Math 341
Group Worksheet #2: Bases, etc.
Friday, February 6, 2015

1. Let $\mathcal{B} = \{(1,1,0,0),(0,2,2,0),(0,0,3,3),(4,0,0,0)\} \subseteq \mathbb{R}^4$.

   (a) Prove that $\mathcal{B}$ is linearly independent. Conclude that $\mathcal{B}$ is a basis for $\mathbb{R}^4$ (say why).

   (b) Since we now know that $\mathcal{B}$ is a basis for $\mathbb{R}^4$, this means that $\mathcal{B}$ is a spans all of $\mathbb{R}^4$. If $v = (a,b,c,d)$ is an arbitrary vector in $\mathbb{R}^4$, show how to write v as a linear combination of vectors in $\mathcal{B}$. (From part (a) you knew that $\mathcal{B}$ spans $\mathbb{R}^4$, but this gives you a separate proof).

   For the next exercise, you may freely use the following theorem (if you want more practice later, try proving it on your own).

   Theorem 1. *Let F be a field, and let a and b be elements of F. If $ab = 0$, then either $a = 0$ or $b = 0$.*

2. Let F be a field, and let $v_1 = (a,b)$ and $v_2 = (c,d)$ be vectors in F^2. Prove that v_1 and v_2 are linearly dependent if and only if $ad - bc = 0$.

3. Let V be a vector space over a field F, and suppse that $\{b_1, b_2, b_3\}$ is a basis for V. Let c be a non-zero element of F. Prove that $\{b_1 + b_2 + b_3, b_2 - b_3, cb_2\}$ is also a basis for V.