Math 341
Group Worksheet #7: Inner products
Monday, May 4, 2015

1. Prove the parallelogram law: if x and y are two vectors in an inner product space V and $\|\cdot\|$ is the norm induced by the inner product $\langle \cdot, \cdot \rangle$, then

$$\|x + y\|^2 + \|x - y\|^2 = 2\|x\|^2 + 2\|y\|^2.$$

2. Let $W = \text{Span}\{(1, 2, 0, 2), (2, -1, 2, 4), (0, 0, 4, 3)\} \subseteq \mathbb{R}^4$. Find an orthonormal basis for W by the following procedure.

   (a) Start with the first vector you were given, and call it v_1. Then replace the second vector by a new vector v_2 that is orthogonal to v_1 but such that v_1 and v_2 span the same subspace as the first two vectors you were given.

   (b) Next, replace the third vector you were given by a vector v_3 orthogonal to both v_1 and v_2, but make sure you have the right span! (Think about orthogonal projections.)

   (c) Finally, make sure all your vectors have length 1.

3. Let S be a subset of an inner product space V, and let

$$S^\perp = \{x \in V \mid x \perp y \text{ for all } y \in S\}.$$

   (a) Prove that $S^\perp$ is a subspace of V.

   (b) If $V = \mathbb{R}^3$ and $S = \{(1, 0, 0)\}$, then what does $S^\perp$ look like?

   (c) If W is a subspace of V, what do you think the dimension of $W^\perp$ should be, in terms of the dimensions of V and W?