Math 341 – Homework #13
Due Wednesday, May 6, 2015

This assignment is based on Section 6.1 of our text.

1. Prove part (b) of Theorem 6.1 from our text: If $\langle \cdot, \cdot \rangle$ is an inner product on V, then for any $x, y \in V$ and $c \in F$ we have $\langle x, cy \rangle = \overline{c} \langle x, y \rangle$.

2. As we did in class (and following p. 331 of our text), define the inner product of two matrices A and B in $M_{n \times n}(F)$ by

$$\langle A, B \rangle = \mathrm{tr}(B^* A),$$

where the *conjugate transpose* (or *adjoint*) B^* of a matrix B is defined by $B^*_{ij} = \overline{B_{ji}}$.

Prove that this really is an inner product, i.e. for all matrices $A, B, C \in M_{n \times n}(F)$ and all scalars $t \in F$, we have

   (a) $\langle A + C, B \rangle = \langle A, B \rangle + \langle C, B \rangle$,

   (b) $\langle tA, B \rangle = t \langle A, B \rangle$,

   (c) $\langle B, A \rangle = \overline{\langle A, B \rangle}$, and

   (d) $\langle A, A \rangle > 0$ if $A \neq 0$.

Parts (a) and (d) are proven at the bottom of page 331, so you only need to prove (b) and (c).

3. Let V denote the vector space of all continuous real-valued functions from the interval $[0, 2\pi]$ to $\mathbb{R}$. (This is a vector space over $\mathbb{R}$. We define an inner product on V by

$$\langle f, g \rangle = \int_0^{2\pi} f(x) g(x) dx.$$

With respect to this inner product, find a function which is orthogonal to $f(x) = \sin(x)$ (and show that the two functions are orthogonal).

4. Prove the Pythagorean Theorem for inner product spaces: If V is an inner product space with norm $\| \cdot \|$ and $x, y \in V$ with $x \perp y$, then

$$\|x + y\|^2 = \|x\|^2 + \|y\|^2.$$

5. Let $W = \mathrm{Span}\{(1, 2, 2), (2, -3, 6)\}$, which is a plane inside of $\mathbb{R}^3$. Find a basis $\{v_1, v_2\}$ for W such that $v_1 \perp v_2$.