Math 341
Group Worksheet #5: Determinants
Friday, March 27, 2015

1. Let A be an upper triangular matrix in $M_{n\times n}(F)$, meaning that $A_{ij} = 0$ for all $j < i$, i.e. all entries below the main diagonal are zero. Prove that $\det(A) = A_{11}A_{22}\cdots A_{nn}$, the product of the entries on the main diagonal.

   Hint: Use cofactor expansion along the bottom row.

2. Find the determinant of the matrix $A \in M_{5\times 5}(\mathbb{R})$, where

$$A = \begin{pmatrix} 2 & 0 & 4 & -6 & 8 \\ 0 & 1 & -2 & -1 & -4 \\ 0 & 0 & 0 & -1 & 1 \\ 1 & 1 & 1 & -1 & 3 \\ 1 & 2 & -3 & 0 & 4 \end{pmatrix}.$$

   Some groups should evaluate the determinant by cofactor expansion, while others should use row reduction, and the result of the previous problem.

3. If E is an elementary $n \times n$ matrix, find its determinant (i.e. explain what the determinant of each type of elementary matrix is). Recall that a type 1 elementary matrix is one obtained by exchanging two rows of I_n, a type 2 elementary matrix is one obtained by scaling a row of I_n, and a type 3 elementary matrix is one obtained from I_n by adding a multiple of one row to another.

4. If E is an elementary matrix, prove that $\det(E^t) = \det(E)$.

   Hint: use the previous problem, and consider each of the three types separately.