Math 341
Group Worksheet #6: Eigenvalues, eigenvectors, and diagonalization
Friday, April 17, 2015

1. Let $A = \begin{pmatrix} -3 & 0 & 0 \\ 5 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \in M_{3\times 3}(\mathbb{R})$. Determine whether or not A is diagonalizable. If it is, find a basis for $\mathbb{R}^3$ consisting of eigenvectors of A. If not, prove it's not.

2. Repeat the previous problem with the matrix $B = \begin{pmatrix} -3 & 0 & 0 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{pmatrix}$.

3. Don't look at the book or your notes for this one. Prove Theorem 5.1 from the book (which we already proved in class). Here it is:

   Theorem. *Let V be a vector space over a field F of dimension n, and let $T : V \to V$ be a linear transformation. Then T is diagonalizable if and only if there is a basis for V consisting entirely of eigenvectors of T.*

   Since you're not looking in your book or notes, here are the definitions you need:

   - An eigenvector of T is any nonzero vector $x \in V$ such that $T(x) = \lambda x$ for some $\lambda \in F$. This scalar lambda is called the eigenvalue of T associated to the eigenvector x.
   - A linear operator is diagonalizable if there is an ordered basis $\mathcal{B}$ for V such that $[T]_{\mathcal{B}}$ (i.e. $[T]_{\mathcal{B}}^{\mathcal{B}}$) is a diagonal matrix.

   If you need to remember how to write $[T]\mathcal{B}$ for a linear operator T and a basis $\mathcal{B}$, ask your groupmates for help!