sim 113.1$     17  $14/3$     19 30    21 2    23  $\frac{4}{15}(212 - 36\sqrt{6})$     25  $e - 1$   
 27  $8/3$     29  $3^8/40$   
 31  $0 \leq x \leq 5, 0 \leq y \leq 5 - x$ , or  $0 \leq y \leq 5, 0 \leq x \leq 5 - y$ ;  $\int_0^5 \int_0^{5-x} f(x, y) dy dx$ , or  $\int_0^5 \int_0^{5-y} f(x, y) dx dy$

- 33  $-2 \leq x \leq 2, -\sqrt{4-x^2} \leq y \leq \sqrt{4-x^2}, \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x, y) dy dx$   
 35  $-4 \leq x \leq 1, 3x \leq y \leq 4-x^2, \int_{-4}^1 \int_{3x}^{4-x^2} f(x, y) dy dx$   
 37  $-1 \leq y \leq 1, 1/2 \leq x \leq 1/(1+y^2), \int_{-1}^1 \int_{1/2}^{1/(1+y^2)} f(x, y) dx dy$     39  $\pi/4$     41  $32/3$   
 43  $\frac{1}{2}\pi ab$

**Section 12.3**

- 1  $3/4$     3 1    5  $4/3$     7  $16/15$     9  $8/3$     11  $8\pi$     13  $8/15$     15 144  
 17  $\frac{4}{3}\pi abc$

**Section 12.4**

- 1 (a)  $4abk$  (b)  $(0, 0)$  (c)  $\frac{4}{3}(ba^3 + ab^3)k$     3 (a)  $\frac{2}{3}k$  (b)  $(\frac{3}{8}, \frac{3}{8})$  (c)  $\frac{44}{105}k$   
 5 (a)  $26/3$  (b)  $(19/13, 151/65)$  (c)  $944/15$   
 7 (a)  $\frac{2}{7} + \frac{1}{6}$  (b)  $\left(\frac{\frac{2}{9} + \frac{2}{15}}{\frac{2}{7} + \frac{1}{6}}, \frac{\frac{1}{11} + \frac{1}{15}}{\frac{2}{7} + \frac{1}{6}}\right)$  (c)  $\frac{2}{11} + \frac{1}{9} + \frac{2}{45} + \frac{1}{28}$   
 9 (a)  $e^2 + e^{-2} - 2$  (b)  $\left(\frac{e^2 + 3e^{-2} - 2}{e^2 + e^{-2} - 2}, \frac{e^4 + e^{-4} - 2}{4(e^2 + e^{-2} - 2)}\right)$   
 (c)  $\frac{1}{9}e^6 + 2e^2 - \frac{2}{9} - 10e^{-2} + \frac{1}{9}e^{-6}$     11 2    13 (a)  $(\sqrt{2}/48)g$  (b)  $g/24$   
 15  $m = \frac{8}{3}, W = \frac{16}{15}g$

**Section 12.5**

- 1  $6\pi$     3  $8\pi$     5  $\frac{81}{2}\pi$     7  $2\pi(1 - \ln 2)$     9  $4/21$     11  $\pi/48$   
 13  $32/9$     15  $8\pi^8 + \frac{64}{15}\pi^5$     17  $5\pi/3$     19  $\pi/3 - 4/9$     21  $\frac{1}{3}(\beta - \alpha)(b^3 - a^3)$   
 23 (a)  $\pi b^3/3$  (b)  $\pi b^5/10$     25  $(\pi k/2)(b^4 - a^4)$     27 (a)  $\bar{x} = 4/(3\pi), \bar{y} = 4/(3\pi)$  (b)  $\pi k/8$   
 29 (a)  $\bar{x} = 0, \bar{y} = \frac{3\sqrt{3} + 8\pi}{6\sqrt{3} + 4\pi}b$  (b)  $\left(\frac{5}{6}\pi + \frac{7\sqrt{3}}{8}\right)b^4k$     31 (a)  $\pi$  (c)  $\sqrt{\pi}$

**Section 12.6**

- 1  $12$     3  $12$     5  $13/56$     7  $16/27$     9 40  
 11  $\frac{763}{594} \times 10^9 = \left(\frac{4}{27} + \frac{100}{88}\right) \times 10^9 \sim 1.3 \times 10^9$   
 13  $\frac{e^6}{24} - \frac{e^4}{15} + \frac{e^{-2}}{24} - \frac{e^{-6}}{60}$     15  $\frac{52}{63} - \frac{12\sqrt{3}}{35}$   
 17 (a) 3 (b)  $(\frac{19}{36}, \frac{5}{9}, \frac{7}{12})$  (c)  $I_z = \frac{9}{4}, I_y = \frac{7}{3}, I_x = \frac{29}{12}$   
 19 (a) 1 (b)  $(\frac{3}{4}, \frac{5}{12}, \frac{7}{12})$  (c)  $I_z = \frac{5}{6}, I_y = \frac{11}{10}, I_x = \frac{11}{15}$   
 21 (a)  $\frac{5}{18}$  (b)  $(\frac{3}{20}, \frac{3}{20}, \frac{65}{972})$  (c)  $I_z = \frac{5}{16}, I_y = \frac{73}{960}, I_x = \frac{73}{960}$   
 23 (a)  $\frac{1}{6}abck$  (b)  $\left(\frac{a}{4}, \frac{b}{4}, \frac{c}{4}\right)$  (c)  $I_z = \frac{abck}{60}(a^2 + b^2), I_y = \frac{abck}{60}(a^2 + c^2), I_x = \frac{abck}{60}(b^2 + c^2)$   
 25 (a)  $abck$  (b)  $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$  (c)  $I_z = \frac{abck}{3}(a^2 + b^2), I_y = \frac{abck}{3}(a^2 + c^2), I_x = \frac{abck}{3}(b^2 + c^2)$

**Section 12.7**

- 1  $\frac{4\pi}{3}$     3  $\frac{\pi}{10}$     5  $\frac{128}{7}$     7  $\frac{2\pi hb^3}{3}$     9  $\frac{\pi hb^4}{10}$     11  $\left(0, 0, \frac{2}{3}\right)$   
 13  $\frac{b^4 ck\pi}{2} + \frac{2b^2 c^3 k\pi}{3}$     15  $\frac{\pi hb^4 k}{10}$     17  $\frac{4\pi b^5}{5}$     19  $\frac{4\pi}{15}$     21  $\frac{4\pi b^4}{3}$   
 23  $\frac{\pi b^5}{10}(1 - \cos(2\alpha))$  or  $\frac{\pi b^5}{5}\sin^2 \alpha$     25  $\frac{4}{3}\pi(b^3 - a^3)$     27  $\frac{\pi b^3}{6}(1 - \cos^4 \beta)$     29  $\pi c^4$   
 31  $\frac{8\pi kb^5}{15}$     33  $b^3\left(\frac{2\pi}{3} - \frac{8}{9}\right)$     35  $\frac{3}{8}b$  from the center

## Extra Problems for Chapter 12

- 1 0 3 0 5  $1/3$  7  $4/5$  9  $\frac{1}{2}(\ln 2)^2 - \frac{1}{6}(\ln 2)^3$  11  $1/6$   
 13  $m = 2k$ ,  $(\bar{x}, \bar{y}) = (\pi/2, \pi/8)$ ,  $I_0 = (\pi^2 - \frac{32}{9})k$  15  $m = \pi r^4/4$ ,  $(\bar{x}, \bar{y}) = (0, 0)$ ,  $I_0 = \pi r^6/6$   
 17  $45\pi$  19  $2\pi a$  23  $311/3960$   
 25  $(\bar{x}, \bar{y}, \bar{z}) = (\frac{1}{5}, \frac{1}{5}, \frac{1}{30})$ ,  $I_z = \frac{k}{60}$ ,  $I_y = I_x = \frac{29k}{3360}$   
 27  $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, \frac{b^2 + c}{2})$ ,  $I_z = \frac{\pi b^4 c k}{2}$  29  $m = \pi(b^4 - a^4)$ ,  $I = \frac{4\pi}{9}(b^6 - a^6)$

## Section 13.1

- 1  $\text{grad } f = 2xi + 2yj$ ,  $f_U = \sqrt{2x} + \sqrt{2y}$  3  $\text{grad } f = 2xy^3i + 3x^2y^2j$ ,  $f_U = \frac{6}{5}xy^3 - \frac{1}{5}x^2y^2$   
 5  $\text{grad } f = -\sin x \sin yi + \cos x \cos yj$ ,  $f_U = (1/\sqrt{5})(-\sin x \sin y + 2 \cos x \cos y)$   
 7  $\text{grad } f = \frac{x}{\sqrt{x^2 + y^2}}i + \frac{y}{\sqrt{x^2 + y^2}}j$ ,  $f_U = \frac{x - y}{\sqrt{2x^2 + 2y^2}}$   
 9  $\text{grad } f = yzi + xzj + xyk$ ,  $f_U = \frac{1}{3}(yz + 2xz - 2xy)$   
 11  $\text{grad } f = -x^{-2}i - 2y^{-2}j - 3z^{-2}k$ ,  $f_U = (1/\sqrt{2})(-x^{-2} + 3z^{-2})$   
 13  $\text{grad } f = \frac{xi + yj + zk}{\sqrt{x^2 + y^2 + z^2}}$ ,  $f_U = \frac{x \cos \alpha + y \cos \beta + z \cos \gamma}{\sqrt{x^2 + y^2 + z^2}}$  15  $7\sqrt{10}/20$   
 17  $9a/\sqrt{a^2 + b^2}$  19  $-1/\sqrt{3}$  21  $(1/\sqrt{2})(i + j)$  23  $\frac{1}{2}(3i + 6j + 2k)$   
 25  $(f_U)_V = \frac{\partial^2 z}{\partial x^2}u_1v_1 + \frac{\partial^2 z}{\partial x \partial y}(u_1v_2 + u_2v_1) + \frac{\partial^2 z}{\partial y^2}u_2v_2$

## Section 13.2

- 1  $2e^3$  3  $\frac{9}{2}e - \frac{3}{2}$  5  $\sin 1 - \cos 1 + 1$  7  $\frac{3}{20} + \frac{5}{7} \ln 4$  9  $-2\pi, 2\pi$   
 11  $-4\pi$  13  $\frac{11}{30}$  15  $\frac{1}{2}$  17  $\frac{1}{2} \ln(\frac{29}{2})$  19  $1243/3$

## Section 13.3

- 1 no potential function 3 no potential function 5 no potential function  
 7  $y \sin x + C$  9  $-2x + 6y + C$  11 no potential function  
 13  $\frac{3}{2}x^2 + 5xy - y^2 + C$  15  $\cosh x \cosh y + C$  23  $\frac{3}{2}x^2 + 4xy - y^2 = C$   
 25  $\frac{2}{3}x^{3/2} + x\sqrt{y} = C$  27  $\frac{1}{3}y^3 + y \arctan x = C$  29  $\cos x \sin y = C$   
 31  $(x^2/2) + (y^2/2) + (2/3)(x + y)^{3/2} = C$  33  $P(x, y) = 2 \sin x \cos x \sin y$

## Section 13.4

- 1 1 3  $\frac{1}{2}(-e^7 + e^{-1} + e - e^{-7})$  5  $-\frac{1}{6}$  7  $\frac{1}{2} - \ln 2$  9  $-17/12$   
 11  $-\pi/2$  13 (a)  $-x - y$  (b)  $-1$  (c)  $y - x$  (d) 0  
 15 (a)  $2bx - 2ay$  (b)  $(2b - a)/3$  (c) 0 (d) 0 17 (a)  $-2$  (b)  $-2\pi$  (c) 0 (d) 0  
 19  $\pi a^2 + \pi/2$

## Section 13.5

- 1  $25\sqrt{21}/4$  3  $(\pi/6)(5\sqrt{5} - 1)$  5  $\pi a^2/2$  7  $(\pi/6)[(1 + 4a^2)^{3/2} - 1]$   
 9  $a^2(\pi - 2)$  11  $A\sqrt{1 + a^2 + b^2}$  13  $8a^2$  15 16 17  $-\pi$  19  $-2e^{-1}$

## Section 13.6

- 1  $\text{curl } F = 0$ ,  $\text{div } F = 2x + 2y + 2z$  3  $\text{curl } F = -i + j - k$ ,  $\text{div } F = 3$   
 5  $\text{curl } F = (ze^{x+y} - ye^{x+z})i + (xe^{y+z} - ze^{x+y})j + (ye^{x+z} - xe^{y+z})k$ ,  
 $\text{div } F = e^{y+z} + e^{x+z} + e^{x+y}$  9 0 11  $A(p(c - b) + q(a - c) + (a - b))$   
 13  $abc(a + b + c)$  15 3 17  $\pi/3$  19  $\pi/2$

## Extra Problems for Chapter 13

- 1  $-\sin x \cos \alpha + \cos y \sin \alpha$     3  $\text{grad } f = ye^{xy}\mathbf{i} + xe^{xy}\mathbf{j}, f_U = e^{xy}(y \cos \alpha + x \sin \alpha)$   
 5  $\frac{1}{\sqrt{14}}(-2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$     7  $\ln\left(\frac{6\sqrt{2}}{5}\right)$     9 0    11  $x \ln x + xy \ln y - x$   
 13  $\frac{3}{2}y^2 - e^{-y} \cos x = C$     15  $\frac{131}{60}$     17  $\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{6}$     19  $2\pi$   
 21  $\text{curl } \mathbf{F} = (x^2z - 3xy^2z^2)\mathbf{i} + (xy - 2xyz)\mathbf{j} + (y^2z^3 - xz)\mathbf{k}, \text{div } \mathbf{F} = yz + 2xyz^3 + x^2y$

## Section 14.1

- 1  $y = -1/2 \cos(t^2) + C$     3  $y = \ln(t + C)$     5  $y = \frac{1 + Ce^{2t}}{1 - Ce^{2t}}, y = 1, y = -1$   
 7  $y = -\frac{1}{0.5t^2 + C}, y = 0$     9  $y = \tan(e^t + C)$     11  $y = C \sec t, y = 0$   
 13  $y = \left(\frac{t^2}{4} + 9\right)^{1/2}$     15  $y = -\sqrt{2(t \ln t - t) + 6}$     17  $y = 2$

## Section 14.2

- 1  $y = C \cdot e^{-5t}$     3  $y = C \cdot e^{\arctan t}$     5  $y = 4 \cdot e^{-t}$     7  $y = e \cdot e^{\cos t}$     9  $y = 0$   
 11  $y = 4t^2$     13  $y = e^{1-t^2}$     15  $y = 100 \cdot e^{-t \cdot \ln 5}$     17  $y = 1,000,000 \cdot e^{t \cdot \ln 1.5}$

## Section 14.3

- 1  $y = 2 + Ce^{-4t}$     3  $y = 5 + Ce^{-(1/2)t^2}$     5  $y = -(t^2 + 2t + 2) \cdot Ce^{-4t}$   
 7  $y = \frac{1}{3}t^{-1} + Ct^2$     9  $y = \sin t + C \cos t$     11  $y = e^{-\cos t} \left( \int_0^t s e^{\cos s} ds + C \right)$   
 13  $y = e^{-\int_0^t \cos(e^r) dr} \left[ \int_0^t e^{\int_0^s \cos(e^r) dr} ds + C \right]$   
 15  $-[400,000t + 15,840,000] + 15,940,000e^{0.025t}$   
 17  $15,000e - 25,000$  dollars, or \$15,774.23

## Section 14.4

- |   | $t$    | 0.0 | 0.1   | 0.2   | 0.3   | 0.4   | 0.5   | 0.6   | 0.7   | 0.8   | 0.9   | 1.0   |
|---|--------|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1 | $Y(t)$ | 1.0 | 1.0   | 1.01  | 1.030 | 1.059 | 1.097 | 1.142 | 1.195 | 1.253 | 1.317 | 1.386 |
| 3 | $Y(t)$ | 1.0 | 1.054 | 1.095 | 1.122 | 1.137 | 1.140 | 1.133 | 1.117 | 1.093 | 1.061 | 1.023 |
- 5 Apply the lemma with  $M = 4$ .    7 Apply the lemma with  $M = \pi/2$ .  
 9  $y(t) = 0$  for  $0 \leq t \leq b$ ,  $y(t) = [(2/3)(t - b)]^{3/2}$  for  $b < t < \infty$ .

## Section 14.5

- 1  $10 - i2$     3  $i10$     5  $0.1 - i(0.7)$     7 4    9  $\pm i5$     11  $-1 \pm i2$   
 13  $5 \text{ cis}(\pi/2)$     15  $3\sqrt{2} \text{ cis}(-3\pi/4)$     17  $2 \text{ cis}(-\pi/6)$     19  $-i2/3$   
 21  $2 \text{ cis}(-\pi/12)$     23  $-4 + i4$     25  $\pm 2^{1/4} \text{ cis}(\pi/8)$     27  $\pm 2 \text{ cis}(-\pi/4)$ ,  
 or  $\pm \sqrt{2}(1 - i)$     29  $ie^{-3}$     31  $e\sqrt{2}/2 - ie\sqrt{2}/2$     33  $(5 - i3)e^{(5-i3)t}$   
 35  $(3 + i2)e^{(2-i7)+(3+i2)t}$     37  $Ce^{(-2+i3)t}$     39  $Ce^{(3-i5)t}$   
 41  $e^{i+(2-i)t}$ , or  $e^{2t} \text{ cis}(1 - t)$     43  $4e^{(2-i)t}$ , or  $4e^{2t} \text{ cis}(-t)$

**Section 14.6**

- 1  $Ae^{2t} + Be^{-3t}$     3  $Ae^{5t} + Bte^{5t}$     5  $A \cos(4t) + B \sin(4t)$   
 7  $e^t[A \cos t + B \sin t]$     9  $-\frac{1}{4}e^{-5t} + \frac{5}{4}e^{-t}$     11  $5e^{-6t} + 20te^{-6t}$   
 13  $-2 \cos(\sqrt{5}t) + \sqrt{5} \sin(\sqrt{5}t)$     15  $e^{-6t}[4 \cos t + 24 \sin t]$   
 17  $2 \cos(2t - \pi/6)$ , amplitude = 2, frequency = 2, phase shift =  $\pi/6$   
 19  $e^{-2t}\sqrt{2} \cos(3t - \pi/4)$ , amplitude =  $\sqrt{2}e^{-2t}$ , frequency = 3, phase shift =  $\pi/4$   
 21  $e^{-2t}[2 \cos(5t) + \sin(5t)]$

**Section 14.7**

- 1  $(24/676) \cos t - (10/676) \sin t$     3  $(1/2)t^2 + (3/16)t - (3/8)$     5  $0.5e^{2t}$   
 7  $-0.2te^{-3t}$     9  $0.125t \sin(4t)$     11  $3t^2e^{-6t}$     13  $Ae^{-3t} + Be^{-2t} + 0.8$   
 15  $A \cos(\sqrt{5}t) + B \sin(\sqrt{5}t) + 2 \sin(2t)$     17  $4e^t + e^{-t} - 3t - 5$   
 19  $e^{-6t}[2 \cos t + 20 \sin t] + 2e^{-4t}$   
 21  $e^{-2t}[A \cos(5t) + B \sin(5t)] + 25 \cos(2t) + 8 \sin(2t)$ , steady state =  $25 \cos(2t) + 8 \sin(2t)$   
 25 5

**Extra Problems for Chapter 14**

- 1  $y = -1/(\sin t + C)$     3  $y = -\ln(-\ln|t| + e^{-2})$     5  $y = Ce^{-5t^2}$     7  $y = e^{-6t}$   
 9  $y = (2/3)t - 2/9 + Ce^{-3t}$     11  $y = 3,500,000e^{0.02t} - 2,500,000$   
 15  $y = Ae^{4t} + Be^t$     17  $y = e^{2t}[A \cos(2t) + B \sin(2t)]$     19  $y = -(1/6)e^{-5t} + (1/6)e^t$   
 21  $y = e^{-(3/2)t}[10 \cos(t/2) + 30 \sin(t/2)]$     23  $y = Ae^{4t} + Be^t + 13/16 + t/4$   
 25  $y = -2 \cos t - 3 \sin t - (1/6)e^{-5t} + (13/6)e^t$



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