Math 341 – Homework #10
Due Wednesday, April 15, 2015

For this assignment, you should refer to the definitions and theorems of Sections 1.2-1.6, 2.1-2.5, 3.1-3.4, 4.1-4.4, and 5.1 of our textbook, as well as notes from lecture. (You might also check out Section 4.4).

1. For the following matrices over the field $\mathbb{R}$, find all eigenvalues, and find an eigenvector associated to each eigenvector.

   (a) $A = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$

   (b) $B = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$

   (c) $C = \begin{pmatrix} 2 & 0 & -1 \\ 4 & -1 & -4 \\ 2 & 0 & -1 \end{pmatrix}$

2. Let A be a matrix in $M_{n\times n}(F)$.

   (a) Prove that A is invertible if and only if zero is not an eigenvalue of A.

   (b) Assuming A is invertible, prove that if λ is an eigenvalue of A, then $\frac{1}{\lambda}$ is an eigenvalue for A^{-1}.

3. For any $A \in M_{n\times n}(F)$, prove that A and A^t have the same eigenvalues.

   Hint: first prove that they have the same characteristic polynomial.

   Definition. A square matrix of the form λI, where λ is a scalar and I denotes the identity matrix, is called a *scalar matrix*. For example, $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ is a scalar matrix. They are called this because multiplying λI times a column vector simply scales that vector by λ.

4. (a) Show that a scalar matrix has only one eigenvalue.

   (b) Suppose that A is a diagonalizable matrix with exactly one eigenvalue. Prove that A is a scalar matrix.

   (c) Prove that $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ is not diagonalizable.