Math 341
Notes on Gaussian Elimination*

For starting out, you should read section 3.4. I recommend you skip to the corollary at the bottom of p. 182, and then read up to but not including Theorem 3.15 on p. 189, working through the examples carefully (the second one is the one we did in class).

To summarize what was said in lecture that is *not* in the book, once you are considering an augmented matrix $(A|b)$ which is in RREF:

- The first non-zero entry of each row is called a *pivot*†, and each column containing a pivot is called a *pivot column*.
- The variables corresponding to pivot columns are called *basic variables*.
- The other variables are called *free variables*.
- Since the matrix is in RREF, you'll notice each row only contains one nonzero entry corresponding to a basic variable. This means all basic variables can now be easily expressed in terms of the free variables. The paragraph in the middle of the page on p. 188 describes how to do this. The form in which the solution set is written near the bottom of this page is the *general solution*, or *parametric solution* to the equation $Ax = b$.

The way to think about the solution set, is that the free variables are "free" to take on any values from F, and the basic variables are totally determined by the free variables. For example, the general solution in the example from class (and the book) is‡

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -2t_1 + 2t_2 + 3 \\ t_1 - t_2 + 1 \\ t_1 \\ 2t_2 + 2 \\ t_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \\ 2 \\ 0 \end{pmatrix} + t_1 \begin{pmatrix} -2 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t_2 \begin{pmatrix} 2 \\ -1 \\ 0 \\ 2 \\ 1 \end{pmatrix}.$$

By the way, right after this point the authors of the book go on to use some words like "homogeneous system of equations." We'll explain what these words mean soon.

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*This algorithm is named after C. F. Gauss, one of the greatest mathematicians of all time. However, according to Wikipedia, the algorithm was known to Chinese mathematicians over 1600 years earlier. (But don't let this affect your opnions of Gauss!)

†The entries in pivot positions are all equal to 1. A matrix which satisfies all the conditions of reduced row echelon form except that the pivot values are 1 is said to be in *row echelon form*. The word "reduced" means the pivot values are all 1.

‡I used t and s instead of t_1 and t_2 as in the book.