Math 341
Group Worksheet #4: Systems of Linear Equations
Friday, February 27, 2015

1. Solve the following system of equations using Gaussian Elimination:

$$\begin{aligned} x_1 + 2x_3 &= 3 \\ 2x_1 + x_2 + 7x_3 &= 0 \\ x_1 + 2x_2 + 8x_3 &= -2 \end{aligned}$$

   Is $\begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}$ in the image of the linear transformation associated to the matrix $\begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 7 \\ 1 & 2 & 8 \end{pmatrix}$?

2. Consider the following two closely related systems of equations over the real numbers:

$$\begin{aligned} x_1 + 2x_2 - x_3 + 3x_4 &= 0 \\ 2x_1 + 4x_2 - x_3 + 6x_4 &= 0 \\ x_2 - x_4 &= 0 \end{aligned}$$

$$\begin{aligned} x_1 + 2x_2 - x_3 + 3x_4 &= 2 \\ 2x_1 + 4x_2 - x_3 + 6x_4 &= 5 \\ x_2 - x_4 &= 3 \end{aligned}$$

   The first system is called *homogeneous*, because it has all zeroes on the right-hand-side The second system is called *inhomogeneous*, because it is not homogeneous.

   (a) Find the general solutions for both systems of equations using Gaussian elimination.

   (b) Let K denote the set of solutions to the homogeneous system. Explain why K is the kernel of the linear transformation L_A associated to the matrix $\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & -1 & 6 \\ 0 & 1 & 0 & -1 \end{pmatrix}$.

   (c) Since K is the kernel of L_A, we know that K is a subspace of $\mathbb{R}^4$. Find a basis for the kernel. (You have already found it.)

   (d) One solution to the inhomogeneous equation should be $s = (-3, 3, 1, 0)$. Verify that every solution to this system is equal to s plus a vector in K.